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On the Generation of Equivalent 'Black Hole' Metrics: A Review

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Abstract: Various line-elements purporting different types of black hole universes have been advanced by cosmologists but a means by which the required infinite set of equivalent metrics can be generated has evaded them. Without such a means the theory of black holes is not only incomplete but also ill-posed. Notwithstanding, the mathematical form by which the infinite set of equivalent metrics is generated was first revealed in 2005, from other quarters and it has in turn revealed significant properties of black hole universes which cosmology has not realised. The general metric ground-form from which the infinite set of equivalent 'black hole' metrics can be generated is presented herein and its consequences explored.

Keywords: Black Hole, Metric, Big Bang Cosmology, General Relativity

Introduction

The simplest black hole solution to Einstein's field equations is the 'Schwarzschild solution'. A means for generating the required infinite set of equivalent metrics for spherically symmetric Schwarzschild spacetime was recently sought by (Fromholz *et al*., 2013), without success (Crothers, 2014a). In this case the fundamental issue is the vacuum state, described by the Einstein field equations $R_{\mu\nu} = 0$, for a static gravitational field in the absence of matter (Einstein, 1916). Cosmology has failed to find a mathematical generator for the necessary infinite set. However, such a generator exists and was first revealed in 2005 (Crothers, 2005a). Moreover, purported black hole metrics are not restricted to spherical symmetry or the absence of matter, the latter being, according to Einstein, everything except his gravitational field (Einstein, 1916). Accordingly, a means for generating an infinite set of equivalent solutions for black hole universes must not be restricted to spherical symmetry in the absence of matter; the restriction applied by (Fromholz *et al*., 2013). The sought for metric ground-form must encapsulate all types of supposed black hole universes; otherwise it is incomplete. Such a means was also first revealed in 2005 (Crothers, 2005b).

Cosmology has often rendered the Schwarzschild solution in isotropic coordinates. This involves a conformal transformation on the usual line-element for Schwarzschild spacetime. Any isotropic solution must also be produced by an isotropic metric ground-form for an infinite set of equivalent solutions. Cosmology has never found such means. However, the isotropic metric ground-form was first revealed in 2006 (Crothers, 2006).

With the discovery of the necessary metric ground-forms for generation of infinite sets of equivalent solutions, the methods of metric extension, by which all types of black hole universes are produced, have been proven to violate the metric ground-form and are consequently inadmissible. This is perhaps most easily seen by the fact that if any one metric of the infinite set is not extendible then none are extendible, owing to equivalence. A comprehensive analysis of this issue was recently presented (Crothers, 2014b), as an elaboration on a number of previous publications over a period of ten years.

The Static Einstein Field Equations in the Absence of Matter

Einstein's field equations couple his gravitational field to its *material sources* so that there is a causal connexion between matter and spacetime geometry. Einstein's field equations:

> *"Couple the gravitational field (contained in the curvature of spacetime) with its sources."(Foster and Nightingale, 1995)*

> *"Since gravitation is determined by the matter present, the same must then be postulated for*

> *geometry, too. The geometry of space is not given a priori, but is only determined by matter"(Pauli, 1981)*
>
> *"Again, just as the electric field, for its part, depends upon the charges and is instrumental in producing mechanical interaction between the charges, so we must assume here that the metrical field (or, in mathematical language, the tensor with components g_{ik}) is related to the material filling the world" (Weyl, 1952)*
>
> *"In general relativity, the stress-energy or energy-momentum tensor T^{ab} acts as the source of the gravitational field. It is related to the Einstein tensor and hence to the curvature of spacetime via the Einstein equation"(McMahon, 2006)*
>
> *"Mass acts on spacetime, telling it how to curve. Spacetime in turn acts on mass, telling it how to move." (Carroll and Ostlie, 2007)*

The *material sources* of Einstein's gravitational field are described by the energy-momentum tensor $T_{\mu\nu}$ and the gravitational field, manifest in curved spacetime geometry, is described by the Einstein tensor $G_{\mu\nu} = R_{\mu\nu}$-½$Rg_{\mu\nu}$. His field equations are:

$$G_{\mu\nu} = -\kappa T_{\mu\nu} \tag{1}$$

where, κ is a coupling constant (Einstein (1917) added a 'cosmological term': $G_{\mu\nu} + \Lambda g_{\mu\nu} = -\kappa T_{\mu\nu}$, which led to de Sitter's empty world). Equation (1) is expressed in words by the following relation:

$$spacetime\ geometry\ =\ -\kappa(material\ sources) \tag{2}$$

Einstein contended that if *material sources* = 0, his field equations become:

$$spacetime\ geometry = 0 \tag{3}$$

Which in mathematical form is:

$$R_{\mu\nu} = 0 \tag{4}$$

because in this case it turns out that $R = 0$ in $G_{\mu\nu}$.

Einstein claimed that Equation 4 describes his gravitational field outside a body such as a star, where the $T_{\mu\nu}$ vanish. However, expression (3) clearly shows that there are no *material sources* present in the field Equation 4. Bearing in mind that Einstein's field equations couple his gravitational field (spacetime geometry) to its *material sources*, since matter is the source of his gravitational field, what then is the material source of the gravitational field described by Equation 4? The invariable answer given by cosmology is that it is the body outside of which the supposed gravitational field exists. This is a circular argument because Einstein removed on the one hand all *material sources* from (2) and hence from (1), by setting *material sources* = 0 to get (3) and hence (4) and on the other hand, immediately reinstated the presence of a material source by asserting that Equation 4 describes his gravitational field outside a body such as a star. Since Equation 4 contains no *material sources* the system of nonlinear differential equations resulting from (4), on the assumption of a 4-dimensional pseudo-Riemannian metric with spherical symmetry, also do not contain any *material sources*, as expression (3) emphasizes. Consequently the so-called 'Schwarzschild solution' for Equation 4 cannot contain a *material source*. Thus, when $T_{\mu\nu} = 0$, *material sources* = 0, there are no *material sources* and hence no gravitational field.

That Equation 4 contain no *material sources* and so does not describe a gravitational field outside a body such as a star, is reaffirmed by the static homogeneous cosmological solutions. There are only three possible static homogeneous cosmological universes in General Relativity: (i) Einstein's cylindrical world; (ii) de Sitter's empty world; (iii) empty Minkowski spacetime. In Einstein's cylindrical world $T_{\mu\nu} \neq 0$; in de Sitter's empty world $T_{\mu\nu} = 0$; in Minkowski spacetime $T_{\mu\nu} = 0$ since there is no matter present in its metric, which is given by:

$$ds^2 = c^2dt^2 - dr^2 - r^2\left(d\theta^2 + sin^2\theta\, d\phi^2\right) \tag{5}$$

Now de Sitter's empty world is the solution to the field equations,

$$R_{\mu\nu} = \Lambda g_{\mu\nu} \tag{6}$$

where, Λ is the 'cosmological constant'. There are no *material sources* present in (6) because $T_{\mu\nu} = 0$: which is precisely why de Sitter's empty universe is completely empty. Although (6) contains no *material sources* because $T_{\mu\nu} = 0$, Einstein contended that (4) contains a material source even though $T_{\mu\nu} = 0$ there as well. Thus, $T_{\mu\nu} = 0$ both includes and excludes *material sources*. This however is impossible-*material sources* cannot be both present and absent by the very same mathematical constraint. Equation 4 contains no *material sources* by mathematical construction just as Equation 5 and 6 contain no *material sources* by mathematical construction. Consequently the Schwarzschild solution for Equation 4 contains no material source either; it therefore does not describe any gravitational field; it is physically meaningless precisely because $R_{\mu\nu} = 0$ is physically meaningless. Furthermore, all experiments attest that gravity is an interaction between two or more bodies. General Relativity cannot account for the simple experimental fact that two stationary suspended bodies approach one another upon release.

Metric Ground-Form for Schwarzschild Spacetime

In paragraph 9 of § I of (Fromholz *et al*., 2013) appear the following remarks:

> "*Furthermore, although one solution of the LL equations yields the Schwarzschild metric in the so-called harmonic radial coordinate r_H, related to the standard Schwarzschild coordinate r_S by $r_H = r_S - M$, where M is the mass of the object, …* "

This raises two issues: (i) the correct identity of r_S, (ii) the identity of M as mass. These Authors call r_S the "*Schwarzschild coordinate*" and $r_H = r_S - M$ the "*harmonic radial coordinate*". What really is r_S? Cosmology has variously and vaguely called it "*the distance*", "*the radius*", "*the radius of a 2-sphere*", "*the coordinate radius*", "*the radial coordinate*", "*the radial space coordinate*", "*the Schwarzschild radial coordinate*", "*the areal radius*", "*the reduced circumference*", "*the shortest distance a ray of light must travel to the centre*" and even "*a gauge choice: It defines the coordinate r*". That r_S goes by so many different identities attests to uncertainty. However, in cosmology it is always treated as the radius and this is clear from the fact that cosmologists always call $r = r_S = 2m = 2GM/c^2$ the "*Schwarzschild radius*" or the "*gravitational radius*", which is, they maintain, the 'radius' of a black hole event horizon.

