About it how to space-time was eliminated from the General Relativity

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Abstract: *General Relativity is a theory which a since about 100 years describes the gravitational phenomena as geometric properties of the space-time. What happens if the space-time will be eliminated and replaced with the material medium. We discuss physical consequences of a such exchange.*

keywords: *general theory of relativity; modified theories of gravity; effective mass density tensor; m(GR) theory*

Introduction

General Relativity (GR) is a theory which a since about 100 years describes the gravitational phenomena as geometric properties of the space-time. Although GR is widely accepted as a fundamental theory of gravitation for the many physicists still this is not a perfect theory.

In GR the space-time plays a very important role. The space-time continuum is a mathematical model that joins three-dimensional space and one dimension time into a single idea, the four-dimensional space-time. Under influence outer gravitational field the space-time is curved. The gravitational wave is the ripple in the curvature of the space-time that propagates as a wave.

Alternative description of the gravity

We suppose that there is an alternative description of gravitational phenomena[1]. The arena, where gravitational phenomena take place is *the medium* [1].

We assume that in the absence gravitational field the medium becomes *the bare medium*. This medium with *the bare mass density* ρ^{bare} is homogeneous, isotropic and independent of the time. The bare medium we define by *the bare mass density tensor* $\rho_{\mu\nu}^{bare}$

$$\rho_{\mu\nu}^{bare} = \rho^{bare} \cdot \eta_{\mu\nu} = \mathrm{diag}(-\rho^{bare}, \rho^{bare}, \rho^{bare}, \rho^{bare}) \tag{1}$$

where: $\eta_{\mu\nu}$ is the Minkowski tensor, $\mu, \nu = 0, 1, 2, 3$.

The bare medium is the equivalent, in some sense, the Cartesian coordinate system.

Under influence outer gravitational field *the bare medium* is changes and deformed (is curved) and becomes *the effective medium* with *the effective mass density tensor* $\rho_{\mu\nu}$.

The effective medium is the equivalent, in some sense, the curvilinear coordinate system.

The metric in the effective medium is defined by the formula

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[1] Our theory we will call the *m(GR)* theory – the modified theory of GR.

$$ds^2(\rho_{\mu\nu}) = \frac{\rho_{\mu\nu}}{\rho^{bare}} dx^\mu dx^\nu \tag{2}$$

where the effective mass density tensor $\rho_{\mu\nu}$ is describes the relationship between the effective and bare medium.

The effective mass density tensor $\rho_{\mu\nu}$ is equivalent, in some sense, the metric tensor $g_{\mu\nu}$.

We suppose also that the deformation (the curvature) of *the effective medium* depends on the distribution of the matter and energy and is expressed by the on the stress–energy tensor $T_{\mu\nu}$.

The equations of motion in the effective medium

Let us consider the Lagrangian function

$$L = \rho_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} \tag{3}$$

The equation of motion expressed by $\rho_{\mu\nu}$ has the form

$$\frac{dp_\gamma}{d\tau} - \frac{1}{2} \frac{\partial \rho_{\mu\nu}}{\partial x^\gamma} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = 0 \tag{4}$$

where: $p_\gamma = \rho_{\gamma\nu} \frac{dx^\nu}{d\tau}$ is the density of the four-momentum. If the four-gradient of the effective mass density tensor vanish i.e. $\frac{\partial \rho_{\mu\nu}}{\partial x^\gamma} = 0$ then $\frac{dp_\gamma}{d\tau} = 0$ and finally $p_\gamma = const$.

Equation (4) has the also different equivalent form

$$\rho_{\gamma\nu} \frac{d^2 x^\nu}{d\tau^2} + \Gamma_{\gamma\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = 0 \tag{5}$$

where: $\Gamma_{\gamma\mu\nu} \equiv \frac{1}{2}\left(\frac{\partial \rho_{\gamma\mu}}{\partial x^\nu} + \frac{\partial \rho_{\gamma\nu}}{\partial x^\mu} - \frac{\partial \rho_{\mu\nu}}{\partial x^\gamma} \right)$ is the Christoffel symbols of the first kind.

