Introducing Integral Geometry: Are Notational Flaws Responsible For The Inability to Combine General Relativity and Quantum Mechanics?

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Parallel line segments are the basic graphical foundation for geometrical field theories such as General Relativity. Although the concept of parallel and curved lines have been well researched for over a century as a description of gravity, certain controversial issues have persisted, namely point singularities (Black Holes) and the physical interpretation of a scalar multiple of the metric $\Lambda g_{\mu\nu}$, commonly known as a Cosmological Constant. We introduce a graphical and notational analysis system which we will refer to as Integral Geometry. Through variational analysis of perpendicular line segments we derive equations that ultimately result from the changes in the area bounded by them. Based upon changing area bounded by relative and absolute line segments we attempt to prove the following hypothesis: General Relativity cannot be derived from Integral Geometry. We submit that examination of the notational differences between GR and IG in order to accept the hypothesis could lead to evidence that the inability to merge General Relativity and Quantum Physics may be due to notational and conceptual flaws concerning area inherent in the equations describing them.

I. INTRODUCTION

In this work, we introduce the concept of Integral Geometry (IG). This concept is an examination of relative and absolute areas, the resulting equations from their summations and differences and finally, physical modeling of differential absolute and relative areas based on perfect fluids and spatial and temporal probabilities. We have found that absolute areas seem to be suitable for building absolute coordinate systems for which we can track particles and that relative areas are suitable for tracking relative waves through a perfect fluid.

In order to facilitate appreciation of some of the possibilities of IG, it is necessary to understand the similarities and differences with current physical laws and equations. So as to keep this as simple and compact as possible, we focus on understanding the similarities of the Cosmological Constant (CC or Λ) problem and that of constant relative area within Integral Geometry.

The CC from General Relativity (GR) has several different names: a Cosmological Constant, a scalar multiple of the metric, an Einstein manifold and a postulated energy density of the vacuum. The "problem" [1] stems from the fact that although this constant seems to be present both in quantum mechanics (QM) and GR, the estimated value within QM is over 100 orders of magnitude different than what would seem to work within GR from examination of the actual empirical evidence.

This problem is also illustrated by a central equation within metric field theories where the effect that a metric $g_{\mu\nu}$ has on space-time $dx_\mu dx_\nu$ is described by [2]

$$ds^2 = g_{\mu\nu} dx_\mu dx_\nu. \tag{1}$$

The view within these field theories is that a metric changing from point to point can describe a non-Euclidean "curvature" of space-time. The Einstein field equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{2}$$

would then be a description of how energy ρ and momentum p (within $T_{\mu\nu}$) effect $g_{\mu\nu}$. If there is no momentum or energy present, then the equation

$$R_{\mu\nu} = 0 \tag{3}$$

would describe zero curvature and a summation of the unchanged components of $g_{\mu\nu}$ would each have a magnitude of $|1|$ and be written as

$$R = 4. \tag{4}$$

There has been, however, the conceptual discrepancy of the CC which would would have summed components of

$$R = 4\Lambda. \tag{5}$$

This is normally just added into the Einstein field equation as

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{6}$$

but we stress that there is no known way to calculate a theoretical value for Λ that would match any proposed physical explanation. As an example, what would $ds^2 = \Lambda g_{\mu\nu} dx_\mu dx_\nu$ mean?

For comparison purposes, we derive here a conceptual framework within IG which we have named Line Segment Space (LSS). During our preliminary research we have developed the equation in LSS of

$$\frac{S_v}{dS_v} dS_v dS_h = S_v dS_h. \tag{7}$$

If we investigate defining the first fraction as

$$\frac{S_\mu}{dS_\mu} \equiv g_{\mu\nu}, \mu = v, \nu = h \tag{8}$$

we then examine why we can find LSS solutions for which

$$|S_{00}| + |S_{11}| + |S_{22}| + |S_{33}| = 0 \tag{9}$$

and

$$\frac{(dS_\mu)^2}{(dS_\nu)^2} = 0 \tag{10}$$

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but *no* metric solutions for

$$R = 0 \tag{11}$$

and

$$R_{\mu\nu} = 0. \tag{12}$$

We propose that this investigation may unveil serious questions of whether not only metric field theory is conceptually valid but Euclidean and non-Euclidean geometry itself. For example, if $S_h = f(S_v)$, can we use this geometric model also as a physical model where relative changes in X and T are functions of relative changes in the density ρ and pressure p of the vacuum when treated as a perfect fluid?