However, none of these various and vague concepts of what r is are correct because the irrefutable geometrical fact is that r is the inverse square root of the Gaussian curvature of the spherically symmetric geodesic surface in the spatial section and as such it is neither a radius nor a distance in the metric. To see this consider Hilbert's solution, incorrectly attributed to Schwarzschild (Abrams, 1989):

$$ds^2 = \left(1-\frac{2M}{r}\right)dt^2 - \left(1-\frac{2M}{r}\right)^{-1}dr^2 - \\ - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \quad 0 \le r \tag{7}$$

In metric (7) the constants c and G are both set to unity, according to the general practice of cosmology. This is however very deceptive. Consider therefore Hilbert's metric with c and G explicit, so that nothing is hidden:

$$ds^2 = c^2\left(1-\frac{2GM}{c^2 r}\right)dt^2 - \left(1-\frac{2GM}{c^2 r}\right)^{-1}dr^2 - \\ - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \quad 0 \le r \tag{8}$$

To correctly identify r in Hilbert's metric (8), consider first the spatial section thereof. It is given by:

$$ds^2 = \left(1-\frac{2GM}{c^2 r}\right)^{-1}dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \tag{9}$$

Now consider the surface in the spatial section (9); it is given by:

$$\begin{aligned} ds^2 &= r^2\left(d\theta^2 + sin^2\theta\, d\phi^2\right) \\ &= r^2 d\theta^2 + r^2 sin^2\theta\, d\phi^2 \end{aligned} \tag{10}$$

This is a simple case of the First Fundamental Quadratic Form of a surface. The intrinsic geometry of a surface is entirely independent of any embedding space and so when it is embedded into a higher dimensional space the intrinsic geometry of the surface is not altered in any way. Therefore the intrinsic geometry of the surface (10) is the same in (8):

> "*And in any case, if the metric form of a surface is known for a certain system of intrinsic coordinates, then all the results concerning the intrinsic geometry of this surface can be obtained without appealing to the embedding space*."(*Efimov, 1980)*

A very important aspect of the intrinsic geometry of a surface is its Gaussian curvature.

Gauss' Theorema Egregium

> *The Gaussian curvature K at any point P of a surface depends only on the values at P of the coefficients in the First Fundamental Form and their first and second derivatives.*

The Gaussian curvature K of a surface can be calculated by means of the following relation (Crothers 2008a, 2008b, 2014b):

$$K = \frac{R_{1212}}{g} \tag{11}$$

where, R_{1212} is a component of the Riemann tensor of the first kind and g is the determinant of the metric tensor. Applying (11) to (10) yields:

$$K = \frac{1}{r^2} \tag{12}$$

From (11) and (12), it is clear that r is neither a radius nor a distance in (10) and that (10) does indeed describe a spherical surface (because the Gaussian curvature has a constant positive value -a surface which has a constant positive Gaussian curvature is called a spherical surface; a surface which has a constant negative Gaussian curvature is called a pseudo-spherical surface). The spherical surface described by (10) does not have a

radius because it is a surface, which is entirely independent of any embedding space and therefore retains its character in Hilbert's metric (8). Thus, r in Hilbert's metric is the inverse square root of the Gaussian curvature of the spherically symmetric geodesic surface in the spatial section thereof; it is neither the radius nor a distance in (8) or in (9). Thus, the *Schwarzschild radius* is neither a distance nor a radius of anything in Hilbert's solution. It is therefore not the 'radius' of the event horizon of an associated 'black hole'.

The radius R_p of the spherically symmetric 3-space described by (9) is given by (Crothers, 2005a; 2005b):

$$R_p = \int\left(1-\frac{2GM}{c^2 r}\right)^{-1/2} dr = \sqrt{r\left(r-\frac{2GM}{c^2}\right)} + \\ + \frac{2GM}{c^2} ln\left(\frac{\sqrt{r}+\sqrt{r-\frac{2GM}{c^2}}}{\sqrt{\frac{2GM}{c^2}}}\right) \tag{13}$$

This is the non-Euclidean radius in the spatial section of Hilbert's metric (8). The true geometric identity of r is given by (12), which also determines the nature of the surface (10) as a spherical surface. This is not an interpretation; it is a definite geometric quantity determined from the metric itself, according to Gauss' theory of surfaces. Cosmology has neither correctly identified r in Hilbert's metric nor correctly specified a spherical surface. That is why there are so many different 'interpretations' of Hilbert's r by cosmology.

Note that when $r = 2GM/c^2$ in expression (13), $R_p = 0$ and Hilbert's metric (8) is undefined. This fixes the range on r to $2GM/c^2 \leq r$ in Hilbert's metric, with the equality producing singularity (i.e., an undefined equation). Since r is neither the radius nor a distance in Hilbert's metric, there is no *a priori* reason to even suppose that $0 \leq r$ therein. When $r = 2GM/c^2$:

$$K = \frac{c^4}{4G^2M^2} \tag{14}$$

Compare now to (Schwarzschild, 1916):

$$ds^2 = \left(1-\frac{\alpha}{r}\right)dt^2 - \left(1-\frac{\alpha}{R}\right)^{-1} dR^2 - \\ - R^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \\ R = \left(r^3+\alpha^3\right)^{1/3},\ 0 \leq r \tag{15}$$

Here α is a positive but otherwise indeterminable constant. Note that Schwarzschild also set $c = 1$. He did not assign $\alpha = 2GM/c^2$ to get a mass M to appear. Examination of Schwarzschild's original paper reveals that $r = \sqrt{x^2+y^2+z^2}$, which can never be less than 0 by mathematical construction. Furthermore, Schwarzschild's metric is undefined (or singular) only at $r = 0$.

The Gaussian curvature K of the surface in the spatial section of Schwarzschild's actual solution is:

$$K = \frac{1}{R^2} = \frac{1}{\left(r^3+\alpha^3\right)^{2/3}} \tag{16}$$

The radius is given by:

$$R_p = \int\left(1-\frac{\alpha}{R}\right)^{-1/2} dR = \sqrt{R(R-\alpha)} + \\ + \alpha \ln\left(\frac{\sqrt{R}+\sqrt{R-\alpha}}{\sqrt{\alpha}}\right),\ R = \left(r^3+\alpha^3\right)^{1/3} \tag{17}$$

This has been generalised in order to generate all possible equivalent forms (Crothers, 2005a; 2005b):

$$ds^2 = \left(1-\frac{\alpha}{R_c}\right)dt^2 - \left(1-\frac{\alpha}{R_c}\right)^{-1} dR^2 - \\ - R_c^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \\ R_c = \left(\left|r-r_o\right|^n + \alpha^n\right)^{1/n};\ r, r_o \in R, n \in R^+ \tag{18}$$

where, r_o and n are entirely arbitrary constants. Expressions (18) constitute the metric ground-form for generating the infinite set of equivalent solutions for Schwarzschild spacetime. For instance, setting $r_o = 0$, $n = 3$, $r \geq r_o$, yields Schwarzschild's actual solution. Setting $r_o = \alpha$, $n = 1$, $r \geq r_o$, yields (Droste's, 1917) solution (which is the correct from of Hilbert's 'solution'). Setting $r_o = 0$, $n = 1$, $r \geq r_o$, $\alpha = 2m$, yields (Brillouin's, 1923) solution. Setting $r_o = M$, $n = 1$, $r \geq r_o$, $\alpha = 2M$ yields the correct form of the solution in the so-called "*harmonic radial coordinate*". Equivalent solutions can be generated from (18) in which r is always less than zero. Indeed, (18) is singular at only ever one value of r on the whole real line; at $r = r_o$. None of the equivalent solutions generated by expressions (18) permit $g_{00} = (1-\alpha/R_c)$ to change signature from + to −. To do so would not only violate the geometry but also change the metric into a non-static solution for a static problem and thereby bear no relation to the required static form (Brillouin, 1923).

Invalidity of Black Hole 'Metric Extensions'

It is apparent that (18) is not 'extendible' to produce a black hole universe. Since (18) generates all the possible equivalent solutions in Schwarzschild form, if any one of them is extendible then all of them must be extendible. In other words, if any one from (18) cannot

be extended then none can be extended. Thus, if Hilbert's solution is equivalent it must require that in Schwarzschild's actual solution $-\alpha \leq r$.

Similarly this must require that $-\alpha \leq r$ in Brillouin's solution and $0 \leq r$ in Droste's solution. It is evident from (18) that this is impossible. To amplify this, consider the specific case $r_o = 0$, $n = 2$, for which (18) yields:

$$\begin{aligned} ds^2 &= \left(1-\frac{\alpha}{R_c}\right)dt^2 - \left(1-\frac{\alpha}{R_c}\right)^{-1} dR^2 - R_c^2 d\Omega^2 \\ d\Omega^2 &= \left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \\ R_c &= \left(r^2+\alpha^2\right)^{1/2} \end{aligned} \tag{19}$$

According to Hilbert's solution this would require the range $-\alpha^2 \leq r^2$ in (19). However, although r can now take any real value whatsoever, r^2 cannot take values < 0. Thus, (19) cannot be 'extended' by any means. Therefore, owing to equivalence, no solution generated by (18) can be extended. Consequently, the supposed extension of Droste's solution to values $0 \leq r$ by means of the Kruskal-Szekeres 'coordinates', the Eddington-Finkelstein 'coordinates' and also the Lemaître 'coordinates', are all fallacious. Cosmology most often effects its extensions by means of Kruskal-Szekeres 'coordinates'. Putting R_c from (18) into the Kruskal-Szekeres form yields:

$$\begin{aligned} ds^2 &= \frac{4\alpha^3}{R_c} e^{-R_c/\alpha}\left(dv^2 - du^2\right) - R_c^2 d\Omega^2 \\ d\Omega^2 &= \left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \\ \left(\frac{R_c}{\alpha}-1\right)e^{R_c/\alpha} &= u^2 - v^2,\ R_c = \left(\left|r-r_o\right|^n + \alpha^n\right)^{1/n} \end{aligned} \tag{20}$$

This does not extend Droste's metric to $0 \leq r$ since the minimum value of R_c is $R_c(r_o) = \alpha$ for all r_o for all n. No equivalent solution generated by (18) is extendible to produce a black hole universe (Crothers, (2014b) for a detailed analysis). Hilbert's solution is not equivalent to (18) and therefore not equivalent to Schwarzschild's solution. A further consequence of this is that the 'singularity theorems' of Hawking and Penrose are invalid (Crothers, 2013a).

