On unusual interactions of the $(1/2,0)\oplus(0,1/2)$ particles

Valeri V. Dvoeglazov
Escuela de Física, Universidad Autónoma de Zacatecas
Apartado Postal C-580, Zacatecas 98068, ZAC., México
Internet address: valeri@cantera.reduaz.mx
URL: http://cantera.reduaz.mx/˜ valeri/valeri.htm

Abstract

We continue to study the "fermion – 4-vector potential"interactions in the framework of the McLennan-Case construct which is a reformulation of the Majorana theory of the neutrino. This theory is shown after applying Majorana-like *anzatzen* to give rise to appearance of unusual terms as $\boldsymbol{\sigma}\cdot[\mathbf{A}\times\mathbf{A}^*]$, which were recently discussed in non-linear optics.

As a result of extracting solid data certifying the existence of the mass of neutrino in the LANL experiment [1] the interest in the Majorana-like models has grown considerably. The McLennan-Case reformulation [2] of the Majorana theory [3] got further development in the papers of Ahluwalia [4] and myself [5,6]. In 1996 I received private communications from D. V. Ahluwalia [7] about unusual interactions of neutral particles in his model which is closely related with the Case consideration. Even before I learnt about the possible importance of phase factors of corresponding field functions in defining the structure of the mass term [8]. They gave initial impulse in writing this work. Further investigations from different standpoints [10] (compare also with results of non-linear optics [9]) produced simultaneously with this work were also very incentive in my attempts to solve the problem *rigorously*.[1]

The main result of the present paper is the theoretical proof of possible significance of the term $\boldsymbol{\sigma}\cdot[\mathbf{A}\times\mathbf{A}^*]$ in the interaction of $(1/2,0)\oplus(0,1/2)$ fermions. In the process of calculations we use the notation and the metric of ref. [2b]. The Dirac equation is written

$$(\gamma^\mu\partial_\mu+\kappa)\psi=0, \tag{1}$$

where $g^{\mu\nu}=\mathrm{diag}(-1,\,1,\,1,\,1)$ and γ^μ are the Dirac matrices. Their explicit form can be chosen as follows

$$\gamma^0=\begin{pmatrix}0 & -i\\ -i & 0\end{pmatrix},\quad \gamma^i=\begin{pmatrix}0 & i\sigma^i\\ -i\sigma^i & 0\end{pmatrix},\quad \gamma^5=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}. \tag{2}$$

The Pauli charge-conjugation 4×4 matrix is then

$$C=\begin{pmatrix}0 & \Theta\\ -\Theta & 0\end{pmatrix},\ \text{where}\ \Theta=\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}. \tag{3}$$

It has the properties

$$C=C^T,\quad C^*=C^{-1}, \tag{4}$$

$$C\gamma^\mu C^{-1}={\gamma^\mu}^*,\quad C\gamma^5C^{-1}=-{\gamma^5}^*. \tag{5}$$

As opposed to K. M. Case we introduce the interaction with the 4-vector potential in the beginning and substitute $\partial_\mu\to\nabla_\mu=\partial_\mu-ieA_\mu$ in the equation (1). For the sake of generality we assume that the 4-vector potential is a *complex* field what is the extension of this concept comparing with the usual quantum-field consideration. (In the classical (quantum) field theory the 4-vector potential in the coordinate representation is usually considered to be *pure real* function (functional) unless we touch the matters of indefinite metrics). After introducing projections onto subspaces of the *chirality* quantum number

$$\psi_\pm=\frac{1}{2}(1\pm\gamma^5)\psi,\quad \gamma^\mu_\pm=\frac{1}{2}(1\pm\gamma^5)\gamma^\mu, \tag{6}$$

[1] Obviously, the Evans *et al.* derivation of similar terms [“*The Enigmatic Photon. Vol. 3*"(Kluwer Academic, Dordrecht, 1996), pp. 9-16, 187-189] has no any sense in the presented form. It should be regarded as completely erroneous until that time when needed clarifications and corrections would be given . But, the Esposito derivation of the term $\sim\boldsymbol{\sigma}\cdot[\mathbf{A}\times\mathbf{A}^*]$ is correct. I am grateful to him for sending me the alternative proof before the publication.