II. DERIVATION

We keep the derivation of this hypothesis simple and non-robust so as to require consideration of the widest range of interpretations to verify the following hypothesis: General Relativity cannot be derived from Integral Geometry.

IG contains three separate conceptual regions: Graphical, Notational and Physical. We demonstrate on how to examine a single concept as it evolves through all three regions.

A. Graphical

Visual examination of graphical proofs.

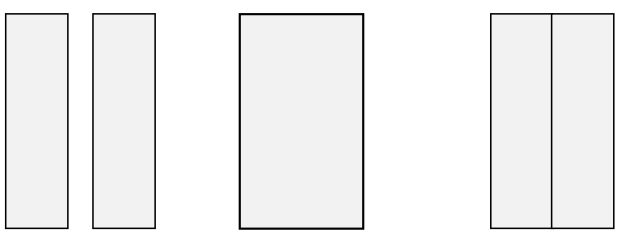

FIG. 1. Summation of two areas on left equal single area in middle

B. Notational

Notational descriptions of graphical proofs.

$$S_v^1 S_h^1 + S_v^2 S_h^2 = S_{total} \tag{13}$$

$$S_{total} - S_v^1 S_h^1 + S_v^2 S_h^2 = 0 \tag{14}$$

In IG, there are two basic notational forms, line segment notation and point notation. S is a single descriptor of a line segment. $A_1 - A_2$ is a point descriptor of a line segment, such that $S = A_1 - A_2$. This distinction will become important in the later discussion of the notation within various historical metric field theories.

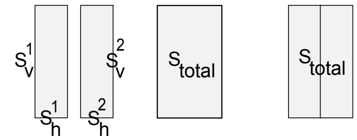

FIG. 2. Summation of two areas on left equal single area in middle

C. Physical

Let us consider the issue of equating notational descriptions to physical models. Current physics cannot find a workable relationship between the Poisson equation describing energy and the Cosmological Constant as the energy density of the vacuum. Through attempts at falsifying our hypothesis, we propose to examine whether this difficulty is due to a notational system within GR that inherently accounts only for parallel and curved lines instead of a more appropriate view of changing relative area.

$$\nabla^2 \phi = \rho \tag{15}$$

$$A g_{\mu\nu} \stackrel{?}{=} \rho_{vac} \tag{16}$$

III. GEOMETRIC SUMMATION OF AREA

A. Quantities of Area Can Be Summed

"Chunks" of area can be summed.

$$\sum S_v^n S_h^n = S_v^1 S_h^1 + S_v^2 S_h^2 + S_v^m S_h^m \tag{17}$$

B. Defining Integration as Summation of Infinitesimal areas

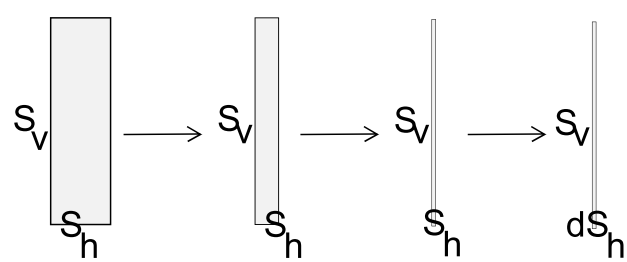

FIG. 3. Rectangle of Area Becomes an Infinitesimal Slice of Area

Summation of blocks of area becomes integration of infinitesimal slices. We specifically point out that *no functional relationship between the line segments is required for either summation or integration of area.* We call the horizontal line segment of zero width dS_h a *point derivative*.

$$\sum S_v^n S_h^n \longrightarrow \int S_v^n dS_h^n \tag{18}$$

$$S_v^n \neq f(S_h^n) \tag{19}$$

$$S_h^n \neq f(S_v^n) \tag{20}$$

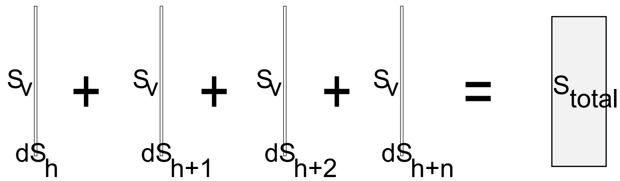

FIG. 4. Integration of Infinitesimal Slices of Area

IV. DIFFERENCES OF INFINITESIMAL SLICES OF AREA ARE NOTATIONALLY DEFINABLE

Differences of area are definable therefore differences of infinitesimal slices of area are definable.

