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A Graviton Condensate Model of Quantum Black Holes and Dark Matter

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Abstract

We propose a model of microscopic black holes and dark matter to reinforce the standard model. We assume that at the center of a black hole there is a spin $\frac{1}{2}$ neutral core field. The core is proposed to replace the singularity of the hole. During Starobinsky inflation gravitons condensate around the core to form a primordial quantum black holes which evolve naturally into abundant dark matter universe.

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1 Introduction and Summary

The motivation behind the model described here is to find an economic way to go beyond the Standard Model (BSM), including gravity. Gravity would mean energies of the Planck scale, which is far beyond any accelerator experiment. This work is hoped to be a step forward in exploring the role on gravity in particle physics and big bang while any complete theory of quantum gravity is beyond the scope of this note.

In particular we pay attention to the nature of microscopic quantum black holes at zero temperature. We made earlier a gedanken experiment of what might happen when exploring a microscopic black hole deep inside with a probe. In [1, 2] we made two assumptions

(i) microscopic black hole has a fermion core field in Kerr metric. The point-like core field of the hole may have a high mass, something like the Planck mass. The core field is called here gravon,

(ii) the core is a point of condensation of gravitons, and also the stable or decaying remnant (possibly without a horizon) of a hole [3]. The black hole singularity is replaced by the core field.

The core is introduced to illustrate the case of singularity free black hole. It is at the same time a good candidate for dark matter by being a condensation point for bosons, gravitons in this model. It may also be a remnant of a radiating black hole with its own interactions. The point of this note is that the *core under Starobinsky inflation* [4] *is the key missing element of the standard model.*

With the Planck scale having its the conventional value 10^{19} GeV finding a gravon is hard. Gamma-ray signals from the sky may be a promising way. A gamma-ray, or particle, with energy half the Planck mass would be a clear signal of the models of this type.

The physical picture of gravity we are looking for in our scheme is as follows. It is closer to quantum chromodynamics rather than quantum electrodynamics. For color and electric charge the relevant objects, hadrons and atoms, respectively, are neutral, of course, as seen from a proper distance while mass cannot be similarly "hidden". Even for a single massless core in vacuum, very close to the core virtual particle pairs are created and destroyed, making a cloud of mass/energy around the core, all objects interacting gravitationally with the core and each other. At shortest scales mainly neighboring gravons and gravitons interact. When the scale is increased gradually larger blocks of gravons and virtual particles interact. With the renormalization group techniques it is expected that an equivalent of curved spacetime is created. In the classical limit general relativity will be obtained from the Starobinsky action. All this happens within the gravitational field of everything else in the universe. It may mean that quantum cosmology needs simultaneously be developed.

2 Why the Core?

While all the details of the present model, like the terms in (1) or (2) below, are not yet known we feel that the model gives at least in principle reasonable answers to a few but important long term problems in theoretical physics.

We propose (i) the core is "virtually" there in the form a M_{Planck}-object which is the nature constant obtained final mass of a classical black hole before it disappears totally. Alternatively, or even more probably, the M_{Planck}-object may be a remnant, either stable or with long lifetime. Remnants have no singularity or information loss problems [2], (ii) the core goes very well together with the Starobinsky inflation model creating a universe with substantial amount of dark matter together with the standard model matter, (iii) we see no problems with unitarity of the present model, and (iv) the model gives a physically appealing interpretation to the nature of Hawking radiation.

More traditional theories have been studied extensively like the various versions of the inflationary model (IM) and the minimally supersymmetric model (MSSM). The IM has an inflaton with a few parameters (as the present model obviously) while the MSSM has a doubled particle spectrum with 120 new parameters. The MSSM has candidates for dark matter. But neither IM or MSSM can solve the singularity or information loss problem, as far as the author is aware. We cannot emphasize enough the physical and philosophical importance of these questions: singularity and information loss/unitarity. It is, of course, ultimately an experimental verification what is needed to accept one model or some other.