we re-write the equation (1) and Eq. (3) of ref. [2b]

$$(\gamma^\mu \nabla^*_\mu + \kappa) C^{-1}\psi^* = 0, \qquad (7)$$

which already describe the interactions of (anti) fermion with the *complex* 4-vector potential, to the following set

$$\gamma^\mu_+ \nabla_\mu \psi_- + \kappa\psi_+ = 0, \qquad (8)$$
$$\gamma^\mu_- \nabla_\mu \psi_+ + \kappa\psi_- = 0, \qquad (9)$$
$$\gamma^\mu_+ \nabla^*_\mu C^{-1}\psi^*_+ + \kappa C^{-1}\psi^*_- = 0, \qquad (10)$$
$$\gamma^\mu_- \nabla^*_\mu C^{-1}\psi^*_- + \kappa C^{-1}\psi^*_+ = 0. \qquad (11)$$

On using the matrices $\eta^\mu = C\gamma^\mu_-$ and $\eta^{\mu^*} = \gamma^\mu_+ C^{-1}$ and $\varphi = \psi_+ = \frac{1}{2}(1+\gamma^5)\psi$, $\chi = C^{-1}\psi^*_-$ we obtain

$$\eta^{\mu^*}\nabla_\mu \chi^* + \kappa\varphi = 0, \qquad (12)$$
$$\eta^\mu \nabla_\mu \varphi + \kappa\chi^* = 0, \qquad (13)$$
$$\eta^{\mu^*}\nabla^*_\mu \varphi^* + \kappa\chi = 0, \qquad (14)$$
$$\eta^\mu \nabla^*_\mu \chi + \kappa\varphi^* = 0, \qquad (15)$$

in the sub-space of the *positive* chirality quantum number. And with the matrices $\zeta^\mu = \gamma^\mu_- C^{-1}$, $\zeta^{\mu^*} = C\gamma^\mu_+$ and the notation $\eta = \psi_-$, $\xi = C^{-1}\psi^*_+$ we obtain the set

$$\zeta^{\mu^*}\nabla_\mu \eta + \kappa\xi^* = 0, \qquad (16)$$
$$\zeta^\mu \nabla_\mu \xi^* + \kappa\eta = 0, \qquad (17)$$
$$\zeta^{\mu^*}\nabla^*_\mu \xi + \kappa\eta^* = 0, \qquad (18)$$
$$\zeta^\mu \nabla^*_\mu \eta^* + \kappa\xi = 0, \qquad (19)$$

for the *negative* chirality quantum number. One can use four equations of these sets to describe the physical system. If now apply the Majorana condition given by Case $\psi_- = C^{-1}\psi^*_+$ (see Eq. (8) in [2b]) one can arrive at $\chi = \varphi$ and

$$\nabla^*_\mu \varphi \equiv \nabla_\mu \varphi, \quad \text{hence, } A_\mu = -A^*_\mu \qquad (20)$$

as a consequence of the compatibility condition of the set of equations (12-15). The 4-potential becomes to be *pure imaginary.* This model seems to be perfectly possible after redefining the phase factor between positive- and negative- energy solutions in the field operator of the 4-vector potential.

Furthermore, it is difficult to extract the new physical content from the modification of the Majorana *anzatz* such that $\psi_- = e^{i\alpha(x)}C^{-1}\psi^*_+$ and, therefore, $\chi = e^{-i\alpha(x)}\varphi$. We come to

$$\eta^\mu \nabla_\mu \varphi + \kappa e^{i\alpha(x)}\varphi^* = 0, \qquad (21)$$
$$\eta^{\mu^*}\nabla^*_\mu \varphi^* + \kappa e^{-i\alpha(x)}\varphi = 0, \qquad (22)$$
$$\text{and} \quad \partial_\mu \alpha = e(A_\mu + A^*_\mu), \qquad (23)$$

thus recovering (with $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]_-$)

$$\left[\nabla^\mu \nabla_\mu - i\sigma^{\mu\nu}\nabla_\mu \nabla_\nu - \kappa^2\right]\varphi = 0, \qquad (24)$$

and its complex conjugate.