$$S_v^1 S_h^1 - S_v^2 S_h^2 \neq 0 \tag{21}$$

$$S_v^1 dS_h^1 - S_v^2 dS_h^2 = (S_v^1 - S_v^2) dS_h \neq 0 \tag{22}$$

$$S_v^1 - S_v^2 \neq 0 \tag{23}$$

V. INFINITESIMAL DIFFERENCES OF INFINITESIMAL SLICES OF AREA ARE DEFINABLE

We can define an infinitesimal limit of when the differences between the lengths of the vertical line segments go to zero. We call the infinitesimal comparison of the vertical line segments dS_v a *line derivative*. It is important to note that neither line segment is required to have, but could have a magnitude of 0.

$$|S_v^1| \neq 0 \tag{24}$$

$$|S_v^2| \neq 0 \tag{25}$$

$$(S_v^1 - S_v^2) dS_h \lim_{\Delta between S_v^1 and S_v^2 \to 0} = dS_v dS_h \tag{26}$$

The special case

$$dS_v dS_h = 0 \tag{27}$$

exists when

$$|S_v^1| = |S_v^2|. \tag{28}$$

VI. THE RELATIVE RATE OF CHANGE OF AREA CAN BE DEFINED THROUGH NORMALIZATION

We can consider values of ratios of line derivatives and point derivatives or consider solutions when we consider these ratios to be constant.

$$\frac{dS_v dS_h}{dS_h dS_h} = \frac{dS_v}{dS_h} \neq 0 \tag{29}$$

$$\frac{dS_v dS_v}{dS_h dS_h} = \frac{(dS_v)^2}{(dS_h)^2} \neq 0 \tag{30}$$

VII. INTRODUCTION OF LINE SEGMENT SPACE

We define Line Segment Space (LSS) as a row of an infinite number of points. We can think of each point having a vertical line segment S_v (see Fig.5).

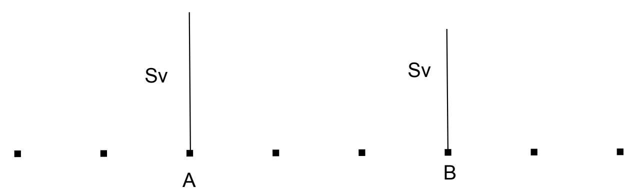

FIG. 5. Infinite Number of Points in a Row Each With Its Own Vertical S

For S_h however, there are two paradigms, absoluteness and relativity.

A. Absolute Line Segment in LSS

Any magnitude or section of S_h is made up of multiple points on the row of points, not necessarily of the same magnitudes. The area bounded by these line segments cannot conceptually overlap (see Figs. 6 and 7).

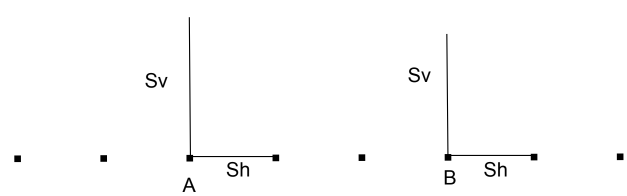

FIG. 6. Any S_h is made up of points in the row.

B. Relative Line Segment in LSS

Each point in the row of points has a separate magnitude or section of S_h, not necessarily of the same magnitudes (see Figs. 8 and 9).

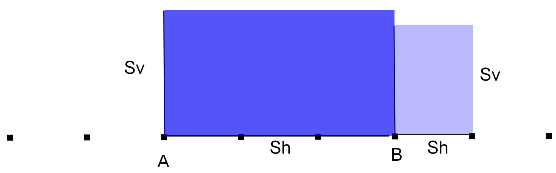

FIG. 7. Areas cannot conceptually overlap.

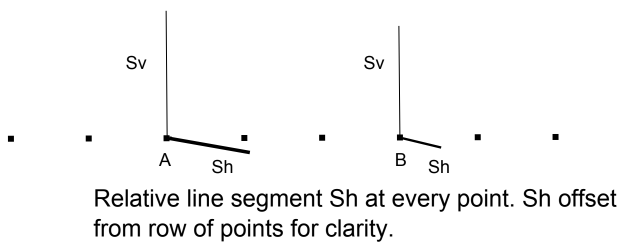

FIG. 8. Every point contains a line segment S_h.