Metric Ground-form for Isotropic Schwarzschild Spacetime

Consider now Hilbert's metric in isotropic coordinates. It is given by:

$$\begin{aligned} ds^2 &= \left(\frac{1-\frac{M}{2r}}{1+\frac{M}{2r}}\right)^2 dt^2 - \\ &-\left(1+\frac{M}{2r}\right)^4\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right],\ 0 \leq r \end{aligned} \tag{21}$$

This is again very deceptive, so rewrite (21) with G and c explicit:

$$\begin{aligned} ds^2 &= c^2\left(\frac{1-\frac{GM}{2c^2 r}}{1+\frac{GM}{2c^2 r}}\right)^2 dt^2 - \\ &-\left(1+\frac{GM}{2c^2 r}\right)^4\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right],\ 0 \leq r \end{aligned} \tag{22}$$

Consider now the spatial section of metric (22):

$$ds^2 = \left(1+\frac{GM}{2c^2 r}\right)^4\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right] \tag{23}$$

The surface in the spatial section is described by:

$$ds^2 = \left(1+\frac{GM}{2c^2 r}\right)^4 r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \tag{24}$$

This is once again a simple First Fundamental Quadratic Form for a surface. Applying expression (11) to expression (24) the Gaussian curvature K of the surface (24) is:

$$K = \frac{1}{\left(1+\frac{GM}{2c^2 r}\right)^4 r^2} \tag{25}$$

The quantity r is thus related to the Gaussian curvature of expression (24) and hence retains that role in (22) and (23). It acts as a parameter for expressions (21) to (25). Equations (5) and (25) reveal the nature of this parameter -it is both the radius and the inverse square root of the Gaussian curvature of the spherically symmetric surface in the spatial section of metric (5) for Minkowski spacetime. Note that the terms within the square brackets in each of expressions (21), (22) and (23) is the metric for Euclidean 3- space. The actual range of r in metric (22) is determined by the radius for (22), thus (Crothers, 2006):

$$\begin{aligned} R_p &= \int\left(1+\frac{GM}{2c^2 r}\right)^2 dr \\ &= r + \frac{GM}{c^2}\ln\frac{2c^2 r}{GM} - \frac{G^2M^2}{2c^4 r} + \frac{GM}{2c^2} \end{aligned} \tag{26}$$

Note that when $r = GM/2c^2$, $R_p = 0$ (a scalar invariant) and the Gaussian curvature is:

$$K = \frac{c^4}{4G^2M^2} \tag{27}$$

This is the very same result (14) for Hilbert's metric (8). This value of K is a scalar invariant.

The metric ground-form to generate an infinite set of equivalent solutions for Schwarzschild spacetime in isotropic coordinates is (Crothers, 2006):

$$\begin{aligned} ds^2 &= c^2\frac{\left(1-\frac{\alpha}{4R_c}\right)^2}{\left(1+\frac{\alpha}{4R_c}\right)^2}dt^2 - \left(1+\frac{\alpha}{4R_c}\right)^4 dR_c{}^2 - \\ &- R_c^2\left(1+\frac{\alpha}{4R_c}\right)^4\left(d\theta^2+\sin^2\theta\, d\varphi^2\right) \\ R_c &= \left[\left|r-r_o\right|^n + \left(\frac{\alpha}{4}\right)^n\right]^{1/n};\ r, r_o \in R, n \in R^+ \end{aligned} \tag{28}$$

If $\alpha = 2GM/c^2$, $n = 1$, $r_o = \alpha/4$, $r_o \leq r$, then the metric (22) is obtained, except that by (26) the range on r is $GM/2c^2 \leq r$, not $0 \leq r$. Note that no matter what values are chosen for n and r_o the Gaussian curvature of the spherically symmetric geodesic surface at $r = r_o$ is always the same; $K = 1/\alpha^2$. This is a scalar invariant for the Schwarzschild forms.

Since (28) can be obtained by a transformation on (18), either set of expressions can be used to generate an infinite set of equivalent solutions for Schwarzschild spacetime.

In (18) and (28) α is an entirely arbitrary constant from the strictly mathematical standpoint. Cosmology contends however that it is associated with a material source, as assigned in Hilbert's solution (7) and (8) above. However, since $R_{\mu\nu} = 0$ contains no matter by mathematical construction, if the constant α is to be associated with *material sources* then it must be that $\alpha = 0$ (no matter). In this case the metric ground-forms (18) and (28) trivially reduce to that for empty Minkowski spacetime, expression (5).

Insinuation of Newton's Escape Velocity

Consider further Hilbert's metric in the revealing form (8) above. According to cosmology, the radius of the event horizon of an associated black hole, the so-called 'Schwarzschild radius' thereof, is given by:

$$r_s = \frac{2GM}{c^2} \tag{29}$$

From this 'radius' expression the escape speed (often called 'escape velocity') of a black hole is determined by solving for c:

$$c = \sqrt{\frac{2GM}{r_s}} \tag{30}$$

This is immediately recognised as Newton's expression for escape speed. From this expression cosmology asserts that the 'escape velocity' of a black hole is the speed of light c. But Newton's expression for escape speed is an implicit two-body relation: One body escapes from another body. It cannot therefore rightly appear in what is supposed to be a solution for a one-body problem (but which is in fact a zero-body problem). Furthermore, if a black hole event horizon has an escape speed c, as cosmology claims by equation (30), then, by definition, light can escape, contrary to the invariable claim of cosmology that it cannot even leave.

The mass appearing in Hilbert's solution is obtained by arbitrarily and inadmissibly inserting, *post hoc*, Newton's expression for escape speed in order to satisfy the false claim that a material source is present in $R_{\mu\nu} = 0$. For example, (McMahon, 2006) says in relation to Hilbert's solution,

> "*… the Schwarzschild radius. In terms of the mass of the object that is the source of the gravitational field, it is given by:*
>
> $$r_s = \frac{2GM}{c^2}$$

In keeping with Equation (30) cosmology claims on the one hand that a black hole has an escape velocity:

> "*black hole A region of spacetime from which the escape velocity exceeds the velocity of light*" (Matzner, 2001)
> "*black hole A massive object so dense that no light or any other radiation can escape from it; its escape velocity exceeds the speed of light*" (Ian, 2001)

> "*A black hole is, ah, a massive object and it's something which is so massive that light can't even escape. … some objects are so massive that the escape speed is basically the speed of light and therefore not even light escapes. … so black holes themselves are, are basically inert, massive and nothing escapes.*" (Bland-Hawthorn, 2013)

Yet on the other hand cosmology also claims that nothing can even leave a black hole:

> "*I had already discussed with Roger Penrose the idea of defining a black hole as a set of events from which it is not possible to escape to a large distance. It means that the boundary of the black hole, the event horizon, is formed by rays of light that just fail to get away from the black hole. Instead, they stay forever hovering on the edge of the black hole* " (Hawking, 2002)

> "*The problem we now consider is that of the gravitational collapse of a body to a volume so small that a trapped surface forms around it; as we have stated, from such a surface no light can emerge*." (Chandrasekhar, 1972)

> "*It is clear from this picture that the surface r = 2m is a one-way membrane, letting future-directed timelike and null curves cross only from the outside (region I) to the inside (region II).*" (d'Inverno, 1992)

> "*Thus we cannot have direct observational knowledge of the region r < 2m. Such a region is called a black hole, because things can fall into it (taking an infinite time, by our clocks, to do so) but nothing can come out*." (Dirac, 1996)

Not only is light not able to even leave the event horizon, neither can ponderable bodies. However, escape velocity does not prevent bodies from leaving, only from escaping, if the speed of a body does not reach escape speed. Thus, according to cosmology, a black hole has and does not have an escape velocity simultaneously at the same place (the event horizon); which is however quite impossible. The very concept of black hole escape velocity is nothing but a play on the words "*escape velocity*" (McVittie, 1978).

Einstein introduced multiple masses into his empty universe, $R_{\mu\nu} = 0$, by applying the Principle of Superposition where the Principle of Superposition is in fact invalid. In relation to Hilbert's solution (Einstein, 1967) contended:

> "*M denotes the sun's mass, centrally symmetrically placed about the origin of co-ordinates; the solution (109a) is valid only outside of this mass, where all the $T_{\mu\nu}$ vanish. If the motion of the planet takes place in the x_1-x_2 plane then we must replace (109a) by:*

$$ds^2 = \left(1 - \frac{A}{r}\right)dt^2 - \frac{dr2}{\left(1 - \frac{A}{r}\right)} - r^2 d\varphi^2 \text{ "}$$

where $A = \kappa M/4\pi$. Notice that Einstein not only introduced a material source M, *post hoc*, he also introduced a planet outside this 'source', by superposing it. However, the Principle of Superposition does not hold in General Relativity because the latter is nonlinear.