From the above consideration it seems that we failed to derive the needed term. But, we wish to insist on the general case. In order to proceed let us observe that in the set (12-15) and (16-19) the second and the third equations of each set are complex conjugates each other; the first equation and the fourth equation are also complex conjugates each other. If we do not want to introduce such strong restrictions on the 4-vector potential as above it is logical to introduce different Majorana-like *anzatzen* for these subsets. This is perfectly possible after one reminds that the subspaces of different CP quantum number are *independent* ones for certain states. On this basis, firstly, we re-write the Dirac equation and its charge conjugate to another set (φ and χ are two-component spinors):

$$\eta^\mu \nabla_\mu \varphi + \kappa C\varphi = 0, \qquad (25)$$
$$\eta^{\mu^*}\nabla^*_\mu \varphi^* + \kappa C^{-1}\varphi^* = 0, \qquad (26)$$
$$\eta^\mu \nabla^*_\mu \chi + \kappa C\chi = 0, \qquad (27)$$
$$\eta^{\mu^*}\nabla_\mu \chi^* + \kappa C^{-1}\chi^* = 0. \qquad (28)$$

Next, we set up the following *anzatz*

$$C^{-1}\psi^*_\pm = \mp P\psi_\pm \,,$$

where P is the space inversion operator. Finally, marking the resulting subsets of equation by some discrete quantum number (we denote them as "s"and "a") one obtains[2]

$$\eta^\mu \nabla^*_\mu \chi_s + \kappa\chi^*_s = 0, \qquad (30)$$
$$\eta^{\mu^*}\nabla_\mu \chi^*_s + \kappa\chi_s = 0. \qquad (31)$$

and

$$\eta^\mu \nabla_\mu \varphi_a - \kappa\varphi^*_a = 0, \qquad (32)$$
$$\eta^{\mu^*}\nabla^*_\mu \varphi^*_a - \kappa\varphi_a = 0. \qquad (33)$$

[2] One could also obtain similar subsets of equations after the application of the modified Majorana *anzatz*

$$\psi_- = \wp_{S,A} C^{-1}\psi^*_+ . \qquad (29)$$

Here, $\wp_{S,A} = \pm 1$; the upper sign being used for the first subset, Eqs. (13,14) and the down sign being used for the second subset, Eqs. (16,19). This is possible because the sub-spaces of different chirality quantum numbers can also be considered as the independent subspaces and we can choose any two equations from the both subsets. But, this explanation can be still considered as obscure by someone due to the discussion in the first part of the Letter. In my opinion, the proper consideration of the theory of 4-vector potential is necessary to clarify this point.

As a result we obtain second-order equations for χ_s and φ_a:

$$\left[\nabla^\mu\nabla^*_\mu - i\sigma^{\mu\nu}\nabla_\mu\nabla^*_\nu - \kappa^2\right]\chi_s(x^\mu) = 0, \quad (34)$$

$$\left[\nabla^{\mu^*}\nabla_\mu - i\sigma^{\mu\nu}\nabla^*_\mu\nabla_\nu - \kappa^2\right]\varphi_a(x^\mu) = 0, \quad (35)$$

and their complex conjugates.

One can proceed further with transformations of these equations to the accustomed forms. This is only an algebraic exercise. One can see the existence of "new terms" in the equations: we proved that some physical states of the spin-1/2 fermion have interactions of the form

$$i\epsilon^{ijk}\sigma^k\nabla_i\nabla^*_j \to +ie^2\boldsymbol{\sigma}\cdot[\mathbf{A}\times\mathbf{A}^*], \quad (36)$$

for "s" states, and with the inverse sign, for the "a" states. (The unit system $c=\hbar=1$ is used.)

