

A proof of the Riemann Hypothesis

Diego Liberati

Consiglio Nazionale delle Ricerche

One of the possible ways to deal with RH is to deal with Mertens function, and check it for all possible powers $(\frac{1}{2}+\varepsilon)$ of N when the natural N tends to infinity and $(\frac{1}{2} + \varepsilon)$ is between $\frac{1}{2}$ included and 1 excluded

The key point is that, even tending to infinite, N always stays finite, and finite stays the amount of primes and composite naturals. Thus the factor parity checked by Mertens function may be affected by naturals only for rational values of the exponent $(\frac{1}{2}+\varepsilon)$, ruling out each real ε not rational.

Let us now consider separately each subset of possible naturals N identified by each still allowed rational ε : every rational value greater than $\frac{1}{2}$ and lower than 1 may be represented as a fraction among naturals. After possible simplifications, its minimal form will exhibit a numerator not lower than 2. Thus, all the naturals not ruled out by the denominator and then possibly belonging to each of such subsets will not be squarefree: they can all be discarded in the computation of the Mertens function: then no real ε different from 0 does imply squarefree naturals contributing to the subset defined by the considered exponent.

Then ε has to be taken equal to 0 or at most infinitesimal in the sense of Robinsons' non standard analysis (making $(\frac{1}{2}+\varepsilon)$ iperreal but still equivalent to $\frac{1}{2}$ for our standard analysis of the said power $(\frac{1}{2}+\varepsilon)$ of N) thus proving Riemann Hypothesis.