Static and Non-Static 'Solutions'

Schwarzschild and Reissner-Nordström spacetimes are static. Kerr and Kerr-Newman spacetimes are stationary. None of them are non-static. All black hole universes however involve a non-static spacetime. Consider Hilbert's metric (7) and (8):

> "*The most obvious pathology at r = 2M is the reversal there of the roles of t and r as timelike and spacelike coordinates. In the region r > 2M, the t direction, ∂/∂t, is timelike ($g_{tt} < 0$) and the r direction, ∂/∂r, is spacelike ($g_{rr} > 0$); but in the region r < 2M, ∂/∂t, is spacelike ($g_{tt} > 0$) and ∂/∂r, is timelike ($g_{rr} < 0$).*

> "*What does it mean for r to 'change in character from a spacelike coordinate to a timelike one'? The explorer in his jet-powered spaceship prior to arrival at r =2M always has the option to turn on his jets and change his motion from decreasing r (infall) to increasing r (escape). Quite the contrary in the situation when he has once allowed himself to fall inside r =2M. Then the further decrease of r represents the passage of time. No command that the traveler can give to his jet engine will turn back time. That unseen power of the world which drags everyone forward willy-nilly from age twenty to forty and from forty to eighty also drags the rocket in from time coordinate r =2M to the later time coordinate r =0. No human act of will, no engine, no rocket, no force (see exercise 31.3) can make time stand still. As surely as cells die, as surely as the traveler's watch ticks away 'the unforgiving minutes', with equal certainty and with never one halt along the way, r drops from 2M to 0.*

> "*At r =2M, where r and t exchange roles as space and time coordinates, g_{tt} vanishes while g_{rr} is infinite*" (Misner *et al*., 1973)

> "*There is no alternative to the matter collapsing to an infinite density at a*

singularity once a point of no-return is passed. The reason is that once the event horizon is passed, all timelike trajectories must necessarily get to the singularity: 'all the King's horses and all the King's men' cannot prevent it."(*Chandrasekhar 1972)*

"*This is worth stressing; not only can you not escape back to region I, you cannot even stop yourself from moving in the direction of decreasing r, since this is simply the timelike direction. (This could have been seen in our original coordinate system; for r < 2GM, t becomes spacelike and r becomes timelike). Thus you can no more stop moving toward the singularity than you can stop getting older.*" *(Carroll, 1997)*

"*For r < 2GM/c*^{2}*, however, the component* g_{oo} *becomes negative and* g_{rr}*, positive, so that in this domain, the role of time-like coordinate is played by r, whereas that of space-like coordinate by t. Thus in this domain, the gravitational field depends significantly on time (r) and does not depend on the coordinate t" (Vladmimirov et al., 1984)*

To amplify this, set $t = r^*$ and $r = t^*$. Then for $0 \leq r < 2M$, Hilbert's solution (7) becomes:

$$\begin{aligned} ds^2 &= \left(1-\frac{2M}{t^*}\right)dr^{*2} - \left(1-\frac{2M}{t^*}\right)^{-1} dt^{*2} - \\ &- t^{*2}\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \\ 0 &\leq t^* < 2M \end{aligned} \tag{31}$$

It now becomes quite clear that this is a time-dependent (i.e., non-static) metric since all the components of the metric tensor are now functions of the timelike t^* and so this metric bears no relationship to the original time-independent (i.e., static) problem that was initially posed (Droste, 1917; Brillouin, 1923; Crothers, 2014b). In other words, this metric is a non-static solution to a static problem: Contra hype! Furthermore, the signature of the metric changes from (+, −, −, −) to (−, +, −, −) and so is no longer Lorentzian.

Metric Ground-form for Kerr-newman Spacetime

The Kerr-Newman spacetime subsumes the Kerr, Reissner-Nordström and Schwarzschild spacetimes. The metric ground-form for generation of the infinite set of equivalent solutions for all these forms was obtained by (Crothers, 2005b). The metric ground-form is:

$$\begin{aligned} ds^2 &= -\frac{\Delta - a^2\sin^2\theta}{\rho^2}dt^2 - \\ &- \frac{2a\sin^2\theta\left(R_c^2 + a^2 - \Delta\right)}{\rho^2}dt\,d\varphi + \\ &+ \frac{\left(R_c^2 + a^2\right)^2 - a^2\Delta\sin^2\theta}{\rho^2}\sin^2\theta\, d\varphi^2 + \\ &+ \frac{\rho^2}{\Delta}dR_c^2 + \rho^2 d\theta^2 \\ \Delta &= R_c^2 - \alpha R_c + a^2 + q^2,\ \rho^2 = R_c^2 + a^2\cos^2\theta \\ R_c &= \left(\left|r - r_o\right|^n + \xi^n\right)^{1/n};\ r, r_o \in R,\ n \in R^+ \\ \xi &= \frac{\alpha}{2} + \sqrt{\frac{\alpha^2}{4} - q^2 - a^2\cos^2\theta}\,,\ a^2 + q^2 < \frac{\alpha^2}{4} \end{aligned} \tag{32}$$

Here r_o and n are entirely arbitrary. Charge is denoted by q and angular momentum is contained in a. If $r_o = \xi$, $n = 1$, $r_o \leq r$, the correct form of the Kerr-Newman 'solution' in Boyer- Lindquist coordinates is obtained. Just as in the case of Schwarzschild spacetime, there is no 'event horizon' and hence no black hole. Since $R_c(r_o) = \xi$ for all r_o for all n, no solution generated by expressions (32) can be extended. This is amplified by the case of $r_o = 0$, $n = 2$ in (32), in which case:

$$R_c = \left(r^2 + \xi^2\right)^{1/2} \tag{33}$$

This is defined for all real values of r and can never be zero. Owing to equivalence, no solution generated by (32) can be extended. A detailed analysis has been presented by (Crothers, 2014b).

Note that if $a = 0$ and $q = 0$, then (32) reduces to the Schwarzschild ground-form.

Similarly, Gaussian curvature for the surface in the spatial section of the Kerr-Newman ground-form (32) has been obtained (Crothers, 2014b), which also reduces to the Schwarzschild form. It is given by:

$$\begin{aligned} K &= \frac{\left[\left(\frac{\partial\beta}{\partial\theta}\sin^2\theta + \beta\sin 2\theta\right)\frac{\partial h}{\partial\theta}\right]}{2hf\sin^2\theta} - \\ &- \frac{\left[h\left(\frac{\partial^2\beta}{\partial\theta^2}\sin^2\theta + 2\frac{\partial\beta}{\partial\theta}\sin 2\theta + 2\beta\cos 2\theta\right)\right]}{2hf\sin^2\theta} - \\ &- \frac{\left(\frac{\partial\beta}{\partial\theta}\sin^2\theta + \beta\sin 2\theta\right)\frac{\partial h}{\partial\theta}}{4hf\sin^2\theta} + \frac{h\left(\frac{\partial\beta}{\partial\theta}\sin^2\theta + \beta\sin 2\theta\right)^2}{4f^2\sin^2\theta} \\ h &= \rho^2 = R_c^2 + a^2\cos^2\theta,\quad f = \left(R_c^2 + a^2\right)^2 - a^2\Delta\sin^2\theta \\ \beta &= \frac{f}{h},\ \Delta = R_c^2 - \alpha R_c + a^2 + q^2,\ R_c = \left(\left|r - r_o\right|^n + \xi^n\right)^{1/n} \\ \xi &= \frac{\alpha}{2} + \sqrt{\frac{\alpha^2}{4} - q^2 - a^2\cos^2\theta},\ a^2 + q^2 < \frac{\alpha^2}{4};\ r, r_o \in R,\ n \in R^+ \end{aligned} \tag{34}$$

Since K of (34) is not a positive constant the surface in the spatial section of (32) is not spherically symmetric. Thus the Kerr and Kerr-Newman spacetimes are not spherically symmetric. However, if $a = 0$ (32) and (34) yield spherical symmetry-Schwarzschild spacetime and Reissner-Nordström spacetime are spherically symmetric. Details can be found in (Crothers 2005a; 2005b; 2014b).

Metric Ground-form for Reissner-Nordström Spacetime

When $a = 0$ expressions (32) reduce to the Reissner-Nordström metric ground-form:

$$\begin{aligned} ds^2 &= c^2\left(1-\frac{\alpha}{R_c}+\frac{q^2}{R_c^2}\right)dt^2-\left(1-\frac{\alpha}{R_c}+\frac{q^2}{R_c^2}\right)^{-1}dR_c^2- \\ &-R_c^2\left(d\theta^2+\sin^2\theta\, d\varphi^2\right) \\ R_c &= \left(\left|r-r_o\right|^n+\xi^n\right)^{1/n};\ r, r_o \in R,\ n\in R^+ \\ \xi &= \frac{\alpha}{2}+\sqrt{\frac{\alpha^2}{4}-q^2},\quad q^2<\frac{\alpha^2}{4} \end{aligned} \tag{35}$$

This is the metric ground-form that generates the infinite set of equivalent solutions for Reissner-Nordström spacetime. It is singular at only one point, $r = r_o$. The values of r_o and n remain entirely arbitrary.

The radius for (35) is (Crothers, 2005b):

$$\begin{aligned} R_p(r) &= \int\left(1-\frac{\alpha}{R_c}+\frac{q^2}{R_c^2}\right)^{-1/2}dR_c \\ &=\sqrt{R_c^2-\alpha R_c+q^2}+ \\ &+\frac{\alpha}{2}ln\left(\frac{R_c-\frac{\alpha}{2}+\sqrt{R_c^2-\alpha R_c+q^2}}{\sqrt{\frac{\alpha^2}{4}-q^2}}\right) \end{aligned} \tag{36}$$

where, $R_c = R_c(r)$ is given in (35). When $r = r_o$, $R_p = 0$.

By means of (34) the Gaussian curvature of the surface in the spatial section is:

$$K=\frac{1}{R_c^2} \tag{37}$$

This proves that the surface is a spherical surface. Also, the invariant Gaussian curvature of the spherical surface occurs at $r = r_o$, in which case:

$$K=\frac{1}{\xi^2}=\frac{1}{\left[\frac{\alpha}{2}+\sqrt{\frac{\alpha^2}{4}-q^2}\right]^2} \tag{38}$$

Note that if $q = 0$ expressions (35), (36), (37) and (38) reduce to those for the Schwarzschild ground-form.

It is evident from (35) that the Reissner-Nordström metric ground-form cannot be extended to produce a black hole. Once again, this is amplified by the case $n = 2$. Consequently, the application of 'Kruskal-Szekeres coordinates' does not extend Reissner-Nordström spacetime to produce a black hole.