The field equation

The Einstein – Hilbert action has form

$$S = \int \left(\frac{c^4}{16\pi G} R(\rho_{\mu\nu}) + L_m(\rho_{\mu\nu}) \right) \sqrt{-\det\left(\frac{\rho_{\mu\nu}}{\rho^{bare}} \right)} \cdot d^4 x \tag{6}$$

where: $R(\rho_{\mu\nu})$ is the Ricci scalar expressed by the $\rho_{\mu\nu}$, G is the Newton's gravitational constant, c is the speed of light in the *medium*, $L_m(\rho_{\mu\nu})$ describing any matter fields, $\det(\rho_{\mu\nu})$ is the determinant of the effective mass density tensor.

The Einstein field equation expressed by the $\rho_{\mu\nu}$ has the form

$$R_{\mu\nu}(\rho_{\mu\nu}) - \frac{1}{2}\frac{\rho_{\mu\nu}}{\rho^{bare}} \cdot R(\rho_{\mu\nu}) = \frac{8\pi G}{c^4} \cdot T_{\mu\nu}(\rho_{\mu\nu}) \tag{7}$$

The left side of the equation (7) represents the deformation (the curvature) of *the effective medium* expressed by the $\rho_{\mu\nu}$. The right side of the equation (7) represents the distribution of the matter and energy expressed by the on the stress–energy tensor $T_{\mu\nu}(\rho_{\mu\nu})$.

The equations of motion in the weak gravitational field

In the weak gravitational field we can decompose $\rho_{\mu\nu}$ in to following simple form $\rho_{\mu\nu} = \rho_{\mu\nu}^{bare} + \rho_{\mu\nu}^{*}$, where $\rho_{\mu\nu}^{*} << 1$ is a small perturbation in the effective density tensor.

Now we assume that: the velocity of the material particles be very small compared to the speed of light, the gravitational field varies so little with the time that the derivatives of the $\rho_{\gamma\mu}^{*}$ by x_4 may be neglected and we additionally assume also $\rho_{\gamma\nu}^{*} \frac{d^2 x^\nu}{d\tau^2} \approx 0$. Finally

$$\frac{d^2 x^i}{dt^2} = -\frac{\partial}{\partial x^i}\left(\frac{1}{2} c^2 \frac{\rho_{00}^{*}}{\rho^{bare}}\right) \tag{8}$$

where i = 1, 2, 3. Equation (8) has the also different equivalent form, (c = *const*)

$$\rho^{bare} \frac{d^2 x^i}{dt^2} = -\frac{1}{2} c^2 \frac{\partial \rho_{00}^{*}}{\partial x^i} \tag{9}$$

Equation (9) is equivalent, in some sense, to Newton's second law. We can see that this equation is different than the classical Newton's equation for the gravity and the principle of equivalence does not make sense. If $\frac{\partial \rho_{00}^{*}}{\partial x^i} = 0$ then $\rho^{bare} \frac{d^2 x^i}{dt^2} = 0$ ($\rho^{bare} \neq 0$). We can see that the bare medium can mimic the inertial frame of reference. The new quality of the understanding, kept in the Mach's spirit, was reached.

The equations of field in the weak approximation

In the weak gravitational field approximation the field equation has the form of the Poisson's equation

$$\nabla^2\left(\frac{\rho_{00}^{*}}{\rho^{bare}}\right) = \frac{8\pi G}{c^2}\rho \tag{10}$$

with solution (in the particular case)

$$\frac{\rho_{00}^{*}(r)}{\rho^{bare}} = \frac{2G}{c^2}\int \frac{\rho dV}{r} = \frac{2GM}{c^2 r} = \frac{2V}{c^2} \tag{11}$$

where V is the well-known gravitational potential.

Substituting the equation (11) to the equation (8) we get well known equation of motion

$$\frac{d^2x^i}{dt^2} = -\frac{\partial V}{\partial x^i} \tag{12}$$

in which the principle of equivalence is satisfy.

The motion in the spherically symmetric gravitational field

In the spherically symmetric gravitational field the Schwarzschild metric takes the form

$$ds^2 = -\left(1 - \frac{\overset{*}{\rho}_{00}}{\rho^{bare}}\right)c^2dt^2 + \frac{dr^2}{1 - \frac{\overset{*}{\rho}_{00}}{\rho^{bare}}} + r^2\left(d\theta^2 + \sin^2\theta \cdot d\varphi^2\right) \tag{13}$$

This solution is expressed by component $\overset{*}{\rho}_{00}$ of the bare mass tensor.