C. Functional Relationships Between S_h and S_v and Physical Interpretations

We consider these questions of the two simplest functional relationships between S_h and S_v:

For absolute line segments, can the Poisson equation

$$\nabla^2\phi = \rho \tag{31}$$

be derived from

$$\frac{dS_v dS_v}{dS_h dS_h} = \frac{(dS_v)^2}{(dS_h)^2} \neq 0 \tag{32}$$

and

$$\phi \equiv S_v \tag{33}$$

via

$$S_v = f(S_h)? \tag{34}$$

For relative line segments can

$$ds^2 = g_{\mu\nu} dx_\mu dx_\nu \tag{35}$$

be derived from

$$\frac{S_v}{dS_v} dS_v dS_h = S_v dS_h \tag{36}$$

and

$$\frac{S_\mu}{dS_\mu} \equiv g_{\mu\nu} \tag{37}$$

via

$$S_\mu = f(S_\nu)? \tag{38}$$

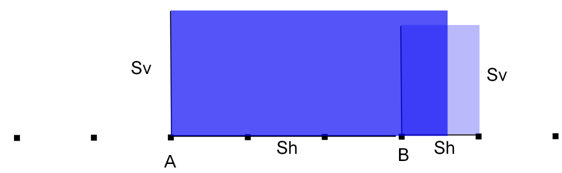

FIG. 9. Areas can conceptually overlap

VIII. MOTIVATION

1. Is an Einstein Manifold the Same as Constant Nonzero Relative Area in LSS? In LSS using relative S_h, for any S_v that is the same at point A and point B in the row,

$$(\frac{S_v}{dS_v} dS_v dS_h)_A - (\frac{S_v}{dS_v} dS_v dS_h)_B = 0. \tag{39}$$

We currently can find no way to falsify this equation from being derived through

$$\frac{S_\mu}{dS_\mu} \equiv \Lambda g_{\mu\nu} \tag{40}$$

into either

$$R g_{\mu\nu} = 0 \tag{41}$$

or

$$R\Lambda g_{\mu\nu} = 0 \tag{42}$$

with the exception that solutions for

$$|g_{\mu\nu}| = 0 \tag{43}$$

with

$$R g_{\mu\nu} = 0 \tag{44}$$

do not conceptually exist in metric field theory. Is allowing the diagonal arguments to be

$$|g_{\mu\nu}| = (1, 1, 1, 1) \tag{45}$$

a *specific* understanding of relativity rather than a *general* one considering that any constant satisfies the equation $R\Lambda g_{\mu\nu} = 0$.

As examples, constants $|g_{\mu\nu}| = (5, 5, 5, 5)$ or even $|g_{\mu\nu}| = (\frac{143521}{23412244}, \frac{143521}{23412244}, \frac{143521}{23412244}, \frac{143521}{23412244})$ satisfy the equation. Why is the notation accepted if the solutions are not unique?

2. Is a Point Singularity in GR Conceptually Equivalent to Running Out of Relative Area in LSS?

3. Is a second set of four time-space components within $ds^2 = g_{\mu\nu} dx_\mu dx_\nu$ (dx_μ first set, dx_μ second set) a demonstration of incorrect notation for a vertical line segment that has its own point derivative included?

4. Is the understanding that length squared is the unit for the Cosmological Constant a demonstration that the concepts of Euclidean and non-Euclidean geometry are a conflation of geometry and physical theory?

5. Is parallel transport of a vector on a manifold equivalent to all solutions of a constant magnitude for a partial geometric derivative in LSS?

6. In 1998 [3, 4] it was discovered that there is unpredicted late inflection point in the expansion (decelerating to accelerating) of the universe. How is this possible at the same moment across the entire universe if nothing can travel faster than light? Would the hypothesis that this is just a property of energy and matter, in that gravitation has a wavelength, be less conceptually offensive?

7. Can we interpret a quantized geometrical wave in LSS (Fig.10) as having a wavelength with regions of relative line segments for which $\frac{(dS_v)^2}{(dS_h)^2} \neq 0$ separated by regions for which $\frac{(dS_v)^2}{(dS_h)^2} = 0$?