Metric Ground-form for Isotropic Reissner-Nordström Spacetime

The metric ground-form for Reissner-Nordström spacetime in isotropic coordinates is (Crothers, 2014b):

$$\begin{aligned} ds^2 &= \frac{c^2\left[1-\frac{\alpha^2}{16R_c^2}+\frac{q^2}{4R_c^2}\right]^2}{\left(1+\frac{\alpha}{4R_c}+\frac{q}{2R_c}\right)^2\left(1+\frac{\alpha}{4R_c}-\frac{q}{2R_c}\right)^2}dt^2- \\ &-\left(1+\frac{\alpha}{4R_c}+\frac{q}{2R_c}\right)^2\left(1+\frac{\alpha}{4R_c}-\frac{q}{2R_c}\right)^2 d\Omega^2 \\ d\Omega^2 &= \left[dR_c^2+R_c^2\left(d\theta^2+\sin^2\theta\, d\varphi^2\right)\right] \\ R_c &= \left(\left|r-r_o\right|^n+\xi^n\right)^{1/n},\quad \xi=\frac{\sqrt{\alpha^2-4q^2}}{4} \\ 4q^2 &< \alpha^2,\quad r\in R,\ n\in R^+ \end{aligned} \tag{39}$$

The radius for (39) is given by:

$$\begin{aligned} R_p &= \int\left(1+\frac{\alpha}{4R_c}+\frac{q}{2R_c}\right)\left(1+\frac{\alpha}{4R_c}-\frac{q}{2R_c}\right)dR_c \\ &= R_c+\frac{\alpha}{2}\ln\left(\frac{4R_c}{\sqrt{\alpha^2-4q^2}}\right)-\frac{\left(\alpha^2-4q^2\right)}{16R_c} \end{aligned} \tag{40}$$

Note that for (40), $R_p(r_o) = 0\ \forall\ r_o\ \forall\ n$, as it must.

The Gaussian curvature of the surface in the spatial section of (39) is:

$$\hat{K}=\frac{1}{R_c^2\left(1+\frac{\alpha}{4R_c}+\frac{q}{2R_c}\right)^2\left(1+\frac{\alpha}{4R_c}-\frac{q}{2R_c}\right)^2} \tag{41}$$

This proves that the surface is a spherical surface.

The invariant Gaussian curvature occurs at $r = r_o$, to yield:

$$\hat{K}=\frac{16\left(\alpha^2-4q^2\right)}{\left[\left(\alpha+\sqrt{\alpha^2-4q^2}\right)^2-4q^2\right]^2} \tag{42}$$

Note that if $q = 0$ then (42) reduces to the invariant Gaussian curvature of the surface in the spatial section of the isotropic Schwarzschild ground-form. The invariant (42) is not the same as for the Reissner-Nordström ground-form (38). This is due to the conformal transformation (Crothers, 2014b). This is most easily seen in the coordinate transformation that changes the Reissner-Nordström ground-form into the isotropic Reissner-Nordström ground-form:

$$R_c(r_o)\left[\left(1+\frac{\alpha}{4R_c(r_o)}\right)^2-\frac{q^2}{4R_c^2(r_o)}\right]= \\ =\frac{\alpha}{2}+\sqrt{\frac{\alpha^2}{4}-q^2} \tag{43}$$

and comparing with (35).

Riemannian Curvature

Riemannian (or sectional) curvature generalises the Gaussian curvature of a surface to dimensions higher than 2. Consequently, in the case of a surface the Riemannian curvature reduces to Gaussian curvature. In general, Riemannian curvature depends upon both position and direction. The Riemannian curvature demonstrates once again that none of the so-called black hole metrics can be extended to produce a black hole.

The Riemannian curvature K_S at any point in a metric space of dimensions $n > 2$ depends upon the Riemann-Christoffel curvature tensor of the first kind R_{ijkl}, the components of the metric tensor g_{ik} and two arbitrary n dimensional linearly independent contravariant direction vectors U^i and V^i, as follows:

$$K_S=\frac{R_{ijkl}U^iV^jU^kV^l}{G_{pqrs}U^pV^qU^rV^s} \tag{44}$$
$$G_{pqrs}=g_{pr}g_{qs}-g_{ps}g_{qr}$$

The Riemannian curvature for the Schwarzschild ground-form is given by (Crothers, 2014b):

$$\mathrm{K_S}=\frac{\left[\begin{array}{l}2\alpha(R_c-\alpha)W_{0101}- \\ -\alpha R_c(R_c-\alpha)^2W_{0202}- \\ -\alpha\mathrm{R}_c(R_c-\alpha)^2W_{0303}sin^2\theta+ \\ +\alpha\mathrm{R}_c^3W_{1212}+\alpha R_c^3W_{1313}sin^2\theta- \\ -2\alpha\mathrm{R}_c^4(R_c-\alpha)W_{2323}sin^2\theta\end{array}\right]}{\left[\begin{array}{l}-2R_c^3(R_c-\alpha)W_{0101}- \\ -2R_c^4(R_c-\alpha)^2W_{0202}- \\ -2R_c^4(R_c-\alpha)^2W_{0303}sin^2\theta+ \\ +2R_c^6\mathrm{W}_{1212}+2R_c^6W_{1313}sin^2\theta+ \\ +2R_c^7(R_c-\alpha)W_{2323}sin^2\theta\end{array}\right]}$$

$$W_{ijkl}=\begin{vmatrix}U^i & U^j\\ V^i & V^j\end{vmatrix}\begin{vmatrix}U^k & U^l\\ V^k & V^l\end{vmatrix}$$
$$R_c=\left(|r-r_o|^n+\alpha^n\right)^{1/n} \tag{45}$$
$$r, r_o \in R,\ n \in R^+$$

Definition

> *If the Riemannian curvature at any point is independent of direction vectors at that point then the point is called an isotropic point.*

$R_c(r_o) = \alpha$ irrespective of the values of r_o and n, in which case (45) reduces to:

$$K_S=\frac{1}{2\alpha^2} \tag{46}$$

Thus, (46) is entirely independent of the direction vectors U^i and V^i and of θ. Hence, $r = r_o$ produces an isotropic point, which again shows that the Schwarzschild form cannot be extended.

Comparing (46) with (27) and (34) for the Schwarzschild form at $r = r_o$, yields:

$$K_S=\frac{K}{2} \tag{47}$$

Hence, at $r = r_o$ the Riemannian curvature of the Schwarzschild form is half the Gaussian curvature of the spherical surface in the spatial section of the Schwarzschild form.

The Riemannian curvature for the Reissner-Nordström ground-form is:

$$K_S=\frac{A+B+C}{D+E+F}$$
$$A=2\left(R_c^2-\alpha R_c+q^2\right)\left(\alpha R_c-3q^2\right)W_{0101}-$$
$$-\left(R_c^2-\alpha R_c+q^2\right)\left(\alpha R_c-2q^2\right)W_{0202}$$
$$B=-\left(R_c^2-\alpha R_c+q^2\right)^2\left(\alpha R_c-2q^2\right)\sin^2\theta W_{0303}+$$
$$+R_c^4\left(\alpha R_c-2q^2\right)W_{1212}$$
$$C=R_c^4\left(\alpha R_c-2q^2\right)\sin^2\theta W_{1313}-$$
$$-2R_c^4\left(\alpha R_c-q^2\right)\left(R_c^2-\alpha R_c+q^2\right)\sin^2\theta W_{2323}$$
$$D=-2R_c^4\left(R_c^2-\alpha R_c+q^2\right)W_{1010}-$$
$$-2R_c^4\left(R_c^2-\alpha R_c+q^2\right)^2\sin^2\theta W_{0202}$$
$$E=-2R_c^4\left(R_c^2-\alpha R_c+q^2\right)^2\sin^2\theta W_{0303}+2R_c^8W_{1212}$$
$$F=2R_c^8\sin^2\theta W_{1313}+2R_c^8\left(R_c^2-\alpha R_c+q^2\right)\sin^2\theta W_{2323}$$

$$W_{ijkl}=\begin{vmatrix}U^i & U^j\\ V^i & V^j\end{vmatrix}\begin{vmatrix}U^k & U^l\\ V^k & V^l\end{vmatrix},\ R_c=\left(\left|r-r_o\right|^n+\xi^n\right)^{1/n} \tag{48}$$

$$\xi=\frac{\alpha}{2}+\sqrt{\frac{\alpha^2}{4}-q^2},\ q^2<\frac{\alpha^2}{4};\ r,r_o\in R,n\in R^+$$

$R_c(r_o) = \xi$ irrespective of the values of r_o and n, in which case (48) reduces to:

$$K_S=\frac{\alpha\xi-2q^2}{2\xi^4} \tag{49}$$

Equation (49) is entirely independent of the direction vectors U^i and V^i and of θ. Thus, $r = r_o$ produces an isotropic point, which again shows that the Reissner-Nordström ground-form cannot be extended. Note that when $q = 0$, (48) and (49) reduce to the corresponding values for the Schwarzschild ground-form (45) and (46).