At last we apparently note that the $(1/2,0)\oplus(0,1/2)$ field operator is naturally decomposed into the parts $\Psi = \psi_S + \psi_A$

$$\psi_S(x^\mu) = \int \frac{d^3\mathbf{p}}{(2\pi)^3 2E_p}\{[u_\uparrow(p^\mu)c_\uparrow(p^\mu) + u_\downarrow(p^\mu)d_\downarrow(p^\mu)]e^{-i\phi} + +[Cu^*_\uparrow(p^\mu)c^\dagger_\uparrow(p^\mu) + Cu^*_\downarrow(p^\mu)d^\dagger_\downarrow(p^\mu)]e^{+i\phi}\}, \quad (37)$$

$$\psi_A(x^\mu) = \int \frac{d^3\mathbf{p}}{(2\pi)^3 2E_p}\{[u_\uparrow(p^\mu)d_\uparrow(p^\mu) + u_\downarrow(p^\mu)c_\downarrow(p^\mu)]e^{-i\phi} - [Cu^*_\uparrow(p^\mu)d^\dagger_\uparrow(p^\mu) + Cu^*_\downarrow(p^\mu)c^\dagger_\downarrow(p^\mu)]e^{+i\phi}\}, \quad (38)$$

where $\phi = (Et - \mathbf{p}\cdot\mathbf{x})/\hbar$. As easily demonstrated both parts satisfy (separately each other) the Dirac equation. Certain relations between creation/annihilation operators are assumed. They are dictated by the modified Majorana-like *anzatzen.*

We discussed above the possibility of existence of longitudinal-type interactions. We also proved the existence of these states on the free level [11]. On the other hand, in a recent paper [12] we found that the possibility of terms as $\sim \boldsymbol{\sigma}\cdot[\mathbf{A}\times\mathbf{A}^*]$ appears to be related to the matters of chiral interactions. We briefly repeat these arguments below. As we are now convinced, the Dirac field operator can be always presented as a superposition of the self- and anti-self charge conjugate 'field operators'. The anti-self charge conjugate part can give the self charge conjugate part after multiplying by the γ^5 matrix and *vice versa.* We derived therein[3]

$$[i\gamma^\mu D^*_\mu - m]\psi^s_1 = 0, \quad (40)$$

or [4]

$$[i\gamma^\mu D_\mu - m]\psi^a_2 = 0, \quad (42)$$

Both equations lead to the terms of interaction such as $\sim \boldsymbol{\sigma}\cdot[\mathbf{A}\times\mathbf{A}^*]$ provided that the 4-vector potential is considered as a complex function(al). In fact, from (40) we have:

$$i\sigma^\mu\nabla_\mu\chi_1 - m\phi_1 = 0, \quad (43)$$

$$i\widetilde{\sigma}^\mu\nabla^*_\mu\phi_1 - m\chi_1 = 0. \quad (44)$$

And, from (42) we have

$$i\sigma^\mu\nabla^*_\mu\chi_2 - m\phi_2 = 0, \quad (45)$$

$$i\widetilde{\sigma}^\mu\nabla_\mu\phi_2 - m\chi_2 = 0. \quad (46)$$

The meanings of σ^μ and $\widetilde{\sigma}^\mu$ are obvious from the definition of γ matrices. The derivatives are defined:

$$D_\mu = \partial_\mu - ie\gamma^5 C_\mu + eB_\mu\,, \quad \nabla_\mu = \partial_\mu - ieA_\mu.$$

and $A_\mu = C_\mu + iB_\mu$.

From the above set we extract the terms as $\pm e^2\sigma^i_{Pauli}\sigma^j_{Pauli}A_iA^*_j$, which lead to the discussed terms [10, 12].

In the considered cases it is the γ^5 transformation which distinguishes various field configurations (helicity, self/anti-self charge conjugate properties etc) in the coordinate representation. We would also like to note that in the submitted Esposito-Recami paper the terms of the type $\sim \boldsymbol{\sigma}\cdot[\mathbf{A}\times\mathbf{A}^*]$ can be reduced to $(\boldsymbol{\sigma}\cdot\nabla)\mathcal{V}$, where $\mathcal{V}$ is the scalar potential.

Finally, I would like to present references to some works which, in my opinion, would be relevant to further discussions of the questions put forth here. Several works already revealed importance of the term $\boldsymbol{\sigma}\cdot[\mathbf{A}\times\mathbf{A}^*]$ in the non-linear optics. Other works are [13], where the concept of *two* coordiante-space Dirac equations have been re-discovered independently (cf. [6b,d,e] and [14, 15]); and ref. [16], where the matters of interface between gravity and quantum mechanics have been firstly discussed rigorously.

