HOW THE GENERALIZED MAXWELL EQUATIONS CAN BE DERIVED FROM THE EINSTEIN POSTULATE*

Valeri V. Dvoeglazov and Jesús A. Cázares Montes

Escuela de Física, Universidad Autónoma de Zacatecas

Apartado Postal C-580, Zacatecas 98068 Zac., México

valeri@ahobon.reduaz.mx

Abstract: We show that the Gersten derivation of Maxwell equations can be generalized. It actually leads to additional solutions of 'S=1 equations.'

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In the recent paper [1] the author (Dr. A. Gersten) studied the matrix representation of the Maxwell equations, both the Faraday and Ampere laws and the Gauss law. His consideration is based on equation (9) which is equivalent to the Einstein postulate on the relation between energy, momentum and mass.

$$((E/c)^2 - \mathbf{p}^2)\, \Psi = (E/c\, I^{(3)} - \mathbf{p}\cdot\mathbf{S}) (E/c\, I^{(3)} + \mathbf{p}\cdot\mathbf{S})\, \Psi$$

$$- \mathbf{p}\, (\mathbf{p}\cdot\Psi) = 0 \qquad \text{(Eq. (9) of ref. [1])}$$

Furthermore, he claimed that the solutions to this equation should be found from the set:

$$(E/c\, I^{(3)} + \mathbf{p}\cdot\mathbf{S})\, \Psi = 0 \qquad \text{(Eq. (10) of ref. [1])}$$

$$(\mathbf{p}\cdot\Psi) = 0 \qquad \text{(Eq. (11) of ref. [1])}$$

Thus, Gersten concluded that his equation (9) is equivalent to Maxwell's equations (10,11). As he also correctly indicated, such a formalism for describing S=1 fields has been considered by several authors before [2-10] with those authors mainly considering the dynamical of Maxwell's equations in the matrix form.

However, we note that equation (9) of [1] is satisfied also under the choice[A]

$$(E/c\, I^{(3)} + \mathbf{p}\cdot\mathbf{S})\, \Psi = \mathbf{p}\, \chi \quad (1)$$

$$(\mathbf{p}\cdot\Psi) = (E/c)\, \chi \quad (2)$$

with some arbitrary scalar function χ at this stage. This is due to the fact that[B]

$$(\mathbf{p}\cdot\mathbf{S})^{jk}\, \mathbf{p}^k = i\, \varepsilon^{jik}\, \mathbf{p}^i\, \mathbf{p}^k \equiv 0 \quad (3)$$

(or after quantum operator substitutions **rot grad** $\chi = 0$). Thus, the generalized coordinate-space Maxwell equations follow after the similar procedure as in [1]:

$$\nabla \times \mathbf{E} = -(1/c)\, \partial \mathbf{B}/\partial t + \nabla\, Im\, \chi \quad (4)$$

$$\nabla \times \mathbf{B} = +(1/c)\, \partial \mathbf{E}/\partial t + \nabla\, Re\, \chi \quad (5)$$

$$\nabla \cdot \mathbf{E} = -(1/c)\, \partial\, Re\, \chi / \partial t \quad (6)$$

$$\nabla \cdot \mathbf{B} = +(1/c)\, \partial\, Im\, \chi / \partial t \quad (7)$$

If one assumes that there are no monopoles, one may suggest that χ (x) is a *real* field and its derivatives play the role of charge and current densities. Thus, surprisingly, by using the Dirac-like procedure[C] of derivation of "free-space" relativistic quantum field equations, Gersten might have in fact come to the *inhomogeneous* Maxwell equations![D] Furthermore, I am not aware of any proofs that the scalar field χ (x) should be firmly connected with the charge and current densities, so there is sufficient room for interpretation. For instance, its time derivative and gradient may also be interpreted as leading to the 4-vector potential. In this case, we need some *mass*/*length* parameter such as in [11a,d]. Both these interpretations were present in the literature [9,11] (cf. also [12]).

It is therefore concluded that the *generalized* Maxwell equations (many versions of which have been proposed during the last 100 years, see, for instance, [13]) should be used as a guideline for proper interpretations of quantum theories.

Annotations

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A. We leave the analysis of possible functional non-linear (in general) dependence of χ and $\partial \chi / \partial x^\mu$ on the higher-rank tensor fields for future publications.

B. See the explicit form of the angular momentum matrices in Eq. (6) of the Gersten paper.

C. That is to say, on the basis of the relativistic dispersion relations

$(E^2 - c^2 \mathbf{p}^2 - m^2 c^4)\, \Psi = 0$, Eq. (1) of ref. [1].

D. One can also substitute $-(2 i h /c)\, \mathbf{j}$ and $-(2 i h)\, \rho$ in the right hand side of (1,2) of the present paper and obtain equations for the current and the charge density

$(1/c)\, \nabla \times \mathbf{j} = 0$ y $(1/c^2)\, \partial \mathbf{j}/\partial t + \nabla \rho = 0$

which coincide with equations (13,17) of [9b]. The interesting question is: whether such defined **j** and ρ may be related to $\partial \chi / \partial x^\mu$.

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