Bifurcation patterns of market regime transition

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The goal of this research was applying a nonlinear approach to the detection of market regime transitions: mean reversion to momentum regimes and vice versa. It has been shown that the transition process has nonlinear scenarios: slow and fast bifurcations. Slow bifurcation assumes that control parameter is changing slowly in relation to the system characteristic time. Gradual absorption of information provides stability loss delay effect. Fast bifurcation has a discrete nonequilibrium nature. Each transition from one attracting cycle to another one is preceded by passing through fixed point state – an effect of precatastophic stabilization exists. Two analytical methods have been developed for recognition of slow and fast bifurcation: *R* analysis and *D* analysis correspondingly. Combined *R*/*D* tool has been incorporated for analysis of world financial crisis of 2008. It turned out that *R* analysis is more convenient for long term investment while *D* analysis suggests middle- and short-term approach. *R/D* analysis has been applied as a filter for currency positional trading system. Slow and fast bifurcation patterns have been applied for the filtering of breakdown signals. Incorporation of a filter allowed to reduce twice the number of trades and to increase system efficiency, Calmar ratio, by seven times. *R*/*D* filter allowed decreasing sensitivity to volatility: duration of equity stagnation has fallen down to two months in relation to one year for the original breakdown system. It has been shown that *R* and *D* patterns may improve the long term efficiency and stability of a momentum quantitative trading model.

Keywords: Switching point; Market transition; Nonlinear analysis; Price action filter; trading system
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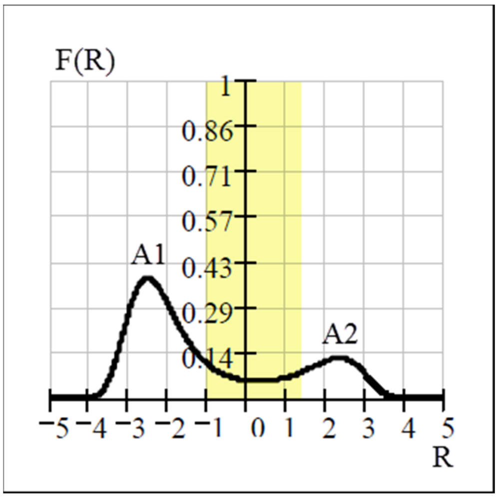
1. Introduction

Among quantitative trading systems two general classes may be considered: mean reversion and momentum based classes. Mean reversion model assumes stationarity of a residual price time series and existence of a high probability benchmark region. Deviations of a price above/below this benchmark disclose reverting trading opportunities of buying 'lows' and selling 'highs'. Normal Gaussian distribution is considered as a preliminary hypothesis which makes traditional statistical techniques of measurement easy to apply: confidence intervals, Sharpe ratio and Kelly capital allocation. Mean reversion includes traditional beta neutrality systems: pairs trading, spread trading, statistical arbitrage, etc. However popularity of these systems caused high competition in algorithmic trading. As a result in short intervals (HFT) arbitrage opportunities tend to collapse faster and faster - over years system profitability may decrease.
This tendency forced some of quantitative funds and individual traders to focus on opposite solution - momentum based models. These models assume market inefficiency, nonstationary behavior of price time series and long term memory of trend motion. Deviations of a price above/below a range disclose trending opportunities of buying 'highs' and selling 'lows'. Normal distribution is not applicable for statistical description – risks are undervalued/overvalued because of distribution function dissymmetry. Momentum patterns cause low probability saturation of distribution: so called fat tailed distributions considered by Vilfredo Pareto (see Mandelbrot (1968)). As a result some of commodity trading advisors (CTA), using a trend following approach prefer Sortino metrics (Rollinger (2013)) instead of Sharpe ratio. Fractal non stationary distribution suggests more reasonable statistics. However it is at the forefront of science and still lacks steady measurements of system efficiency - momentum theory is less refined than its mean reversion competitor.

Besides mathematical difficulties there is another problem – evolving market exists in two modes of mean reversion and momentum. Moreover a fractal random walk can under certain conditions be considered as superposition of momentum and mean reversion. Regime switching and a transition to momentum is a focus of this research. It is an attempt to incorporate nonlinear bifurcation theory in momentum model description. According to General Theory of Reflexivity, introduced by Soros (2008) a tendency is a result of information diffusion combined with interpretation distortion. Collective behavior may move the price under influence of a random factor. However this fluctuation under some circumstances (regime switching) is estimated by investors as new information and forces them to interpret fundamentals with a bullish or bearish bias. This mechanism accelerates the price and adds new information to the system. As a result a critical fluctuation triggers market instability and tendency continuation. This process has a nonlinear instable nature and that's why nonlinear chaos models of econophysics are suggested here for switch point detection.

2. Slow and fast bifurcation scenarios

Let us consider structural shifts of market, caused by fundamental releases, regulation changes or other discrete events. Each of these factors $i = \overline{1,N}$ may be described by control order parameter R_i while the behavior of a system shift is defined by control vector $\vec{R} = \{R_i\}$. Although some insufficient deviation of $d\vec{R}$ can not cause a structural instability, there is an unstable transition threshold area. Let us define bifurcation or transition as a qualitative shift of market regime. It can be represented by the switching from mean reversion to trend motion, from one mean reversion regime to another or from one equilibrium price range to another. Let us assume that the system is described by a set of characteristic coordinates (prices)/momentums (price momentums) $(\vec{q},\vec{p})$ or $\vec{R}$ vector which composes a phase space universe. A selected regime may be described geometrically by the area of extremal density probability (DP) – attraction area. For one dimensional case (1D) two regimes of a system are illustrated in figure 1. Here *A1* and *A2* represent two attractors while *R* is a control parameter like P/E for stocks universe or basic rate for currencies. A bifurcation unstable region is marked by yellow rectangle here. A small deviation may transfer a system into one of attractors– slow bifurcation transition. According to a second scenario a fundamental shift may force a system to make *A1-A2* discrete transition, overcoming attraction. A new equilibrium state is reached then discretely. This scenario may be designated as a fast bifurcation (see figure 2).