3 Inflation, Primordial Black Holes and Big Bang

The inflationary picture of the first phase of the universe is very successful model to explain the evolution of our universe in the Big Bang framework. It has provided us a mechanism to explain among other things the primordial fluctuations observed in the cosmic microwave background (CMB). There exists several models of inflation in the literature, many based on a scalar inflaton field. Perhaps the first model proposed was due to Starobinsky [4, 6], for a review see [7]. Instead of a scalar field he proposed a modified theory of gravity in which the action is written as a general function $f(R)$ of the Ricci scalar R

$$S = \int d^4x\sqrt{-g}f(R) \tag{1}$$

for which a most obvious simple case is

$$S = \int d^4x\sqrt{-g}\Big(\frac{1}{16\pi G}R + \frac{1}{b}R^2\Big) \tag{2}$$

[2]The skeptical reader is advised to consult the recent review [5] before reading on.

with the dimensionless coupling $b = 6M^2/M_{\text{Planck}}^2$, where M is a constant of mass dimension one, $M_{\text{Planck}} = G^{-1/2}$, G is the Newton's constant with scale dependence and g is the determinant of the metric. In this model inflation is driven purely by the gravitational sector, which we find very appealing. The gravitational sector creates gravitons and gravons in spacetime before the standard model particles are created in the bang. Gravons possess the basic Planck length scale, in other words, they are heavy. Therefore they can collect graviton condensates around them which become primordial black holes turning later into dark matter. It is safe to say that the amount of dark matter in the universe can be qualitatively high in this model.

Starobinsky inflation occurs at sufficiently high curvature regime in the early universe when the R^2 term dominates the action resulting in an unstable inflationary period with exponential expansion. As the curvature decreases with time the first, the Einstein-Hilbert term begins to dominate, and inflation ends with graceful exit [6].

Another important property of the Starobinsky action is that making a non-perturbative renormalization group (RG) analysis it leads to asymptotically safe (AS) gravity. There exists a non-trivial, or non-Gaussian, UV fixed point, where G is asymptotically safe and the R^2 coupling b vanishes. This vanishing of the coupling b in the UV turns out to be of great importance for a succesful inflationary behavior.

Asymptotic safety was first introduced by Weinberg [8] and it states that a UV complete theory for gravity is obtained by assuming that gravity is non-perturbatively renormalizable through the existence of a non-trivial interacting fixed point under the RG. The starting point for RG calculations is an exact renormalization group equation (ERGE) in Wilsonian context [6].

4 Condensed Boson Model

4.1 Bose-Einstein Condensate

It likely that one, or more gravon(s) behave as a nucleus around which gravitons condense. Gravons form the seed for black holes and dark matter in our scheme. Dark matter exist abundantly all over the galaxies. We must look for a process for the formation of core surrounded by a graviton condensate.

A Bose-Einstein condensation model has been studied by Dvali and Gomez [9], see also [10]. Freely citing them, we give below a brief summary of their model to see the mechanism of condensation.

Graviton interaction is the starting point. The graviton-graviton interaction dimensionless coupling constant is

$$\alpha = L_{\text{Planck}}^2/L^2 \tag{3}$$

where L is a characteristic wave length of the gravitons participating in the interaction and L_{Planck} is the Planck length. Newton's constant G_{N} is related to the Planck length by $L_{\rm Planck}^2 = \hbar G_{\rm N}$. The physical meaning of the coupling α is understood as the relativistic generalization of Newtonian attraction between two gravitons. The attraction between two non-relativistic massive particles of mass m can be written in terms of α as

$$V(r)_{\rm Newton} = -\hbar\frac{\alpha}{r} \tag{4}$$

with the only difference that for a massive particle m is its Compton wave length, $L = \hbar/m$. The difference for gravitons is that the role of the Compton wave length is replaced by the actual wave length.

