The Twin Paradox Analogy in Classical Physics

Vadim N. Matveev[1], Oleg V. Matvejev

Imagine an *h*-deep flat bottom still water pond. On its water surface, there is a boat fitted with a pendulum clock and instruments activated by signals generated by this clock (in time with this clock). The function of the clock pendulum is performed by a high-speed shuttle continuously moving along a vertical (relative to the boat) line between the boat and the bottom. Each run of the shuttle to the bottom and back takes the time $\Delta t=2h/Vz$, where Vz is the rate of dive and emergence of the underwater shuttle, and the process is accompanied by a change in the clock readings. The shuttle moves at a constant speed V relative to water, and when the boat is at rest, the shuttle moves perpendicular to the bottom. The speed Vz of dive and emergence of the shuttle is equal to V. The time Δt of the shuttle run to the bottom and back is equal to $2h/V$. The speed V of the shuttle exceeds that of the boat (v), i.e. the condition $v<V$ is satisfied.

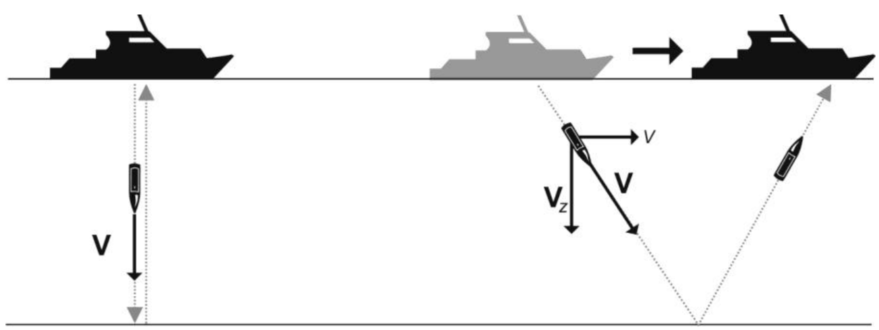

Fig. 1. The ship on the left is at rest on the water surface. A shuttle moves at a velocity of V from a barge to the bottom and back. The ship on the right is moving at a velocity of v along the water body surface. The speed of movement of the shuttle equals V,

[1] matwad@mail.ru

the shuttle's horizontal velocity component equals v, and the vertical component, V_Z, equals $V\sqrt{1-(v/V)^2}$.

If the boat moves at a speed *v*, then the clock tick rate and operating speed of the instruments onboard decrease. This is due to the fact that, with the boat moving at a speed *v*, the speed *Vz* of the dive and the emergence of the shuttle travelling between the boat and the bottom of the pond along the hypotenuses of right triangles, equals $V\sqrt{1-(v/V)^2}$. The operating speed of instruments activated by the signals emitted by the clock is reduced accordingly. The "time" *t'(v)* related to the instruments of the boat moving at a velocity *v* is passing $1/\sqrt{1-(v/V)^2}$ times slower than the time *t* related to the instruments of the boat at rest. Thus, the faster the boat moves on water, the less often the pendulum "swings" and the slower is the performance of the instruments aboard, whose operating speed is proportional to the frequency of the shuttle pendulum. If at some point in time the clock on a boat in motion and at rest had zero readings, then at any time thereafter the ratio is true:

$$t'(v) = t\sqrt{1-(v/V)^2} \qquad (1)$$

Let us assume that the time *t* is the basic one, concurrent with our time, and the time *t'* is the supplementary proper time of the instruments on the boats in motion. The further arguments will be presented in relation to the basic (our) time *t*, referring to the *t'* readings of the clock on the boat in motion only when necessary.

Now the question arises.

What will happen if one of the two boats that are at rest on water will float away at a speed *v* to a distant point, and then after a while it will return to the boat that has remained at rest?

The answer is clear. As shown by the formula (1), the instruments aboard the **non-inertial** boat travelling there and back will reflect $1/\sqrt{1-(v/V)^2}$ less time than those aboard the **inertial** boat at rest, and the former ones, having performed fewer operations, will grow less "old", if the wear of the instruments is proportional to the number of operations performed.

What if two boats are sailing side by side at a speed *v* relative to the water, then one of them halts, waits for a while and then catches up with the boat sailing ahead? The instruments aboard the inertial boat that continues its voyage but is missing information on the presence of water and on the fact of its motion relative to the water perceive such a manoeuvre of the halted boat as a voyage of the latter to a remote point and back.

Let us show that if the proper times of the voyage there and back of the non-inertial boat relative to the inertial one are equal, then again, the instruments of the non-inertial boat will reflect $1/\sqrt{1-(v/V)^2}$ times less of instrument-fixed time than the ones on the inertial boat in motion, and the instruments on the non-inertial boat will grow less "old".

Let at the time of the halt of one of the boats the clocks of the boats show zero. Suppose that having made a certain pause the boat that had lagged behind at the time t_1, when its clock due to the halt showed this time, commenced its voyage at a speed u, such that $v<u<V$, following the boat that was moving away from it. The distance between the boats at the start of the lagging boat is equal to vt_1. Setting off at a time t_2, within a period of time equal to $vt_1/(u–v)$ the lagging ship will catch up with the boat sailing at a constant speed v. During this time the clock of the boat sailing at a speed u after the boat that is floating away will tick away its proper time, which is $1\big/\sqrt{1-(u/V)^2}$ times less than our time and equals $vt_1\sqrt{1-(u/V)^2}\big/(u-v)$. We assume that the velocity u is such that the proper time $t'(u)_2–t'(u)_1$ of the overtaking boat is numerically equal to its halt time t_1, i.e., $t'(u)_2–t'(u)_1= t_1$ or

$$t_1 = vt_1\sqrt{1-(u/V)^2}\big/(u-v) \qquad (2)$$

This equality satisfies the condition under which a twin in the twin paradox spends the same proper time on the way to a remote destination and back. By elementary transformations of the equality (2) we can obtain the value of the velocity u, which equals $\frac{2v}{1+(v/V)^2}$. Substituting this value in the expression of the time $vt_1/(u–v)$ required for the return of the boat, and summing the time $vt_1/(u–v)$ and the time t_1, we obtain the basic (our) time spent by the lagging boat during the halt and return to the sailing boat. This time equals $2t_1\big/(1-v^2/V^2)$. As the clock on the inertial boat sailing at a velocity v goes $1\big/\sqrt{1-(v/V)^2}$ times slower than our clock, the instruments on this boat will determine the time spent by the lagging boat during the halt and return to the sailing boat as a value satisfying the equality

$$t'_2 = 2t_1\big/\sqrt{1-(v/V)^2} \qquad (3)$$

As the time elapsed on the non-inertial boat by the moment of its return is numerically equal to $2t_1$, and the time on the inertial boat numerically equals $2t_1\big/\sqrt{1-(v/V)^2}$, the time that has elapsed on the non-inertial ship during this period is $1\big/\sqrt{1-(v/V)^2}$ times less and the instruments on the non-inertial boat grew less "old" than on the inertial one.

Combining the boats in groups - those resting on water and those sailing - we can obtain all kinematic effects and paradoxes within Special Relativity in the symmetrical form. Within the scope of elementary classical physics we can likewise obtain Lorentz transformations and the four-dimensional "space-time" of the water reservoir [1-2].

References

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