Figire 1. Attraction and transition areas

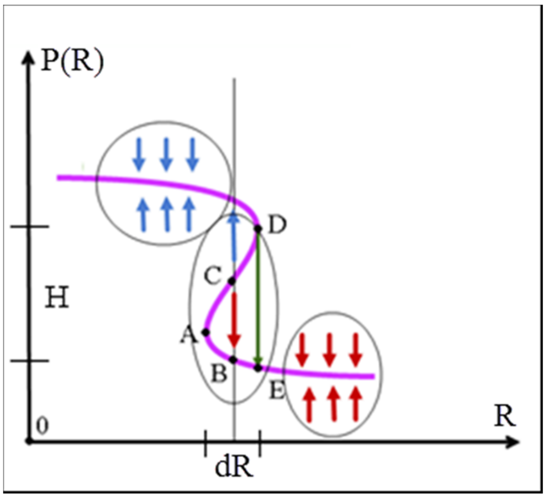


Figure 2. Catastrophic cusp

In vicinity of attractors, when small deviations exist, the distribution function can be described by Normal distribution – a mean reversion approach may be applied. However a large scale transition is a caused by discrete momentum and is modelled according to momentum based system. This transition can be described by a fat tailed class of DP functions. Levy distribution with infinite variance $\langle x^2 \rangle = \infty$ is one of them. According to Zaslavsky (2007) Levy DP can be represented through its characteristic function $F(q,t)$. Here α, β correspond to constant parameters.

$$F(x,t) = \int_{-\infty}^{\infty} F(q,t)dq \qquad F(q,t) = F_0 \cdot \exp(-\alpha t |q|^{\beta}) \tag{1}$$

Bifurcation can be described in terms of catastrophe theory, developed in 1955 by Hassler Whitney (1955). A catastrophic 'cusp' is represented in figure 2. It is formed by deformation of continuously differentiable curve *P*(*R*), where *R* is control order parameter and *P* is some characteristic such as price. Before the curve deformation D, C, A, B correspond to different parameters. However a deformation leads to multiplicity or uncertainty in the unstable area of C – vicinity. According to Whitney the disruption $D \to E$ appears as a fusion of stable and unstable regimes, marked by ovals. It corresponds to fast bifurcation. In terms of a bifurcation theory this one-dimensional evolution corresponds to the saddle-node fusion in a phase space. In case of differentiable, not deformed *P*(*R*) curve a transition between two regimes is realized continuously – slow bifurcation scenario.

3. Market coherence and slow bifurcation

Let us consider slow (adiabatic) bifurcation and its fundamental difference from the fast disruptions such as Levy random walks, so called Levy flights. Adiabatic evolution can be described in a convenient analytical way. One pure analytical effect makes it convenient for the forecast of complex market systems - stability loss delay (SLD) proven by Neishtadt (1987). Adiabatic bifurcation assumes that control parameter is changing slowly in relation to the system characteristic time: $T_R \ll \tau$ and $P_0(R)$ mapping curve is differentiable. Let's consider a nonlinear system (2). We may introduce a new variable $P(t) = P_0 + \xi(t)$.

$$\dot{P} = f(P,R) \qquad \dot{R} = \varepsilon \qquad \varepsilon \ll 1 \tag{2}$$

The equation may be modified using decomposition:

$$\dot{\xi} = A(R)\xi + O\left(\left|\xi^2\right|\right) + \varepsilon h(R) \qquad h \ll 1 \tag{3}$$

A critical R^* corresponds to the positive imaginary part *A*(*R*), while for $R \prec R^*$ $A(R)$ has only negative imaginary solutions. Neishtadt (1987) has shown that a phase point of $P(T_0)$ corresponding to $R^* - dR$ will be attracted to the instability regime eventually. A time needed for instability increase is defined by adiabatic parameter.

$$T_R \sim \dot{R} \qquad \xi(T_R) / \xi(T_0) > 1 \tag{4}$$

This relation means that slow bifurcation saturation is preceded by over critical control parameter change, and stability loss delay exists in adiabatic case.

SLD effect has been explained independently by Vaga (1991) for market systems. He based his explanation on nonlinear Coherent Market Model and constructed this model with use of agent interaction theory. In fact Vaga has extended an Ising model of ferromagnetic magnetization and applied it to social adaptation problems. A market system is considered as a group of clusters (domains) of investors. Each investor may have bullish or bearish bias. An external influence such as new fundamental information causes a gradual collective polarization of the market. A number of 'bears' and 'bulls' are represented by N^+ or $N^-: N^+ + N^- = N$.

Market polarization than can be defined by a following parameter:

$$q = \frac{N^{+} - N^{-}}{N} \quad -1 \le q \le 1 \qquad (5)$$

External influence causes the transition $q_1 \to q_2$. Weidlich (1971) has shown that a transition $q_1 \to q_2$ frequency for bullish polarization is described by the following relation:

$$p_{12} = p_0 \exp\left(\frac{Iq + H}{\theta}\right) \qquad (6)$$

Here I and θ correspond to adaptability of investor and random fluctuations factor correspondingly. H is an external control parameter. In such a way market polarization is defined by relation between the 'noise' and external information background. A transitional probability density $F(q,t)$ is described by nonlinear Fokker-Planck equation (7).

$$\frac{dF(q,t)}{dt} = -\frac{\partial}{\partial q}\big(K(q)F(q,t)\big) + \frac{1}{2}\frac{\partial^2}{\partial q^2}\big(Q(q)F(q,t)\big) \qquad (7)$$

Here members K and Q are nonlinear control parameters. A simple estimation of deviation may be derived for quasi stable transition:

$$\frac{F_0 - F}{T} \simeq -\frac{\partial}{\partial q}\big(K_0(q)F(q,t)\big) + \frac{1}{2}\frac{\partial^2}{\partial q^2}\big(Q_0(q)F(q,t)\big) \qquad (8)$$

Here T is a relaxation time needed for $F \to F_0$ slow transition which occurs after slow parameter change $K \to K_0$, $Q \to Q_0$. The time of relaxation is defined by transport properties K_0, Q_0 of the considered system. Generally these properties are connected with an interaction of domains (Q - noise) and their ability to react when new information is absorbed (K external drift). Gradual absorption of information provides SLD effect which forms a tendency in the market. Finally absorption is completed, fundamentals are taken into account and the new equilibrium state (fair price range) is reached.

4. Disruption and birth of the limit cycle

Unlike adiabatic bifurcation fast bifurcation is represented by disruption mechanism $D \to E$ in figure 2. Continuous transition is absent and a new state is formed not as a result of evolution or the merging of new attractor with a previous one. It is composed by a strong influence of new information expressed by discrete shift of the control parameter. These phase transition are irreversible. It means that $P(R)$ mapping is no longer differentiable function and the nonlinear problem can not be considered in analytical decomposition (2)-(3). In fact an external shift is realized discretely: $\dot{R} \gg 1,\ T_R \ll \tau$. In this case we may refer to qualitative research methods of nonlinear dynamics. Let us assume that the considered system can be described by two dynamic parameters such as $(\vec{P}, \dot{\vec{P}})$. Here $\vec{P}(t)$ is dynamic characteristic, for example price of an assets group. For two dimensional maps a behavior of a system is defined by vector trajectory in $\vec{\Gamma} = (P, \dot{P})$ plane. In general case the regular motion is defined by two types of attractors: fixed points $(P_0, \dot{P}_0)$ and closed cycles. It means that any dynamic state can be modeled as combination of fixed points, cycles (periodic or quasi periodic) and transitional unstable trajectories (separatrixes) connecting these attractors. The following effect, proved by Poincare–Bendixson (1901) theorem, is related only to 2D types of surfaces and continuous vector field $\vec{\Gamma}(t)$.