The Riemannian curvature for the isotropic Schwarzschild ground-form is given by:

$$\hat{K}_S=\frac{A+B}{C+D}$$

$$A=\frac{16\alpha\left(4R_c-\alpha\right)^2}{R_c\left(4R_c+\alpha\right)^4}\hat{W}_{0101}-$$

$$-\frac{8\alpha\left(4R_c-\alpha\right)^2}{\left(4R_c+\alpha\right)^4}\hat{W}_{0202}-$$

$$-\frac{8\alpha R_c\left(4R_c-\alpha\right)^2\sin^2\theta}{\left(4R_c+\alpha\right)^4}\hat{W}_{0303}$$

$$B=\frac{\alpha\left(4R_c+\alpha\right)^2}{32R_c^3}\hat{W}_{1212}+$$

$$+\frac{\alpha\left(4R_c+\alpha\right)^2\sin^2\theta}{32R_c^3}\hat{W}_{1313}-$$

$$-\frac{\alpha\left(4R_c+\alpha\right)^2\sin^2\theta}{16R_c}\hat{W}_{2323}$$

$$C=\frac{-\left(4R_c-\alpha\right)^2\left(4R_c+\alpha\right)^2}{4^4R_c^4}\hat{W}_{0101}-$$

$$-\frac{\left(4R_c-\alpha\right)^2\left(4R_c+\alpha\right)^2}{4^4R_c^2}\hat{W}_{0202}-$$

$$-\frac{\left(4R_c-\alpha\right)^2\left(4R_c+\alpha\right)^2\sin^2\theta}{4^4R_c^2}\hat{W}_{0303}$$

$$D=\frac{\left(4R_c+\alpha\right)^8}{4^8R_c^6}\hat{W}_{1212}+$$

$$+\frac{\left(4R_c+\alpha\right)^8\sin^2\theta}{4^8R_c^6}\hat{W}_{1313}+$$

$$+\frac{\left(4R_c+\alpha\right)^8\sin^2\theta}{4^8R_c^4}\hat{W}_{2323}$$

$$\hat{W}_{ijkl}=\begin{vmatrix}\hat{U}^i & \hat{U}^j\\ \hat{V}^i & \hat{V}^j\end{vmatrix}\begin{vmatrix}\hat{U}^k & \hat{U}^l\\ \hat{V}^k & \hat{V}^l\end{vmatrix}$$

$$R_c=\left[\left|r-r_o\right|^n+\left(\frac{\alpha}{4}\right)^n\right]^{1/n} \tag{50}$$

$$r,r_o\in R\,n\in R^+$$

When $r = r_o$, $R_c = \alpha/4$, for all r_o and for all n and the Riemannian curvature becomes:

$$\hat{K}_S=\frac{\left[\begin{array}{r}8\left(\hat{W}_{1212}+\hat{W}_{1313}\sin^2\theta\right)-\\ -\alpha^2\hat{W}_{2323}\sin^2\theta\end{array}\right]}{\left[\begin{array}{r}16\alpha^2\left(\hat{W}_{1212}+\hat{W}_{1313}\sin^2\theta\right)+\\ +\alpha^4\hat{W}_{2323}\sin^2\theta\end{array}\right]} \tag{51}$$

Note that (51) differs from the Schwarzschild form (46) due only to the terms in $\hat{W}_{2323}$, due to the conformal transformation. Moreover, (51) also depends upon θ. At $\theta = 0$ and $\theta = \pi$, (51) reduces to the exact value for the Schwarzschild ground-form (46). Hence, at $r = r_o$, $\theta = 0$ and $\theta = \pi$ produce the isotropic point of the Schwarzschild ground-form. This shows, once again, that the Schwarzschild and isotropic Schwarzschild ground-form cannot be extended.

At $\theta = \pi/2$ expression (51) becomes:

$$\hat{K}_S=\frac{\left[\begin{array}{r}8\left(\hat{W}_{1212}+\hat{W}_{1313}\right)-\\ -\alpha^2\hat{\mathrm{W}}_{2323}\end{array}\right]}{\left[\begin{array}{c}16\alpha^2\left(\hat{W}_{1212}+\hat{W}_{1313}\right)+\\ +\alpha^4\hat{W}_{2323}\end{array}\right]} \tag{52}$$

The Riemannian curvature for the isotropic Reissner-Nordström ground-form is much more complicated. It is given by (Crothers, 2014b):

$$\hat{K}_S=\frac{\left[\begin{array}{l}\hat{R}_{0101}\hat{W}_{0101}+\hat{R}_{0202}\left(\hat{W}_{0202}+\hat{W}_{0303}\sin^2\theta\right)+\\ +\hat{R}_{1212}\left(\hat{W}_{1212}+\hat{W}_{1313}\sin^2\theta\right)+\hat{\mathrm{R}}_{2323}\hat{W}_{2323}\end{array}\right]}{\left[\begin{array}{l}\hat{G}_{0101}\hat{W}_{0101}+\hat{\mathrm{G}}_{0202}\left(\hat{W}_{0202}+\hat{W}_{0303}\sin^2\theta\right)+\\ +\hat{G}_{1212}\left(\hat{W}_{1212}+\hat{W}_{1313}\sin^2\theta\right)+\hat{G}_{2323}\hat{W}_{2323}\end{array}\right]}$$

$$\hat{W}_{ijkl}=\begin{vmatrix}\hat{U}^i & \hat{U}^j\\ \hat{V}^i & \hat{V}^j\end{vmatrix}\begin{vmatrix}\hat{U}^k & \hat{U}^l\\ \hat{V}^k & \hat{V}^l\end{vmatrix}$$

$$R_c=\left[\left|r-r_o\right|^n+\xi^n\right]^{1/n}$$

$$\xi = \frac{\sqrt{\alpha^2 - 4q^2}}{4} \qquad 4q^2 < \alpha^2$$
$$r, r_o \in R,\ n \in R^+$$

$$\hat{R}_{0101} = \frac{64L(F+H) - IJ - 64L^2}{Z} + \frac{KL}{R_c Z}$$

$$F = \left(16R_c^2 - \alpha^2 + 4q^2\right)\left(4R_c + \alpha\right)$$
$$H = 4R_c\left(4R_c + \alpha + 2q\right)\left(4R_c + \alpha - 2q\right)$$
$$I = 64\left(4R_c + \alpha + 2q\right)\left(4R_c + \alpha - 2q\right)$$
$$J = \left(16R_c^2 - \alpha^2 + 4q^2\right)\left(4\alpha R_c + \alpha^2 - 4q^2\right)$$
$$K = 16\left(16R_c^2 - \alpha^2 + 4q^2\right)\left(4q^2 - 4\alpha R_c - \alpha^2\right)$$
$$L = \left[\alpha(4R_c + \alpha)^2 - 4q^2(8R_c + \alpha)\right]$$
$$Z = \left(4R_c + \alpha + 2q\right)^4\left(4R_c + \alpha - 2q\right)^4$$

$$\hat{R}_{0202} = \frac{-8R_c\left(16R_c^2 - \alpha^2 + 4q^2\right)L}{64Z}$$

$$\hat{R}_{1212} = -\frac{(N-O)}{4^4 R_c^4}$$

$$\mathrm{N} = \left(16R_c^2 - \alpha^2 + 4q^2\right)\left[(4R_c + \alpha)(12R_c + \alpha) - 4q^2\right]$$
$$O = \left[(4R_c + \alpha)^2 - 4q^2\right]\left(48R_c^2 - \alpha^2 + 4q^2\right)$$

$$\hat{R}_{2323} = -\frac{(P-Q)\sin^2\theta}{4^4 R_c^2}$$

$$P = \left[(4R_c + \alpha)^2 - 4q^2\right]^2$$
$$Q = \left(16R_c^2 - \alpha^2 - 4q^2\right)^2$$

$$\hat{G}_{0101} = -\frac{\left(16R_c^2 - \alpha^2 + 4q^2\right)^2}{4^4 R_c^4}$$
$$\hat{G}_{0202} = -\frac{\left(16R_c^2 - \alpha^2 + 4q^2\right)^2}{4^4 R_c^2} \qquad (53)$$
$$\hat{G}_{1212} = \frac{\left(4R_c + \alpha + 2q\right)^4\left(4R_c + \alpha - 2q\right)^4}{4^8 R_c^6}$$
$$\hat{G}_{2323} = \frac{\left(4R_c + \alpha + 2q\right)^4\left(4R_c + \alpha - 2q\right)^4 \sin^2\theta}{4^8 R_c^4}$$

When $r = r_o, R_c = \xi = \sqrt{\alpha^2 - 4q^2}\,/\,4$ for all r_o and for all n and the Riemannian curvature (53) becomes:

$$\hat{K}_S = \frac{\hat{A}\hat{W}_{0101} + \hat{C}\left(\hat{W}_{1212} + \hat{W}_{1313}\sin^2\theta\right) - \hat{E}\hat{W}_{2323}}{\hat{H}\left(\hat{W}_{1212} + \hat{W}_{1313}\sin^2\theta\right) + \hat{I}\hat{W}_{2323}}$$

$$\hat{A} = \frac{\hat{J}\hat{L}\hat{M} - 64\hat{L}^2}{\hat{M}^4}$$
$$\hat{J} = 64\sqrt{\alpha^2 - 4q^2}$$
$$\hat{L} = \alpha\left(\sqrt{\alpha^2 - 4q^2} + \alpha\right)^2 - 4q^2\left(2\sqrt{\alpha^2 - 4q^2} + \alpha\right)$$
$$\hat{M} = \left(\sqrt{\alpha^2 - 4q^2} + \alpha + 2q\right)\left(\sqrt{\alpha^2 - 4q^2} + \alpha - 2q\right)$$
$$\hat{C} = \frac{4\left(\alpha^2 - 4q^2 + \alpha\sqrt{\alpha^2 - 4q^2}\right)}{\left(\alpha^2 - 4q^2\right)}$$
$$\hat{E} = \frac{\left(\alpha^2 - 4q^2 + \alpha\sqrt{\alpha^2 - 4q^2}\right)^2 \sin^2\theta}{4\left(\alpha^2 - 4q^2\right)}$$
$$\hat{H} = \frac{\left[\left(\sqrt{\alpha^2 - 4q^2} + \alpha\right)^2 - 4q^2\right]^4}{4^2\left(\alpha^2 - 4q^2\right)^3}$$
$$\hat{I} = \frac{\left[\left(\sqrt{\alpha^2 - 4\mathrm{q}^2} + \alpha\right)^2 - 4\mathrm{q}^2\right]^4 \sin^2\theta}{4^4\left(\alpha^2 - 4\mathrm{q}^2\right)^2}$$

$$\hat{W}_{ijkl} = \begin{vmatrix} \hat{U}^i & \hat{U}^j \\ \hat{V}^i & \hat{V}^j \end{vmatrix} \begin{vmatrix} \hat{U}^l & \hat{U}^l \\ \hat{V}^k & \hat{V}^l \end{vmatrix} \qquad (54)$$

