There are infinitely many sextuplets of primes

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Every sextuplet of primes is centered on an odd multiple of 15, like (7 11 13 17 19 23), (97 101 103 107 109 113) and so on, but not all the odd multiples of 15 are the center of a prime sextuplet: this is because some of them are sieved by primes higher than 5. Let us call the odd multiples of 15 the candidate centers of a sextuplets of prime.

In fact, every prime p higher than 5 does sieve one of such candidate sextuplet, in at least one of its six positions, 6 times every p candidates, thus not sieving p-6 sextuples every p candidates.

Then, in order to compute how many candidate sextuplets are not sieved by any of such infinitely many primes greater than 5, one needs to compute the product of the infinitely many fractions p-6/p over all the primes p greater than 5.

When p tends to the infinity, both the numerator and the denominator of such product tend to the infinity with the same strength (even if their fraction tends to be infinitesimal towards zero, quite slowly indeed, because p-6/p tends to increase toward 1 when p increases).

Thus, when the denominator tends to the infinity, also the numerator does, proving that there are infinitely many sextuplets of primes not sieved by any lower prime, then proving the conjecture.

It follows that there are infinitely many prime quadruplets, being a quadruplet, like (11 13 17 19) the core of every sextuplet, like (7 11 13 17 19 23)

It also follows that there are infinitely many twin pairs, being a quadruplet, like (11 13 17 19) formed by a pair of twin pairs, like (11 13) and (17 19). Thus **Polignac conjecture is true for n=2** and then **twin primes conjecture is true**.

Moreover, it follows that **Polignac conjecture is also true for n=4**, being every center of the quadruple an isolated composite, thus centering a consecutive prime gap of 4, like the 'cousins' (13 17). (Besides that, also the extreme pairs of every sextuplet, like (7 11) and (19 23) are cousins).