In 1939 Andronov (1937) and Leontovich have shown that there are 4 types of fast bifurcations for Poincare–Bendixson systems. Each type corresponds to an increasing of control parameter R over its critical threshold:

1.4. Stable fixed point $\Rightarrow$ unstable fixed point $\Rightarrow$ stable cycle;
2.4. Unstable cycle $\Rightarrow$ Stable fixed point $\Rightarrow$ unstable fixed point;
3.4. Unstable fixed point $\Rightarrow$ stable fixed point;
3.5 Stable cycle $\Rightarrow$ unstable cycle.

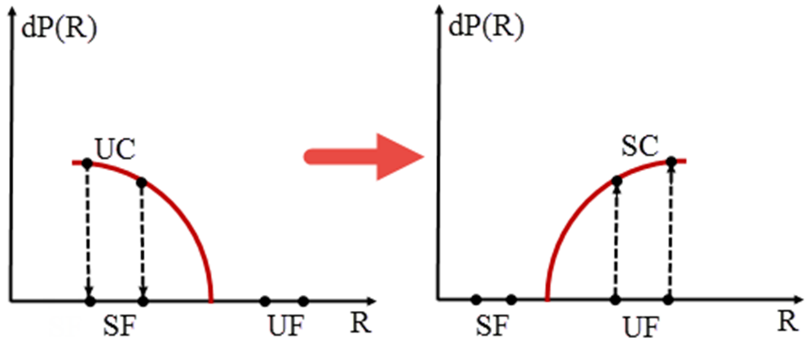


Figure 3. Scenario 1 and Scenario 2 of fast bifurcation. SF – stable fixed point, UF – unstable fixed point, SC – stable cycle, UC – unstable cycle.

Firs two scenarios are represented by figure 3. Their combination composes the disruption $D \rightarrow E$, discussed above. It is worth to note that each transition from one attracting cycle, expressed by mean reversion to another one is preceded by passing through fixed point state – an effect of precatastophic stabilization (PS - effect). Fixed point state may be described by small cycle with negligible scale – equilibrium price range. PS – effect is observed in mechanical systems such as turbulent flows – intermittency before large scale disturbance. It is observed in market systems as well and is known as a particular case of volatility clustering (see Lux (2000)). Narrowing of pair spreads and volatility decrease are precursors of fast fundamental shifts. That is why PS effect can be naturally used for the searching of mean reversion/momentum switching point. Returning to Market Coherence Model this effect can be qualitatively illustrated as well. A discrete switching polarization of domain is possible only under certain condition: random fluctuation θ of domains is very small in relation to external information H: $H \rightarrow \theta$. It means that domains simultaneously are polarized according to the external influence. Small fluctuations naturally correspond to small dispersion of investor's biases and preliminary system stabilization.

5. *R* analysis and identification of slow bifurcation

After classification of basic transitions and corresponding precursors such as SLD and PS we may focus on detailed quantitative analysis of these patterns. As it was mentioned above SLD effect is defined by delay between control parameter shift and the stability loss. However the choice of generalized control parameter is evidently a contentious issue itself. We base the choice of control parameter on correspondence between turbulent system of whirls and an arbitrary system of agents (investors). Stokes (1851) introduces dimensionless Reynolds number for prediction of similar flow patterns. For certain type of system this parameter allowed quantitative analysis of laminar-turbulent transition. Reynolds number R may be defined in the following way:

$$\mathrm{R}(t) = \frac{l \cdot u_0}{D(t)} = \frac{q^+}{q^-(t)} \tag{9}$$

Here l is spatial scale of system, u_0 is velocity of input flow which is assumed to be constant in this example. $D(t)$ corresponds to non stationary viscosity of system which defines the energy dissipation. The numerator of relation (9) expresses an intensity of energy injection. In fact Reynolds number can be considered as a measure of relation between input q^+ and dissipated power $q^-(t)$. If we generalize R into the space of arbitrary market systems than it may be considered as dimensionless interpretation of I/θ relation of Coherence Market Model. We introduce a following assumption: bifurcation necessarily corresponds to the condition $R(t) \neq 1$ of misbalance, while equilibrium state is realized for $R = 1$. In given description bifurcation corresponds to the transition of an equilibrium state $R = 1$:

$$R(t_0) = 1 \Rightarrow \uparrow q^+(t) \Rightarrow \uparrow R(t) \Rightarrow R(t) \succ 1 \Rightarrow \uparrow q^-(t) \Rightarrow \downarrow R(t) \Rightarrow R(t_1) = 1 \tag{10}$$

$$R(t_0) = 1 \Rightarrow \downarrow q^+(t) \Rightarrow \downarrow R(t) \Rightarrow R(t) \prec 1 \Rightarrow \downarrow q^-(t) \Rightarrow \uparrow R(t) \Rightarrow R(t_1) = 1 \tag{11}$$

Here $\uparrow$ and $\downarrow$ show a finite increase and decrease of corresponding parameter for $t_1 \succ t \succ t_0$. The delay between a new cycle appearance and macro scale excitation is defined by the inertial properties of system domains – SLD effect. An appearance of a new qualitative state is represented in the following steps:

1.5. New portion of energy/information is injected into the system: $\uparrow q^+(t)$;
2.5. Control parameter passes its stable threshold $R \succ 1$;
3.5. Adaptation of the system leads to a gradual absorption of energy/information: $\uparrow q^-(t)$;
4.5. New equilibrium state is reached: $R = 1$.