Expressions (54) depend upon the direction vectors $\hat{U}^i$ and $\hat{V}^i$ and also upon the curvilinear coordinate θ. Accordingly, at $\theta = 0$ and $\theta = \pi$, (53) becomes:

$$\hat{K}_S = \frac{\hat{R}_{0101}\hat{W}_{0101} + \hat{R}_{0202}\hat{W}_{0202} + \hat{R}_{1212}\hat{W}_{1212}}{\hat{G}_{0101}\hat{W}_{0101} + \hat{G}_{0202}\hat{W}_{0202} + \hat{G}_{1212}\hat{W}_{1212}} \qquad (55)$$

With all quantities therein given by (53). Then when $r = r_o$ (54) reduces to:

$$\hat{K}_S = \frac{\hat{A}\hat{W}_{0101} + \hat{C}\hat{W}_{1212}}{\hat{H}\hat{W}_{1212}} \qquad (56)$$

With all quantities therein given by (53).
Similarly, at $\theta = \pi/2$, (53) becomes,

$$\hat{K}_S = \frac{\begin{bmatrix} \hat{R}_{0101}\hat{W}_{0101} + \hat{R}_{0202}\left(\hat{W}_{0202} + \hat{W}_{0303}\right) + \\ + \hat{R}_{1212}\left(\hat{W}_{1212} + \hat{W}_{1313}\right) + \hat{R}_{2323}\hat{W}_{2323} \end{bmatrix}}{\begin{bmatrix} \hat{G}_{0101}\hat{W}_{0101} + \hat{G}_{0202}\left(\hat{W}_{0202} + \hat{W}_{0303}\right) + \\ + \hat{G}_{1212}\left(\hat{W}_{1212} + \hat{W}_{1313}\right) + \hat{G}_{2323}\hat{W}_{2323} \end{bmatrix}} \qquad (57)$$

With all quantities therein given by (53).

If $q = 0$, (53) to (57) reduce to the values for the isotropic Schwarzschild form, which again shows that the Reissner-Nordström ground-form cannot be extended.

For a full analysis (Crothers, 2014b).

The Acceleration Invariant

Doughty (1981) obtained the following expression for the acceleration ω of a point along a radial geodesic for the static spherically symmetric line-elements

$$\omega = \frac{\sqrt{-g_{11}}\left(-g^{11}\right)\left|\frac{\partial g_{00}}{\partial r}\right|}{2g_{00}} \tag{58}$$

By means of (32) for the spherically symmetric spacetimes, the acceleration is given by:

$$\omega = \frac{\sqrt{-g_{11}}\left(-g^{11}\right)\left|\frac{\partial g_{00}}{\partial R_c}\right|}{2g_{00}}$$

$$R_c = \left(\left|r - r_o\right|^n + \xi^n\right)^{1/n};\ r, r_o \in R,\ n \in R^+ \tag{59}$$
$$\xi = \frac{\alpha}{2} + \sqrt{\frac{\alpha^2}{4} - q^2},\quad q^2 < \frac{\alpha^2}{4}$$

where, the components of the metric tensor are functions of $R_c(r)$. Consequently, the acceleration for the Reissner-Nordström ground-form is given by:

$$\omega = \frac{\left|\alpha R_c - 2q^2\right|}{2R_c^2\sqrt{R_c^2 - \alpha R_c + q^2}} \tag{60}$$

Since $q^2 < \alpha^2/4$, (60) becomes:

$$\omega = \frac{\alpha R_c - 2q^2}{2R_c^2\sqrt{R_c^2 - \alpha R_c + q^2}}$$

$$R_c = \left(\left|r - r_o\right|^n + \xi^n\right)^{1/n};\ r, r_o \in R,\ n \in R^+$$

$$\xi = \frac{\alpha}{2} + \sqrt{\frac{\alpha^2}{4} - q^2} \qquad q^2 < \frac{\alpha^2}{4} \tag{61}$$

When $q = 0$ (61) reduces to the acceleration for the Schwarzschild ground-form:

$$\omega = \frac{\alpha}{2R_c^2\sqrt{1 - \frac{\alpha}{R_c}}} \tag{62}$$

$$R_c = \left(\left|r - r_o\right|^n + \alpha^n\right)^{1/n};\ r, r_o \in R,\ n \in R^+$$

According to (61) and (62), whether or not $q = 0$, $r \to r_o \Rightarrow \omega \to \infty$, which constitutes an invariant condition and therefore reaffirms that the Schwarzschild and Reissner-Nordström forms cannot be extended and hence do not to produce black holes.

For the isotropic Schwarzschild ground-form the acceleraion is given by:

$$\omega = \frac{128R_c^2}{\left(4R_c + \alpha\right)^3\left(4R_c - \alpha\right)}$$

$$R_c = \left[\left|r - r_0\right|^n + \left(\frac{\alpha}{4}\right)^n\right]^{1/n} \tag{63}$$

$$r, r_0 \in R n \in R^+$$

The isotropic Schwarzschild acceleration invariant is then:

$$\left(r \to r_o\right) \Rightarrow \left(R_c \to \frac{\alpha}{4}\right) \Rightarrow \left(\omega \to \infty\right) \tag{64}$$
$$\forall r_o\ \forall n$$

just as for the case of the Schwarzschild ground-form.

For the isotropic Reissner-Nordström ground-form the acceleration is given by:

$$\omega = \frac{8R_C^2\left(A + B\right)}{CD}$$
$$A = 64\left(4R_c + \alpha + 2q\right)\left(4R_c + \alpha - 2q\right)R_c$$
$$B = -16\left(16R_c^2 - \alpha^2 + 4q^2\right)\left(4R_c + \alpha\right)$$
$$C = \left(4R_c + \alpha + 2q\right)^2\left(4R_c + \alpha - 2q\right)^2$$
$$D = \left(16R_c^2 - \alpha^2 + 4q^2\right)$$
$$\mathrm{R}_c = \left(\left|r - r_0\right|^n + \xi^n\right)^{1/n}$$

$$\xi = \frac{\sqrt{\alpha^2 - 4q^2}}{4} \qquad q^2 < \frac{\alpha^2}{4} \tag{65}$$

The isotropic Reissner-Nordström acceleration invariant is then:

$$\left(r \to r_o\right) \Rightarrow \left(R_c \to \xi\right) \Rightarrow \left(\omega \to \infty\right) \tag{66}$$
$$\forall r_o\ \forall n$$

just as for the case of the isotropic Schwarzschild ground-form, the Schwarzschild ground form and the Reissner-Nordström ground-form.

The Kretschmann Scalar

Cosmology argues, without proof, that a 'physical' singularity can only occur where the Riemann tensor

scalar curvature invariant, also called the Kretschmann scalar, is unbounded (i.e., infinite spacetime curvature). The Kretschmann scalar f is given by $f = R_{\mu\nu\sigma\tau}R^{\mu\nu\sigma\tau}$. However, since the 'black hole' metrics cannot be extended, there is no such curvature singularity. The Kretschmann scalar is always a finite scalar invariant corresponding to a metric ground form.

The Kretschmann scalar for the Kerr-Newman ground-form is:

$$f = \frac{8}{\left(R_c^2 + a^2\cos^2\theta\right)^6}\left(Y_1 - Y_2 + Y_3\right)$$

$$Y_1 = \frac{3\alpha^2}{2}\left(\begin{array}{r} R_c^6 - 15a^2R_c^4\cos^2\theta + \\ +15a^4R_c^2\cos^4\theta - \\ -a^6\cos^6\theta \end{array}\right)$$

$$Y_2 = 6\alpha q^2 R_c\left(\begin{array}{r} R_c^4 - 10a^2R_c^2\cos^2\theta + \\ +5a^4\cos^4\theta \end{array}\right)$$

$$Y_3 = q^4\left(7R_c^4 - 34a^2R_c^2\cos^2\theta + 7a^4\cos^4\theta\right)$$

$$R_c = \left(\left|r - r_0\right|^n + \xi^n\right)^{1/n}$$

$$\xi = \frac{\alpha}{2} + \sqrt{\frac{\alpha^2}{4} - q^2 - a^2\cos^2\theta} \quad , \quad a^2 + q^2 < \frac{\alpha^2}{4}$$

$$r, r_o \in R, n \in R^+ \tag{67}$$

According to (67), at $r = r_o$, $R_c = \xi$ and so (67) becomes:

$$f = \frac{8}{\left(\xi^2 + a^2\cos^2\theta\right)^6}\left(Y_1 - Y_2 + Y_3\right)$$

$$Y_1 = \frac{3\alpha^2}{2}\left(\begin{array}{r} \xi^6 - 15a^2\xi^4\cos^2\theta + \\ +15a^4\xi^2\cos^4\theta - \\ -a^6\cos^6\theta \end{array}\right) \tag{68}$$

$$Y_2 = 6\alpha q^2\xi\left(\begin{array}{r} \xi^4 - 10a^2\xi^2\cos^2\theta + \\ +5a^4\cos^4\theta \end{array}\right)$$

$$Y_3 = q^4\left(7\xi^4 - 34a^2\xi^2\cos^2\theta + 7a^4\cos^4\theta\right)$$

The Kretschmann scalar (68) is finite, irrespective of the values of r_o and n. Note that (68) depends upon θ. When $\theta = 0$ and when $\theta = \pi$:

$$f = \frac{8}{\left(\xi^2 + a^2\right)^6}\left(Y_1 - Y_2 + Y_3\right)$$

$$Y_1 = \frac{3\alpha^2}{2}\left(\xi^6 - 15a^2\xi^4 + 15a^4\xi^2 - a^6\right) \tag{69}$$

$$Y_2 = 6\alpha q^2\xi\left(\xi^4 - 10a^2\xi^2 + 5a^2\right)$$

$$Y_3 = q^4\left(7\xi^4 - 34a^2\xi^2 + 7a^4\right)$$

When $\theta = \pi/2$ (68) reduces to:

$$f = \frac{8}{\xi^8}\left[\frac{3\alpha^2\xi^2}{2} - 6\alpha q^2\xi + 7q^4\right] \tag{70}$$

Note that (70) does not contain the 'angular momentum' term a and that (70) is precisely that for the Reissner-Nordström ground-form (Crothers, 2005b; 2014b).