According to our theory, the effective mass density tensor of the body is change in the gravitational field[2] then effective mass tensor of the body is also changed (volume does not change).

$$m_{\mu\nu}^{Schwarzschild} = m^{bare}\begin{bmatrix} -\left(1 - \frac{2GM}{c^2r}\right) & 0 & 0 & 0 \\ 0 & \left(1 - \frac{2GM}{c^2r}\right)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \cdot \sin^2\theta \end{bmatrix} \tag{14}$$

Because the mass anisotropy depends on the mass M of the star and the planet orbit radius r, we should observe the effective mass change for the elliptical orbit, also in *the Solar System*.

Annual the biggest relative changes (for example from perihelion to perihelion) for the m_{tt} and for m_{rr} should be measured in the Solar System and the estimated value for the planet Earth is equal to $6.6 \cdot 10^{-10}$ [2]. This small change in the effective mass disturbs Newtonian orbit, causing the anomalous motion in the planet's perihelion.

Conclusion

In this paper it was proposed a new approach to the gravitational phenomena. The *m(GR)* theory is the new theory of gravitation, where the space-time (with metric tensor $g_{\mu\nu}$) was eliminated and replaced with *the medium* (with the effective mass density tensor $\rho_{\mu\nu}$).

Theory proposes a new look for **the (effective) mass density**, which **is now the tensor**. Also predicts the mass anisotropy, which cause the anomalous perihelion motion of the planet.

The *m(GR)* theory satisfied classical tests of the GR but their the physical interpretation is different, e.g. under the influence of the gravitational field only physical properties of the

[2] We assumed that equation (11) is satisfied.

- rods was changed, but not the space properties,
- clocks was changed, but not the time properties [2].

Theory also predicts different than the currently used paradigm for the propagation and detection of the gravitational waves [2].

Preliminary theoretical studies are promising and perhaps will eliminate the space-time from the classical and quantum gravity and opens a new way for the missing mass in the Universe.

References

[1]. M. J. Kubiak, submitted to The African Review of Physics, vol. 10, 2015.

[2]. M. J. Kubiak, The African Review of Physics, **vol. 9, 2014**, pp. 123 – 126, http://www.aphysrev.org/index.php/aphysrev/article/view/879/359 .

Appendix

The *m(GR)* vs. GR

Table below includes comparison of both theories.

GR	***m(GR)***
Space-time with metric tensor $g_{\mu\nu}$	*Medium with effective mass density tensor* $\rho_{\mu\nu}$
Metric $ds^2(g_{\mu\nu}) = g_{\mu\nu} dx^\mu dx^\nu$	*Metric* $ds^2(\rho_{\mu\nu}) = \frac{\rho_{\mu\nu}}{\rho^{bare}} dx^\mu dx^\nu$
Equation of motion $\frac{d^2x^\gamma}{d\tau^2} + \Gamma^\gamma_{\mu\nu}(g_{\mu\nu}) \frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = 0$ where: $\Gamma^\gamma_{\mu\nu}(g_{\mu\nu})$ is the Christoffel symbols of the second kind expressed by $g_{\mu\nu}$	*Equation of motion* $\rho_{\gamma\nu}\frac{d^2x^\nu}{d\tau^2} + \Gamma_{\gamma\mu\nu}(\rho_{\mu\nu}) \frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = 0$ where: $\Gamma_{\gamma\mu\nu}(\rho_{\mu\nu})$ the Christoffel symbols of the first kind expressed by $\rho_{\mu\nu}$
Equation of field $R_{\mu\nu}(g_{\mu\nu}) - \frac{1}{2} g_{\mu\nu} \cdot R(g_{\mu\nu}) = \frac{8\pi G}{c^4} \cdot T_{\mu\nu}(g_{\mu\nu})$	*Equation of field* $R_{\mu\nu}(\rho_{\mu\nu}) - \frac{1}{2}\frac{\rho_{\mu\nu}}{\rho^{bare}} \cdot R(\rho_{\mu\nu}) = \frac{8\pi G}{c^4} \cdot T_{\mu\nu}(\rho_{\mu\nu})$