As it follows from 1-4 a positive feedback for input/output mechanisms is the compulsory condition for slow bifurcation. Without loss of generality one can be represented in the following way: $\partial q^+(t) / \partial q^-(t) = 1$. A generalized control parameter R expresses the open boundary nature of a considered system. It is worth to note that a consideration of a market system leads to replacement of energy by information. We will show below an equivalence of these approaches. We may remark that an adaptation of system is a response to an external influence. This adaptation leads to a gradual change of a market microstructure and its ability to absorb information – Step 3. It has been shown (see Kamenshchikov (2013)) that in general case R parameter transition defines necessary but not sufficient requirement of slow bifurcation.
Let's introduce an absolute value of fluctuation $|dP(t)|$ and acceleration of a fluctuation change $\ddot{P} = \delta^2 |dP(t)| / \delta t^2$. In such a way systems may be delimited into accelerator type $\ddot{P} \succ 0$ and decelerator type $\ddot{P} \prec 0$. Than a slow bifurcation (see Kamenshchikov (2013)) corresponds to $R(t) \succ 1$ condition for decelerator and $R(t) \prec 1$ for accelerator. A bifurcation condition may be represented than in the following way:

$$sign\left(\ddot{P}\right) sign\left(R - 1\right) = -1 \tag{12}$$

Decelerator type bifurcation corresponds to Step 1 and Step 2 while accelerator type – to Step 3 and Step 4. If we consider attractor – mean reversion zone, than we shall focus on decelerator type of motion. The considered condition provides positive dynamic entropy of Kolmogorov – Sinai (see Zaslavsky (1988)). One is used as a measure of order in nonlinear systems with slowly changing control parameter. We have following relation for the system with K degrees:

$$h = \left\langle \sum_{i=N}^{K} h_i^+ \right\rangle = \left\langle \ln\left(\prod_{i=N}^{K} \sigma_i^+ \right) \right\rangle \quad \sigma(t) = \frac{|\delta P(t)|}{|\delta P_0|} \tag{13}$$

Here $\langle\ \rangle$ designates spatial averaging, while σ_i^+ is a positive Lyapunov increment.

According to its definition dynamic entropy is a first derivative of entropy $h = \partial S / \partial t$., which will be represented in the informational form. According to Shannon (1948) information per agent/investor can be represented in the following way:

$$S_i = -\sum_j \nu_j \ln \nu_j = -\nu_+ \ln \nu_+ - \nu_- \ln \nu_- \tag{14}$$

Frequency ν_j here is a frequency of j type appearance. This relation is sufficiently simplified if we consider only two types of investors: 'bulls' and 'bears'. Informational entropy is connected to classical Boltzmann entropy in a simple form: $S = S_i k_B$. That is why normalization $k_B = 1$ allows to state equivalence of these concepts: $h = \partial S_i / \partial t$. As it follows from this relation instability $h \succ 0$ ($R(t) \succ 1, \ddot{P} \prec 0$) preceding slow bifurcation, leads to a gradual absorption of information. In such a way we come to correspondence between energy injection and new information absorbed. In relation to financial markets this can be interpreted in such a way: new fundamental information injects volatility into the system. Finally a new equilibrium state is reached: $h = 0, dS_i = 0$. This period may be considered as a tendency completion. We will show below that a preliminary analysis of control parameter therefore may be used to predict a slow regime switching.

6. *D* analysis and identification of fast bifurcation

As it was shown above PS effect may be used for fast bifurcation pattern recognition. However a classical nonlinear approach of phase trajectories needs algorithmic formalization. One of these approaches can be reached through the Markovian random walk model of Fokker-Planck–Kolmogorov combined with fractal description of Mandelbrot (1968).The basis of this model is short memory Chapman - Kolmogorov equation. If we consider a one dimensional case then a transitional probability for price $P(t)$ random walks $P_1, t_1 \to P_3, t_3$ formally satisfies the following markovian relation:

$$W(P_3, t_3 | P_1, t_1) = \int dP_2 W(P_3, t_3 | P_2, t_2) \cdot W(P_2, t_2 | P_1, t_1) \tag{15}$$

Here $W(P, t | P_0, t_0)$ is a conditional probability density. Fokker – Planck – Kolmogorov (FPK) equation has been received based on two basic assumptions.

1.6. $W(P', t' | P, t) = W(P', P, t' - t) = W(P', P, {}_\Delta t)$. A transitional probability doesn't depend on initial time point;
2.6. $F(P', t) = F(P', P, t)$. A final probability doesn't depend on the initial coordinate.
FPK equation than may be represented in the following way:

$$\frac{\partial F(P,t)}{\partial t} = \frac{1}{2} \cdot \frac{\partial}{\partial P} \left(D(P) \cdot \frac{\partial F(P,t)}{\partial P} \right) \quad D(P) = \lim_{{}_\Delta t \to 0} \left(\frac{\langle\langle {}_\Delta P^2 \rangle\rangle}{{}_\Delta t} \right) = \frac{\langle\langle {}_\Delta P^2 \rangle\rangle}{{}_\Delta t_{\min}} \tag{16}$$

Here double brackets designate an averaging of an initial coordinate:

$$\langle\langle {}_\Delta P^2 \rangle\rangle = \int (P - P_0)^2 \cdot W(P, P_0, {}_\Delta t) dP_0 \tag{17}$$

Let us modify the given nonlinear diffusion *D*. In general mean reverting case the correspondence between price and time is ambiguous: diffusion has an implicit time parameter. It can than be modified in the following way:

$$D(P,t) = \frac{\langle\langle {}_\Delta P^2 \rangle\rangle}{{}_\Delta t_{\min}} = {}_\Delta t_{\min} \cdot \langle\langle \varepsilon(P,t) \rangle\rangle \quad \varepsilon(P,t) = \left(\frac{{}_\Delta P(P,t)}{{}_\Delta t_{\min}} \right)^2 \tag{18}$$

Now we may consider price *P* as parameter taken a one way correspondence between time and price into account. Let's introduce a variable time lag $T = t - t_0$.

Than a power regression of diffusion will be expressed as following:

$$D(t_0, T) = D_0(t_0) \cdot T^{\kappa} \tag{19}$$

Then an expectation of the shift can be modified:

$$E\left([x(t_0 + T) - x(t_0)]^2\right) = D_0(t_0) \cdot T^{\kappa+1} \tag{20}$$

According to Mandelbrot (1968) an expectation of fractal Brownian motion deviation is self-affine:

$$E\left([B_H(t+T) - B_H(t)]^2\right) = V_H \cdot T^{2H} \tag{21}$$

A comparison of this relation with an equation (20) allows expressing the Hurst factor H through a stability coefficient $\kappa : H = (\kappa + 1)/2$. Cases of $H \succ 0.5$ ($\kappa \succ 0$) and $H \prec 0.5$ ($\kappa \prec 0$) correspond to the momentum and mean reverting motion correspondingly. A critical value $H_{cr} = 0.5$ corresponds to a Brownian random walk – no tendency, no mean reversion when traditional trading strategies are not applicable. Levy flights and "thick tails" of distribution correspond to $H \succ 0.5$ ($\kappa \succ 0$) – diffusion instability. A sense of a Hurst factor can be clarified with a use of spectral description. If we introduce a characteristic frequency $T = 1/f$, then according to the relation (19) a following relation may be derived (see Kamenshchikov (2014)):