Expressions (67) reduce to the Kerr form when $q = 0$, thus:

$$f = \frac{4}{\left(\xi^2 + a^2cos^2\theta\right)^6}\left[3\alpha^2\left(\begin{array}{r} \xi^6 - 15a^2\xi^4cos^2\theta + \\ +15a^4\xi^2cos^4\theta - \\ -a^6cos^6\theta \end{array}\right)\right]$$

$$R_c = \left(\left|r - r_0\right|^n + \xi^n\right)^{1/n}$$

$$\xi = \frac{\alpha}{2} + \sqrt{\frac{\alpha^2}{4} - a^2\cos^2\theta}, \quad a^2\cos^2\theta < \frac{\alpha^2}{4} \tag{71}$$

$$r, r_o \in R, n \in R^+$$

This too depends upon the value of θ. When $\theta = 0$ and when $\theta = \pi$, (71) becomes:

$$f = \frac{4}{\left(\xi^2 + a^2\right)^6}\left[3\alpha^2\left(\xi^6 - 15a^2\xi^4 + 15a^4\xi^2 - a^6\right)\right] \tag{72}$$

When $\theta = \pi/2$ (71) reduces to:

$$f = \frac{12}{\alpha^4} \tag{73}$$

which is precisely the scalar curvature invariant for the Schwarzschild ground-form. Indeed, when $a = 0$ and $q = 0$, expressions (68) reduce to those for the Schwarzschild ground-form (Crothers, 2005a; 2014b):

$$f = \frac{12}{\alpha^4}$$

The Kretschmann scalar is finite in every case and so there are in fact no curvature singularities anywhere.

Black Hole Universes and Big Bang Universes in Contrast

There are four different types of black hole universes alleged by cosmology; (a) non-rotating charge neutral, (b) non-rotating charged, (c) rotating charge neutral, (d) rotating charged. Black hole masses or 'sizes', are not types, just masses or sizes of the foregoing types. There are three purported types of big bang universes and they are characterised by their constant k curvatures; (a) k = -1, negative spacetime curvature and spatially infinite, (b) k = 0, flat spacetime and spatially infinite, (c) k = 1, positive spacetime curvature and spatially finite. Compare now the generic defining characteristics of all black hole universes with those of all big bang universes (Crothers, 2013b).

All black hole universes:

- Are spatially infinite
- Are eternal
- Contain only one mass
- Are not expanding (i.e., are not non-static)
- Are either asymptotically flat or asymptotically curved

All big bang universes:

- Are either spatially finite (1 case; k = 1) or spatially infinite (2 different cases; k = -1, k = 0)
- Are of finite age (~13.8 billion years)
- Contain radiation and many masses
- Are expanding (i.e., are non-static)
- Are not asymptotically anything

Note also that no black hole universe even possesses a big bang universe k-curvature.

Comparison of the defining characteristics of all black hole universes with all big bang universes immediately reveals that they are contradictory and so they are mutually exclusive; they can't co-exist. No proposed black hole universe can be superposed with any other type of black hole universe, with any big bang universe, or with itself. Similarly, no proposed type of big bang universe can be superposed with any other type of big bang universe, with any black hole universe, or with itself.

Furthermore, General Relativity is a nonlinear theory and so the Principle of Superposition is invalid therein. Let $\boldsymbol{X}$ be some black hole universe and $\boldsymbol{Y}$ be some big bang universe. Then the linear combination (i.e., superposition) $\boldsymbol{X} + \boldsymbol{Y}$ is not a universe. Indeed, $\boldsymbol{X}$ and $\boldsymbol{Y}$ pertain to completely different sets of Einstein field equations and so they have absolutely nothing to do with one another.

Despite the contradictory nature of the defining characteristics of black hole universes and big bang universes and despite the fact that the Principle of Superposition is invalid in General Relativity, cosmology superposes to produce multiple unspecified black holes within an unspecified big bang universe.

According to cosmology the finite mass of a black hole is concentrated in its 'singularity', where volume is zero, density is infinite and spacetime curvature is infinite. This singularity is said to be not merely a place in the equations where the equations are undefined, but is a real physical object. Now gravity is not a force in General Relativity, because it is spacetime curvature. Thus, according to cosmology, a finite mass produces infinite gravity. However, no finite mass can have zero volume and infinite density and no finite mass can produce infinite gravity anywhere.

A black hole constitutes an independent universe because its spacetime is spatially infinite; its spacetime is not contained within its 'event horizon'. The spacetime of a black hole is either asymptotically flat or asymptotically curved. There is no bound on asymptotic, for otherwise it would not be asymptotic. The Schwarzschild, Reissner-Nordström, Kerr and Kerr-Newman spacetimes are all asymptotically flat. Without the asymptotic condition the black hole equations do not even obtain.

Cosmology routinely claims that Newton's theory predicts black holes, derived by Michell and Laplace.

> "*Laplace essentially predicted the black hole…*" *(Hawking and Ellis, 1973)*

> "*Eighteenth-century speculators had discussed the characteristics of stars so dense that light would be prevented from leaving them by the strength of their gravitational attraction; and according to Einstein's General Relativity, such bizarre objects (today's 'black holes') were theoretically possible as end-products of stellar evolution, provided the stars were massive enough for their inward gravitational attraction to overwhelm the repulsive forces at work*" (*Michael and Hoskin, 1997)*

> "*Two important arrivals on the scene: The neutron star (1933) and the black hole (1795, 1939)*" *(Misner et al., 1973)*

> "*That such a contingency can arise was surmised already by Laplace in 1798. Laplace argued as follows. For a particle to escape from the surface of a spherical body of mass*

> *M and radius R, it must be projected with a velocity v such that* $\frac{1}{2}v^2 > GM/R$; *and it cannot escape if* $v^2 < 2GM/R$. *On the basis of this last inequality, Laplace concluded that if* $R < 2GM/c^2 = R_s$ *(say) where c denotes the velocity of light, then light will not be able to escape from such a body and we will not be able to see it!*
>
> "*By a curious coincidence, the limit* R_s *discovered by Laplace is exactly the same that general relativity gives for the occurrence of the trapped surface around a spherical mass.*" (*Chandrasekhar, 1972)*

But it is not "*a curious coincidence*" that General Relativity gives the same R_s "*discovered by Laplace*" because the Newtonian expression for escape speed was deliberately inserted *post hoc* into Hilbert's solution in order to make a mass appear in equations that contain no *material sources* by mathematical construction.

The theoretical Michell-Laplace dark body is not a black hole; it possesses an escape velocity at its surface, but the black hole has both an escape velocity and no escape velocity simultaneously at its 'surface' (i.e. event horizon); masses and light can leave a Michell-Laplace dark body, but nothing can leave a black hole; it does not require irresistible gravitational collapse to form, whereas a black hole does (unless it is 'primordial'); it has no (infinitely dense) singularity, but a black hole does; it has no event horizon, but a black hole does; it has 'infinite gravity' nowhere, but a black hole has infinite gravity at its singularity; there is always a class of observers that can see a Michell-Laplace dark body, but there is no class of observers that can see a black hole (McVittie, 1978); the Michell-Laplace dark body persists in a space which can contain other Michell-Laplace dark bodies and other matter, but the spacetimes of all types of black hole universes permit no other black holes and no other masses; the Principle of Superposition holds for Michell-Laplace dark bodies but not for black hole universes; the space of a Michell-Laplace dark body is 3-dimensional and Euclidean, but a black hole universe is a 4-dimensional non-Euclidean (pseudo-Riemannian) spacetime; the space of a Michell-Laplace dark body is not asymptotically anything whereas the spacetime of a black hole is asymptotically flat or asymptotically curved; a black hole constitutes an independent universe, but a Michell-Laplace dark body does not; a Michell-Laplace dark body does not 'curve' a spacetime, but a black hole does (a Michell-Laplace dark body exerts a force of gravity but a black hole does not possess a gravitational force). Therefore, a Michell-Laplace dark body does not possess the characteristics of the black hole and so it is not a black hole.

Conclusion

Cosmology has failed in its few attempts to produce a means by which an infinite set of equivalent metrics that lead to black holes can be generated. Nonetheless, such metric ground-forms have in fact been obtained. These ground-forms prove that no metric can be extended by any means to produce a black hole universe. All methods used to extend metrics to produce black hole universes are invalid.

Black hole universes are produced by invalid mathematical operations and inadmissible insinuation of the Newtonian expression for escape speed, from which the notion of black hole escape velocity was obtained along with the black hole event horizon and its 'radius'. Black holes have and do not have an escape velocity simultaneously at the same place, the event horizon; but this is impossible.

Multiple black holes within a big bang universe have been produced by applying the Principle of Superposition. However, the Principle of Superposition does not hold in General Relativity. Such superpositions are therefore invalid. Black hole universes and big bang universes are mutually exclusive by their very definitions.

The finite mass of a black hole produces infinite gravity (infinite spacetime curvature) at the black hole singularity. However, no finite mass can produce infinite gravity anywhere. Similarly, no finite mass can have zero volume and infinite density.

Newton's theory does not predict black holes. The Michell-Laplace dark body is not a black hole because, other than mass, it shares none of the properties of a black hole.

The metric ground-forms herein prove that black holes have no scientific basis. All reports of black holes being discovered have no scientific merit.

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Dedication

I dedicate this paper to my brother.

Paul Raymond Crothers

12th May 1968-25th December 2008

Ethics

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