Gravitons can self-condense into black holes. To see this let us localize as many soft gravitons as possible around a core within a region of space of size L. We try to form a condensate of gravitons of characteristic wave length L by gradually increasing the occupation number N. For small N the gravitons behave like photons, and the condensate requires external binding forces. As we increase N the effects of the graviton interaction become large. Individual gravitons feel strong collective binding potential and for the critical occupation number

$$N = N_c = \frac{1}{\alpha} \tag{5}$$

the graviton condensate becomes self-sustained. The condition for this can be obtained by equating the kinetic energies of individual gravitons, $E_k = \hbar/L$, with the collective binding potential $V = -\alpha N\hbar/L$

$$E_k + V = (1 - \alpha N)\frac{\hbar}{L} = 0 \tag{6}$$

The concept of maximal packing is that the system is so densely packed that its defining characteristics becomes simply N. In particular

$$L = \sqrt{N}L_{\rm Planck}, \quad \alpha = \frac{1}{N}. \tag{7}$$

For gravitons being in an overpacked point means that further increase of N without increasing L becomes impossible. Any further increase of N results in the increase of the wave length in such a way that the system stays at the maximal packing point (7). Equation (6) indicates that the critical point (7) can be achieved for arbitrary N, but decrease of L beyond $L < \sqrt{N}L_{\rm Planck}$ would result into an even stronger bound system. This collapse of L can happen but it cannot take the system out of the critical point (7). The reason is that the decrease of L is balanced by the decrease of N due to quantum depletion and leakage of the condensate. The condensate slowly collapses and it looses gravitons at the same rate. So the systems always stays at the critical point (7).

The reason for the leakage is that due to the interaction with the other gravitons some of the gravitons get excited above the ground state. The ground state energy is within $1/N$ from the escape level and the gravitons gaining energies above this tiny gap leave the condensate for the continuum. The condensate starts to leak with a depletion rate essentially given by

$$\Gamma_{leakage} = \frac{1}{\sqrt{N} L_{\text{Planck}}} + L_{\text{Planck}}^{-1} \mathcal{O}(N^{-3/2}) \tag{8}$$

This can be understood from the following. Since the graviton-graviton coupling in the condensate is $1/N$ the probability for any pair of gravitons to scatter is suppressed by the factor $1/N^2$, but this suppression is compensated by a combinatoric factor $\sim N^2$ counting the number of available graviton pairs.

The quantum depletion rate translates into the following leakage law

$$\dot{N} = -\frac{1}{\sqrt{N} L_{\text{Planck}}} + L_{\text{Planck}}^{-1} \mathcal{O}(N^{-3/2}) \tag{9}$$

where the dot means time derivative. This quantum leakage of the graviton condensate becomes Hawking radiation in the semi-classical limit, which is defined as the following double scaling limit

$$N \to \infty, \quad L_{\text{Planck}} \to 0, \quad L = \sqrt{N} L_{\text{Planck}} = \text{finite}, \quad \hbar = \text{finite} \tag{10}$$

Thus the semi-classical limit is the limit in which all the quantum physics of the condensate decouples as $1/N \to 0$ and becomes impossible to resolve. The condensate becomes now a collection of infinite number of infinitely soft non-interacting bosons.

The thermality of Hawking radiation follows from the leakage law. Rewriting N in terms of the black hole mass we get the Stefan-Boltzmann law for a black hole with Hawking temperature $T = \hbar/L$

$$\dot{M} = -\frac{\hbar}{L^2}. \tag{11}$$

The exponential suppression of higher frequencies, usually attributed to the thermality of the source, follows from the combinatorics of the quantum depletion. The underlying quantum physics of this thermal-like spectrum has nothing to do with the thermality of the source, since condensate is in fact cold, but with the underlying quantum physics of BEC being at the overpacked critical point.