$$D(t_0, f) = D_0(t_0) \cdot f^{-\kappa} \tag{22}$$

We may note that the case of $H \succ H_{cr}$ corresponds to the large scale transport, while if $H \prec H_{cr}$ a micro scale energy absorption is more intense. The extension of this interpretation means that a generalized Hurst parameter has no obligatory preliminary limitations $-\infty \prec H \prec \infty$. It is worth to note that instability of diffusion is directly connected with a fractal scaling invariance:

$$\Lambda_D = \sqrt{\left\langle\left\langle {}_{\Delta}x^2 \right\rangle\right\rangle} = \sqrt{D_0(t_0)} \cdot T^H \tag{23}$$

$$\Lambda_D(kT) = k^H \Lambda_D(T) \tag{24}$$

A precatastophic stabilization effect analysis can be based upon the simple deviation of diffusion, which expresses scattering of price in the vicinity of initial value P_0. As it was shown above a fast bifurcation (2D case) is preceded by transition to focus – a very small area of deviations: stable cycle$\Rightarrow$unstable cycle$\Rightarrow$stable fixed point$\Rightarrow$unstable fixed point$\Rightarrow$stable cycle. According to the formalism, suggested above it corresponds to the following conditions:

$$D(T, t_0) - D(T, t_0 + {}_{\Delta}t) \prec 0 \quad \Lambda(T, t_0) \prec \Lambda(T, t_0 + {}_{\Delta}t) \tag{25}$$

7. R/D analysis – global financial crisis

In the current section we apply a combination of R and D price action analysis for detection of slow and fast market shifts correspondingly. It means that we take into account both signals as precursors of regime shifts. A nonlinear approach to switching point detection is applied. The most trivial modification of analysis is suggested - the principle of 'refined simplicity' is believed to be the most attractive for building flexible and effective trading algorithms. However more complex formalizations of PS and SLD effects are certainly possible and may be investigated in the future.

We may consider the uniform historical series of price $P = \{P_k\}$ $k = \overline{1, M}$. Slow bifurcation precursor is built on the basis of R analysis. The condition (12) for *SLD* effect can be modified for uniform series:

$$sign\left(\ddot{P}_i\right) sign\left({}_{\Delta}\varepsilon_i(t_i)\right) = -1 \quad \varepsilon_i = \frac{1}{2}\left(P_i - P_{i-1}\right)^2 \tag{26}$$

Here the sign of (R-1) relation coincides with energy increase per one step. Acceleration is represented in the following residual form:

$$\ddot{P}_i = P_i - 2P_{i-1} + P_{i-2} \tag{27}$$

For D analysis diffusion is expressed in discrete form:

$$D(i,N)=\frac{\sum_{j=i}^{j=i+N}\left(P_j-P_i\right)^2}{N} \qquad {}_{\Delta}D(i,N)=D(i+1,N)-D(i,N) \tag{28}$$

PS effect for fast bifurcation can be formalized in the following way $sign\left[{}_{\Delta}D(i,N)\right]=-1$. N is a parameter to be optimized in each considered case.

First qualitative example refers to the financial crisis of 2008 triggered by mortgage collapse in the US. Let us consider a period of 16.07.2006-21.02.2010 which includes a beginning of financial crisis and recovering stages. A preliminary origin of the crisis corresponds to the falling of bank liquidity in August 2007. In September 2008 it caused a failure of greatest American mortgage agencies: Lehman Brothers, Fannie Mae, Freddie Mac. In January 2009 US Federal Reserve has started the fourth supporting program of financial stabilization (QE4) that led to a preliminary recovering of a financial system.

We may choose S&P500 index as the marker of global investment activity. Let's analyze a time series of weekly close prices – an index price was defined at the trading end of each Friday – the end of a trading week. Here the following bifurcation functions are analyzed:

$$B_R(i)=-\ddot{P}_i\,{}_{\Delta}\varepsilon_i \qquad B_D(i,N)=-{}_{\Delta}D(i,N) \tag{29}$$

According to relations (26), (28) these functions reach positive values if PS or SLD effects are observed. We use directly bifurcation functions B_R and B_D, to estimate the probability of slow and fast transition correspondingly. In figure 4 the price action and bifurcation function B_R are represented. To represent both time series at one chart a preliminary normalization has been done: $P_i \to P_i/\max(P_i)$, $B_R^i \to B_R^i/\max(B_R^i)$. It is worth to note that clusters of positive B_R typically correspond to the periods of range breakdowns. Moreover maximums of bifurcation function verify price range breakdowns – both are marked by green color at the chart. We define the bearish mood when downward breakdown is confirmed by bifurcation maximum. First two selected bearish periods are started at August 2007 (bank liquidity falling) and October 2008 (failure of mortgage agencies). As it was mentioned above R analysis discloses slow systematic market shifts when the mood of investors is gradually 'turned' to the new fundamental course: R analysis is more convenient for long term investment. Here we assume that long term investors analyze the tendencies of fundamentals rather than one particular signal. D analysis suggests middle- and short-term approach (Fig.5). Signals are therefore more frequent. It is hard to select certain clusters, however maximums precede breakdowns. Here 'four points' parameter (1 month) has been used: N=4.

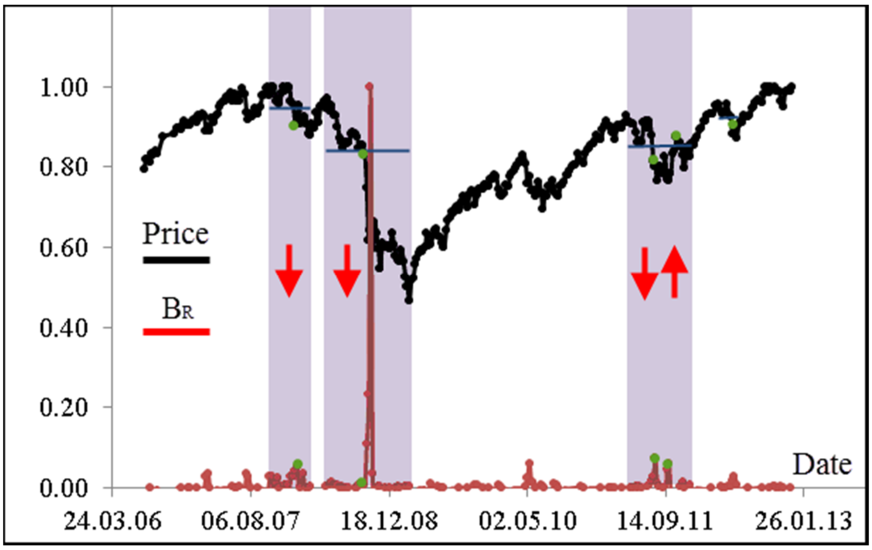


Figure 4. Weekly closes of S&P500 (black) and function of slow bifurcation. Green points correspond to extremal points and breakdowns.