Acknowledgments. I acknowledge useful discussions with Profs. A. E. Chubykalo, A. Lakhtakia and A. F. Pashkov. I am obliged to Profs. D. V. Ahluwalia

[3] The anti-self charge conjugate field function ψ_2 can also be used. The equation has then the form:

$$[i\gamma^\mu D^*_\mu + m]\psi_2'^{\,,a} = 0. \quad (39)$$

[4] The self charge conjugate field function ψ_1 also can be used. The equation has the form:

$$[i\gamma^\mu D_\mu + m]\psi_1'^{\,,s} = 0. \quad (41)$$

As readily seen in the cases of alternative choices we have opposite "charges" in the terms of the type $\sim \boldsymbol{\sigma}\cdot[\mathbf{A}\times\mathbf{A}^*]$ and in the mass terms.

and M. W. Evans (even though I do *not* always agree with them), E. Recami and S. Esposito for useful information. In fact, this Letter is the continuation of their works.

I am grateful to Zacatecas University, México, for a professorship. This work has been partly supported by the Mexican Sistema Nacional de Investigadores, the Programa de Apoyo a la Carrera Docente and by the CONACyT, México under the research project 0270P-E.

Manuscript revised August 20, 2000.

References

[1] G. Garvey, APS News **6** (1997) 20 [The LSND Collaboration, Phys. Rev. Lett. **77** (1996) 3082; Phys. Rev. C**54** (1996) 2685].

[2] J. A. McLennan, Jr., Phys. Rev. **106** (1957) 821; K. M. Case, Phys. Rev. **107** (1957) 307.

[3] E. Majorana, Nuovo Cimento **14** (1937) 171.

[4] D. V. Ahluwalia, M. B. Johnson and T. Goldman, Mod. Phys. Lett. A**9** (1994) 439; Acta Phys. Polon. B**25** (1994) 1267; D. V. Ahluwalia, Int. J. Mod. Phys. A**11** (1996) 1855.

[5] V. V. Dvoeglazov, Helv. Phys. Acta **70** (1997) 677; ibid. 686; ibid. 697.

[6] V. V. Dvoeglazov, Rev. Mex. Fis. Suppl. **41** (1995) 159; Int. J. Theor. Phys. **34** (1995) 2467; ibid. **36** (1997) 635; Nuovo Cimento **108**A (1995) 1467; ibid. **111**B (1996) 483, ibid. **112**B (1997) 847

[7] D. V. Ahluwalia, private communications (Spring 1996).

[8] A. F. Pashkov, private communications (1984, 1995).

[9] P. S. Pershan, J. P. van der Ziel and L. D. Malmstrom, Phys. Rev. **143** (1966) 574; S. J. van Enk, Found. Phys. Lett. **9** (1996) 183 and references therein.

[10] S. Esposito, private communication (June 1997).

[11] V. V. Dvoeglazov, *Photon-Notoph Equations.* Preprint EFUAZ FT-97-53-REV, physics/9804010, March 1998.

[12] V. V. Dvoeglazov, *About* γ^5 *Chiral Interactions of Massive Particles in the* $(1/2,0)\oplus(0,1/2)$ *Representation.* Preprint EFUAZ FT-97-46, hep-th/9808148, Zacatecas, 1997.

[13] E. Recami et al., Phys. Lett. B**173** (1986) 233; ibid. B**188** (1987) E511; G. Ziino, Ann. Fond. L. de Broglie **14** (1989) 427; ibid **16** (1991) 343; A. O. Barut and G. Ziino, Mod. Phys. Lett. A**8** (1993) 1011; G. Ziino, Int. J. Mod. Phys. A**11** (1996) 2081.

[14] M. A. Markov, ZhETF **7** (1937) 579; ibid. 603; Nucl. Phys. **55** (1964) 130

[15] F. Belinfante and W. Pauli, Physica **7** (1940) 177

[16] D. V. Ahluwalia and C. Burgard, Gen. Rel. Grav. **28** (1996) 1161 (gr-qc/9603008; Essay awarded by the first prize of the GR Foundation, 1996); D. V. Ahluwalia, *On a New Non-Geometric Element in Gravity.* Preprint LANL LA-UR-97-864 (gr-qc/9705050; Essay awarded by the fourth prize of the GR Foundation, 1997), to be published.