4.2 Microscopic Model

In [9] a simple prototype model, based on standard theory of Bogoliubov-de Gennes equation, is considered, which captures the key features of the phenomenon. Let $\Psi(x)$ be a field operator that describes the order parameter of the Bose gas. The particle number density is given by the correlator $n(x) = \langle \Psi(x)\Psi(x) \rangle$. A simple Hamiltonian that takes into account the self-interaction of the order parameter is

$$H = -\hbar L_0 \int d^3x \Psi(x) \nabla^2 \Psi(x) - g \int d^3x \Psi(x)^+ \Psi(x)^+ \Psi(x) \Psi(x) \tag{12}$$

where L_0 is a parameter of length dimensionality and g is an attractive interaction coupling constant of dimensionality $[\text{length}]^3[\text{mass}]$. We set the system in a finite box of size R with periodic boundary conditions $\Psi(0) = \Psi(2\pi R)$. The normalization condition is

$$\int d^3x \Psi^+ \Psi = N. \tag{13}$$

Performing the plane-wave expansion

$$\Psi = \sum_k \frac{a_k}{\sqrt{V}} \exp^{\mathrm{ikx/R}} \tag{14}$$

where $V = (2\pi R)^3$ is the volume and $a_k, a_k{}^+$ are particle creation and annihilation operators, The rewritten Hamiltonian is

$$\mathcal{H} = \sum_k k^2 a_k{}^+ a_k - \frac{1}{4}\alpha \sum_k a^+_{k+p} a^+_{k'-p} a_k a_{k'} \tag{15}$$

where $\alpha = \frac{4gR^2}{\hbar V L_0}$ and $\mathcal{H} = \frac{R^2}{\hbar L_0} H$.

We will now study the spectrum of low lying excitatons about a uniform BEC. We assume that most particles occupy the $K = 0$ level and study the small quantum fluctuations about this state. The spectrum of fluctuations is determined by the Bogoliubov-De Gennes equation. In a first approximation we can use the Bogoliubov replacement

$$a_0{}^+ = a_0 = \sqrt{N_0} \sim \sqrt{N} \tag{16}$$

of the ground state creation and annihilation operators into classical c-numbers. This approximation relies on taking $N \gg 1$ and $\hbar \neq 0$. Keeping only terms up to quadratic order in a_k^+, a_k for $k \neq 0$, and taking into account the normalization condition (13)

$$a_0 a_0 + \sum_{k\neq 0} a_k^+ a_k = N \tag{17}$$

leads to the following Hamiltonian describing the small quantum fluctuations

$$\mathcal{H} = \sum_{k\neq 0} (k^2 + \alpha N/2) a_k^+ a_k - \frac{1}{4}\alpha N \sum_{k\neq 0} (a_k^+ a_{-k} + a_k a_{-k}) \tag{18}$$

In order to diagonalize the Hamiltonian we perform a Bogoliubov transformation

$$a_k = u_k b_k + v_k^* b_k^+ \tag{19}$$

The Bogoliubov coefficients are given by

$$u, v = \pm \frac{1}{2} \Big(\frac{k^2 - \alpha N/2}{\epsilon(k)} \pm 1 \Big) \tag{20}$$

leading to the following spectrum of the Bogoliubov modes

$$\epsilon(k) = \sqrt{k^2(k^2 - \alpha N)} \tag{21}$$

The Hamiltonian in terms of b-particles is diagonal and has the following form

$$\mathcal{H} = \sum_k \epsilon(k) b_k^+ b_k + \text{constant} \tag{22}$$

As is seen in (21) the first Bogoliubov energy vanishes for

$$N = N_c = \frac{1}{\alpha} \tag{23}$$

and the system undergoes a quantum phase transition. The essence of this phase transition is that fot $N > N_c$ the first Bogoliubov level becomes tachyonic and the uniform BEC is no longer a ground state. Taking into account $\frac{1}{N}$-corrections it is clear that the gap between the uniform ground state and the Bogoliubov modes collapses to $\frac{1}{N}$ and becomes extremely cheap to excite these modes. So by quantum fluctuations the system starts to be populated by Bogoliubov modes easily. This means that the condensate starts to undergo a very efficient quantum depletion. The number density of the depleted a-particles to each k-levels are given by