The direction of price action is defined by the breakdown following a bifurcation maximum (green marks). First bearish signals confirm a long trend tendency disclosed by *R*-analysis. It is important to note that generally fast bifurcations are more frequent and can be detected long before by nonlinear *D* analysis. However we suppose that diversified analytical tool has to take into account both types of transitions and time horizons. Although represented results have empirical nature they help to better understand *R*/*D* analysis applications in the area of quantitative algorithmic trading.

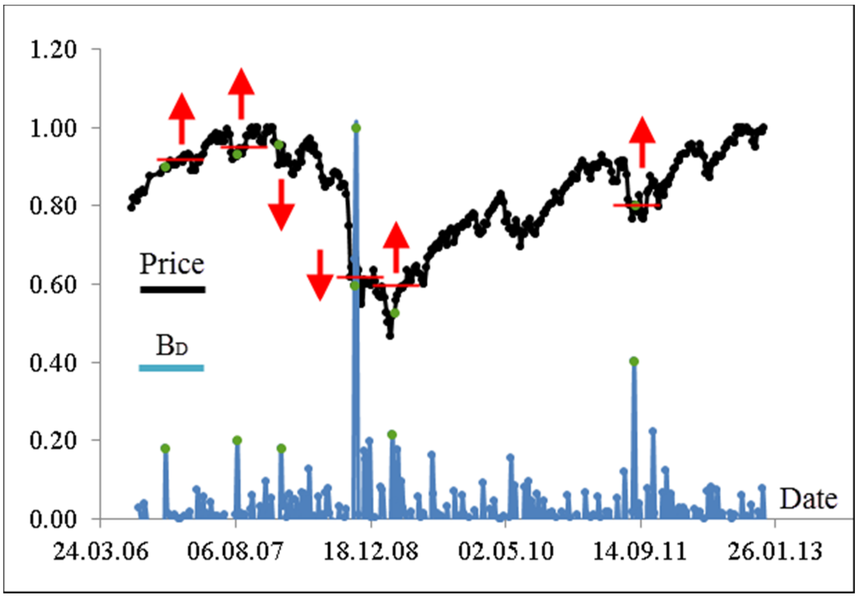


Figure 5. Weekly closes of S&P500 (black) and function of fast bifurcation.
Green points correspond to extremal points and breakdowns.

8. Backtest of R/D precursors – breakdown trading algorithm

In the current section we would like to present backtest results of a simple breakdown trading system for 07.11.2011-30.06.2015 period. These results are represented for the evaluation of *R*/*D* analytical applications. The considered system is only a first step to the application of a nonlinear approach for bifurcation detection. It doesn't contain risk management and capital allocation schemes as well.

The currency EUR/USD spot market is selected. We preserve the equal trading volume of 1 Lot=$ 100 000. All types of instant operations are available: Buy/Sell, Buy Close/Sell Close. Stop orders are not used in the considered scheme. The considered period contains 5750 bars of four hour time frame (H4). Four digits quotes are provided by Dukascopy Bank with average transaction costs for the considered period: bid/ask spread=1.8 pips, overnight swap long=-0.38 pips, overnight swap short=-0.11 pips. We should recall that 1 pip corresponds to $10 for 1 Lot trading volume. Initial balance of $10000 and leverage of 1:50 are used. In the figure below a close price (00:00 Central European Time) time series is represented. This signal is combined with corresponding volatility which is defined as number of ticks – elementary price actions, per one bar.

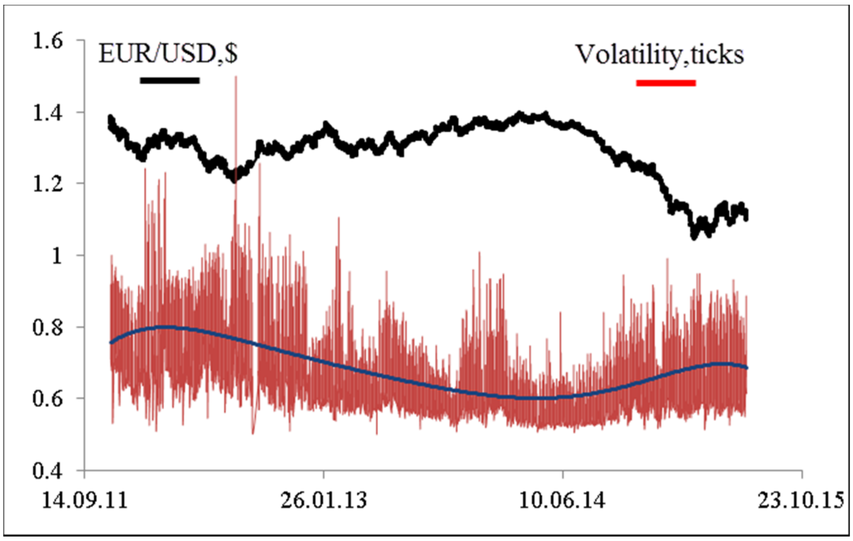


Figure 6. H4 closes of EUR/USD spot (black) and volatility 6 degree approximation (blue).

A six degree polynomial function allows detection of decreasing volatility period: 26.07.2012-12.03.2014. This notion will be used below - we assume that a typical breakdown momentum strategy should represent worse results in this period. The first considered system, System I, is built on the basis of well-known channel breakdown.

System I:

- The upper boundary of a channel corresponds to the maximum of recent N close prices: $P_i^{\max} = \max(P_{i-N}, P_i)$;
- The low boundary of a channel corresponds to the minimum of recent N close prices: $P_i^{\min} = \min(P_{i-N}, P_i)$;
- If a price is closed above the recent $P_{\max}$ level than a long position is opened: $P_i > P_{i-1}^{\max}$. At the same time a short position is closed, if exists;
- If a price is closed below the recent $P_{\min}$ level than a short position is opened: $P_i < P_{i-1}^{\min}$. At the same time a long position is closed, if exists.