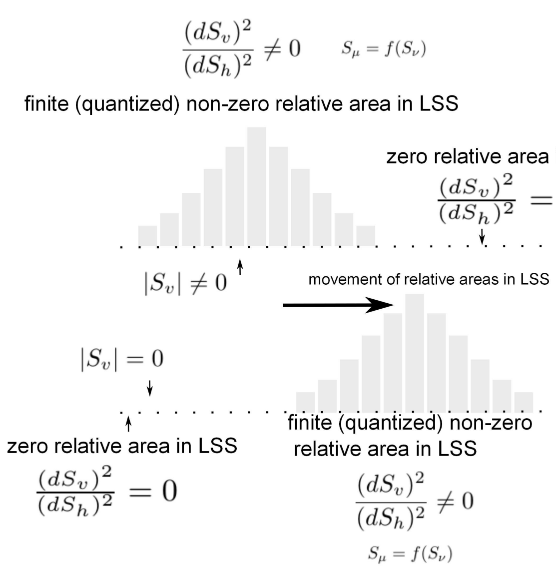

FIG. 10. Proposed quantized waves in LSS. Relative area overlap not shown for clarity.

8. Can Gunnar Nordström's first metric theory [5] be derived from absolute line segments in Line Segment Space?

$$\frac{d^2\phi}{dx_1^2} + \frac{d^2\phi}{dx_2^2} + \frac{d^2\phi}{dx_3^2} + \frac{d^2\phi}{dx_4^2} = \mu \tag{46}$$

9. Can Gunnar Nordström's second metric theory [5] with variable mass be derived from either absolute or relative line segments in Line Segment Space?

$$\frac{d^2\phi}{dx_1^2} + \frac{d^2\phi}{dx_2^2} + \frac{d^2\phi}{dx_3^2} + \frac{d^2\phi}{dx_4^2} = e^\mu m \tag{47}$$

10. Can bi-metric gravity theory be derived from using point notation for relative line segments in Line Segment Space where the end points for $S_v = A_1 - A_2$ are conceptually separated into their own line segments so that one can be considered constant while the other is dynamic?

$$ds^2 = g_{\mu\nu} dx_\mu dx_\nu \tag{48}$$

$$ds^2 = h_{\mu\nu} dx_\mu dx_\nu \tag{49}$$

$$A_1 - 0 \longrightarrow \frac{A_1 - 0}{dA_1} = g_{\mu\nu} \tag{50}$$

$$0 - A_2 \longrightarrow \frac{0 - A_2}{dA_2} = h_{\mu\nu} \tag{51}$$

11. Physical Modeling: Interpretation of the Cosmological Constant

    What we find most troubling is that bringing in the concept of the CC as the energy density of the vacuum into Line Segment Space would seem to make more physical and equational sense than in GR. Infinitesimal slices of area in IG can be called Geometric One-Forms. For the relative functional relationship $S_h = f(S_v)$, it would seem that relative changes in dS_v can be interpreted as dt and dx which are functions of relative changes of S_h (dS_h) which would physically correspond to the relative changes of density and pressure of the vacuum treated as a perfect fluid. Moreover, the one form $S_h dS_v$ would seem to correspond to the integral definition of probability at a point, a foundational assumption of QM.

We consider refuting Integral Geometry worthwhile considering these possible coincidences and the intractableness of the CC problem.

[1] S. M. Carroll, W. H. Press, and E. L. Turner, Annual Review of Astronomy and Astrophysics **30**, 499 (1992).
[2] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (W.H. Freeman, San Francisco, 1973).
[3] S. Perlmutter, G. Aldering, G. Goldhaber, R. Knop, P. Nugent, P. Castro, S. Deustua, S. Fabbro, A. Goobar, and D. Groom, The Astrophysical Journal **517**, 565 (1999).
[4] A. G. Riess, A. V. Filippenko, P. Challis, A. Clocchiatti, A. Dier-

cks, P. M. Garnavich, R. L. Gilliland, C. J. Hogan, S. Jha, and R. P. Kirshner, The Astronomical Journal **116**, 1009 (1998).

[5] M. Janssen, J. Norton, J. Renn, M. S. T. Sauer, and J. Stachel, *The Genesis of General Relativity* (Boston Studies in the Philosophy of Science, 2007).