$$n_k = |v_k|^2 \tag{24}$$

Since n_k decreases as $1/|k|^4$ for large $|k|$, the total number of depleted particles is well approximated by the first -level depletion

$$\Delta N \sim n_1 = \big(\frac{1 - \alpha n/2}{\sqrt{1 - \alpha N}} - 1\big) \sim \sqrt{N} \tag{25}$$

The striking similarity of the above BEC physics with the black hole quantum picture suggests that in both cases we are dealing with one and the same physics of a quantum phase transition. Indeed the physics of the graviton condensate is reproduced for the particular case of $L_0 = R = L$ and $g = \hbar L_{\text{Planck}}^2$.

The criticality condition (23) is nothing but the self-sustainability condition (5) which implies that the graviton condensate is maximaly packed (7). The energy gap too the first Bogoliubov level is given by

$$\epsilon_1 = \frac{\hbar}{L\sqrt{N}} = \frac{\hbar}{N L_{\text{Planck}}} \tag{26}$$

This expression summarizes the remarkable property of maximally packed systems: The energy cost of a collective excitation can be made arbitrarily low by increasing the occupation number of bosons in the condensate.

Thus by increasing N one can encode essentially unlimited amount of information in these modes. In the semi-classical limit (10) the energy gap collapses to zero and the BEC, the black hole, becomes an infinite capacitor of information storage.

This is a very general property of overpacked BEC's which are at the critical point of quantum phase of quantum phase transition. In both cases the cold atomic system [11, 12] versus the graviton condensate the key point is the maximal packing. The overpacking of the system results in the collapse of the mass gap and the Bogliubov modes become degenerate within an $1/N$ window. These almost degenerate Bogoliubov modes are the quantum holographic degrees of freedom that are responsible both for the entropy as well as for the efficient depletion of the system.

4.3 Effect of Core in BEC

We first consider the effect of a highly localized δ-impurity on the BEC in one dimension [13]. This approach indicates that the density of the BEC is substantially increased in the vicinity of an attractive impurity, which enhances inelastic collisions and may result in the loss of the impurity atom. In addition, a scaling argument has been given to show that attractive impurity-BEC interactions can lead to a point-like ground state of the impurity in 2D and 3D.

It is reasonable to assume that at the Planck scale no point-like ground state is formed (that's what we want to avoid) because of uncertainty relations and possible repulsive action of gravity at the shortest didtances. Rather it is expected that the gravons help to provide seeds for graviton condensation.

5 Discussion and Conclusions

The present note contains some tentative thoughts, and references elsewhere, how to go a short but important step beyond the standard model towards a model of Planck scale phenomena, assuming the standard model is valid up to that scale. At the Planck scale black holes are the key objects of quantum gravity to study. Unfortunately existing model calculation results concerning Planck mass region black holes are still unreliable. And a key idea is still missing. On the other hand, ERGE based calculations provide rather solid results for $f(R)$ gravity.

The next task is to find a real quantum action for the model of this note as a field theory, first for pure gravity later one and more standard model particles included. Pure gravity should be taken in this model as gravon and graviton terms in a quantum condensed state that will correspond the Einstein equation (5). A realistic model of quantum gravity should start from the microscopic entities operating at the quantum scale, the Planck scale. Then the methods of the new model theory, be it quantum field theory or something else, will be introduced to calculate the properties of the model like the UV behavior of the interaction.

The scheme discussed here can be summarized as having the gravon and the graviton the fundamental elementary particles of quantum gravity, to be included to the standard model. The BEC dressed gravon, going through Starobinsky inflation and Bose-Einstein condensation, is a natural candidate for black holes and dark matter in the universe.

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