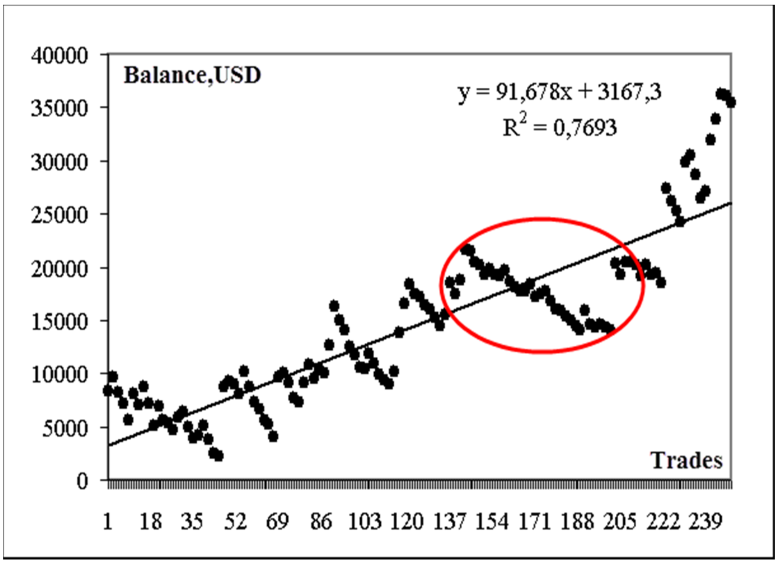


Figure 7. System I: trades - 125, profitable fraction - 37%, maximum drawdown - 85%, Cumulated profit - 241%, PL - 2.37, Calmar ratio=2.84, maximum loss series - 7

The system has been optimized according to the cumulated profit objective function. Optimization led to N=25 for the channel parameter. Backtest results, represented in figure 7, allow noting of continuous drawdown period, marked by red oval: 154- 222 trades. This period corresponds to the time frame of 04.10.2012-18.03.2014 and is comparable to low volatility period, noted above. Here we use a number of measurement metrics tools:

1.8. Profitable fraction – a relative fraction of profitable trades: $PF = N_{prof} / N$;
2.8. Maximum drawdown (MD) – drawdown between consequent maximum and minimum of yield in relation to the initial balance – estimation of risk;
3.8. Accumulated capital is preserved along the backtest and cumulated yield metrics is used;
4.8. PL – profit/loss ratio is a relation between average profitable and unprofitable trades: $PL = \langle P \rangle / \langle L \rangle$;
5.8. Calmar ratio is metric used for evaluation of system efficiency. It is expressed through the relation between cumulated yield and maximum drawdown: $Calmar = Y(\%) / MD(\%)$;
5.9. Maximum loss series defines the longest series of consequent losses.

Though results are quite attractive according to Calmar ratio, strategy efficiency is non uniform. The considered system is sensitive to volatility. Our system obviously needs modification to provide more smooth equity growth. Firstly a precursor of a short term D-analysis has been applied:

System II:

- The upper boundary of a channel corresponds to the maximum of recent N close prices: $P_i^{\max} = \max(P_{i-N}, P_i)$;
- The low boundary of a channel corresponds to the minimum of recent N close prices: $P_i^{\min} = \min(P_{i-N}, P_i)$;
- If a price is closed above the recent $P_{\max}$ level and the D precursor is switched than a long position is opened: $P_i > P_{i-1}^{\max}$ and $B_D(i, \overline{N}) \succ 0$. At the same time a short position is closed, if exists;
- If a price is closed below the recent $P_{\min}$ level and the D precursor is switched than a short position is opened: $P_i < P_{i-1}^{\min}$ and $B_D(i, \overline{N}) \succ 0$. At the same time a long position is closed, if exists.

This system suggests two degrees: $N, \overline{N}$. Optimization is than realized with account of these two parameters. An optimal set corresponds to $N = 29, \overline{N} = 15$.

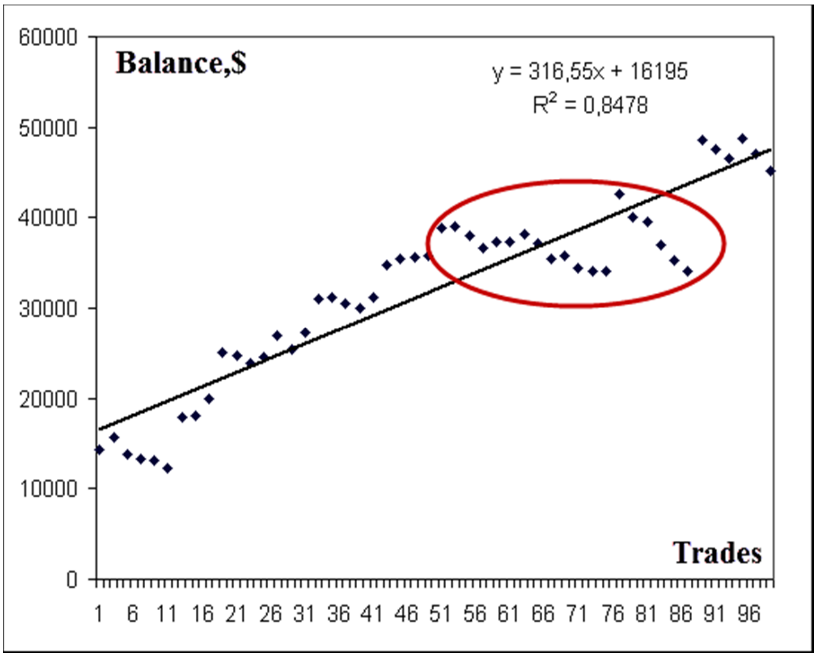


Figure 8. System II: trades - 50, profitable fraction - 52%, maximum drawdown - 41%, Cumulated profit - 351%, PL - 2.07, Calmar ratio=8.56, maximum loss series - 5

The backtest results (figure 8) present high filtration efficiency: 50 trades against 125 for System I. Its psychological attractiveness increased as well for profitable fraction has grown up by 15%. Period of equity drawdown has been replaced by flat range. This modification allowed improving Calmar ratio from 2.84 to 8.56 – more than three times. However we still miss long term market shifts, which can be disclosed by *R* analysis. The corresponding precursor has been added to the existing scheme:

System III:

- The upper boundary of a channel corresponds to the maximum of recent *N* close prices: $P_i^{\max} = \max(P_{i-N}, P_i)$;
- The low boundary of a channel corresponds to the minimum of recent *N* close prices: $P_i^{\min} = \min(P_{i-N}, P_i)$;
- If a price is closed above the recent $P_{\max}$ level and *D* or *R* precursors are switched than a long position is opened: $P_i > P_{i-1}^{\max}$ and ($B_D(i,\overline{N}) \succ 0$ or $B_R(i) \succ 0$). At the same time a short position is closed, if exists;
- If a price is closed below the recent $P_{\min}$ level and *D* or *R* precursors are switched than a short position is opened: $P_i < P_{i-1}^{\min}$ and ($B_D(i,\overline{N}) \succ 0$ or $B_R(i) \succ 0$). At the same time a long position is closed, if exists.

Given algorithm allows detection of both types of bifurcation when at least one of precursors is switched on. Backtest results show that a profitable fraction and loss series length have been preserved. However the Calmar ratio has been significantly improved by risks mitigation.

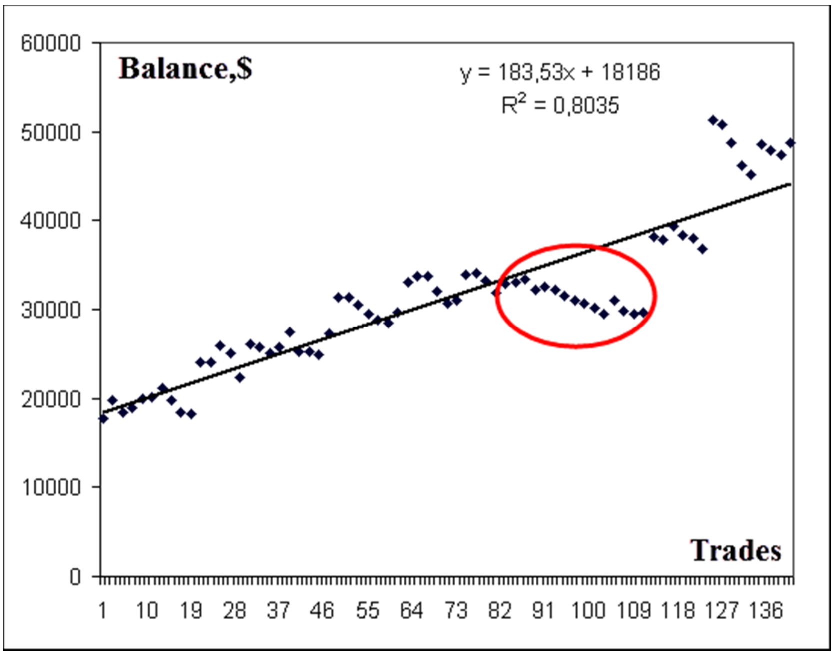


Figure 9. System III: trades - 71, profitable fraction - 45%, maximum drawdown - 27%, Cumulated profit - 388%, PL - 2.54, Calmar ratio=14.37, maximum loss series – 6

It is worth to note that the smoother equity growth is based on time frame diversification: algorithm adapts to prevailing market horizon and switches from long term to short term filters. As a result the sensitivity to market volatility has been decreased as well: the period of equity mean reversion now corresponds to 07.02.2013-22.04.2013. That is a definite improvement of efficiency - two months of low profitability in comparison to 1 year of System I. Finally a comparative table of two systems is represented.

Metrics	Trades number	Profitable fraction	Maximum drawdown	Cumulative profit	Profit/ Loss	Calmar ratio	Max. loss series
System I	125	37%	85%	241%	2.37	2.84	7
System II	71	45%	27%	388%	2.54	14.37	6

Table.1. Comparison of characteristics: simple channel breakdown (I) and breakdown with R/D filtering (II)

This preliminary estimation of *R*/*D* analysis allows making a qualitative conclusion: *R* and *D* precursors may improve the long term efficiency and stability of a quantitative trading model. However a logical scheme of precursors may be extended with use of a quantitative measurement of bifurcation functions (Section 7). This modification may add additional degrees and thus dangerous overfitting or date bias. At the same time it may discover potential for more refined and sensitive tuning of the *R*/*D* bifurcation nonlinear analysis.

9. Conclusions

The goal of this research was applying a nonlinear approach to the switching point detection. The analytical and numerical investigation of bifurcation analysis tools has been made. It has been shown that the transition process has a nonlinear nature. It has been proven that two scenarios of bifurcations are possible: slow and fast bifurcations. In first case a small deviation may transfer a system into one of attractors by unstable transition. According to a second scenario a fundamental shift may force a system to make nonequilibrium transition, overcoming attraction.

Slow bifurcation assumes that control parameter is changing slowly in relation to the system characteristic time - a switching delay exists. Gradual absorption of information provides stability loss delay effect which forms a tendency in the market.

Unlike slow bifurcation fast bifurcation has a discrete nonequilibrium nature. It has been shown by catastrophe theory that each transition from one attracting cycle to another one is preceded by passing through fixed point state – an effect of precatastophic stabilization. Small fluctuations than naturally correspond to small dispersion of investor's biases and preliminary system stabilization.

Two analytical methods have been developed for recognition of slow and fast bifurcation: *R* analysis and *D* analysis correspondingly. Each of these methods refers to the nonlinear effects of stability loss delay and preliminary stabilization. *R* analysis is based on introduction of extended Reynolds control parameter. Methods used for analysis of laminar-turbulent transition have been applied. Recognition of fast bifurcation is built on the basis of fractal random walk model – *D* analysis. The diffusion law has been explained in terms of Hurst stability description. We assumed that a fast bifurcation is preceded by transition to focus – a very small area of deviations. The nonlinear diffusion decreasing is considered as a precursor of fast transition.

Combined *R*/*D* analysis has been incorporated for the analysis of world financial crisis of 2008. S&P500 price action is analyzed with use of slow and fast bifurcation functions. It turned out that *R* analysis is more convenient for long term investment while *D* analysis suggests middle- and short-term approach. First bearish signals confirm a long and middle term tendencies of mortgage failure.

R/D analysis has been applied as a filter for currency positional trading system. Slow and fast bifurcation precursors have been applied for the filtering of breakdown signals. Incorporation of a filter allowed to reduce twice the number of trades and to increase system efficiency, Calmar ratio, by seven times. Another improvement regards adaptability of a system. An original breakdown system had high volatility sensitivity – equity curve stagnation corresponded to low market volatility and had 1 year duration. *R*/*D* filter allowed decreasing of sensitivity: duration of equity stagnation has fallen down to two months. The preliminary estimation of *R*/*D* analysis allowed making a qualitative conclusion: *R* and *D* patterns may improve the long term efficiency and stability of a momentum quantitative trading model. However a logical scheme of precursors may be extended with use of a quantitative measurement of bifurcation functions.

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