Galactic disk dynamics from galactocentric potential-simple harmonic motion

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ABSTRACT
Galactocentric gravitational force is used to model disk stars' kinematics in the vertical direction. Stellar oscillation perpendicular to the galactic plane is modeled as simple harmonic motion (SH) made by the centrifugal force (Mach principle) opposite to the gravitational pull by the enclosed mass within the star's galactic orbit. It was found that the galactocentric force of rotationally supported disks produces the observed stellar velocity ellipsoid tilt at all disk vertical distances in the solar neighborhood. The observed galactic rotational velocity at the solar location is used to obtain the galactocentric gravitational field of $g_r = 2 \times 10^{-10}\, m/s^2$. The velocity ellipsoid tilt angles for the sun as well as stellar mid-plane velocities vs. maximum heights computed from SH model were compared with those computed from mass models of the Milky Way. It was found that a galactocentric potential alone that produces the stellar rotational velocity, with no adjusting parameters, describes well the kinematics of the disk in the vertical direction.

Key words: galactocentric gravitational potential, simple harmonic motion, stellar perpendicular motion, dark matter.

1 INTRODUCTION

The mass (baryonic and dark) and its distribution within a galaxy governs the kinematics of stars within it. An ideal galactic potential may be approximated by assuming that all the mass of the galaxy is a point mass concentrated in the galactic center. For stellar kinematics close to or within the galactic disk we usually take into account the detailed mass distribution, i.e., we include local mass distribution. In this paper we generalize a global glactocentric point-like potential to describe the stellar kinematics anywhere in the galaxy including within a galactic disk. Specifically, we describe disk stellar kinematics in the solar neighborhood by approximating a galactocentric point-like potential and derive from it the disk kinematics by considering only stellar rotational velocity as the sole determining factor.

The ratio of two scale parameters that describe stellar spatial distribution are the radial scale-length and the vertical-scale height which describe the axis ratio of stellar velocity ellipsoid (Van der Kruit & Grijs 1999). In the radial direction the velocity dispersion is related to the epicyclic frequency through the Toomre parameter Q for local stability. The Tully-Fisher relation then relates the stability criterion to the integrated magnitude of the disk scale-length. The vertical velocity dispersion relates directly to the scale-height through hydrostatic equilibrium. While Stability Criterion for disk galaxies (Toomre 1964) involves parameters that relate to local disk (surface density) and a global parameter (orbital frequency) there is no consensus regarding the origin of the orientation of the axes of the velocity ellipsoid (tilt) in a stable disk, but is thought to depend on the galactic mass distribution, as the shape and orientation of the velocity ellipsoid are believed to be connected to the symmetry of the underlying galaxy potential (Lindblad 1930; Lynden-Bell 1962; Amendt & Cuddeford 1991). In the current gravitational paradigm of galactic dynamics, the shape of the dark matter halo plays a role in defining the velocity ellipsoid parameters. In that line, we relate the stellar velocity ellipsoid tilt to the galactic radial gravitational field that originates from the "total" mass enclosed by the star's orbit and approximated as a point source mass located at the galactic center. In other words, the ellipsoid tilt is approximated as due to stellar movement about the center in the perpendicular plane of the disk due to the proposed global galactocentric potential.

Stellar velocity ellipsoid shape can be determined from the stellar kinematics in our Galaxy (Chereul et. al. 1999; Soubiran et al. 2008; Siebert et al. 2008). For gravitationally supported disk galaxies, disk stars are confined to small regions of the potential. Kuijken & Gilmore (1989) models galactic disk potential with two distinct components; one originating from the local galactic disk (surface density) and the other one from local contribution from a corona which is defined by the author as roughly a spherical mass component which includes baryonic and dark matter. Here we extend the concept of the corona as a global contribution by galactocentral mass (including dark matter) enclosed by stellar galactic orbits including the local disk contribution to the potential by introducing a simple harmonic motion (SH) model of a gravitational pendulum made by the rotating star and sustained by the gravitational field of the galactocentric potential.

The aim of this paper is two folds: one is to estimate the velocity ellipsoid tilt angles at all heights in the solar neighborhood as uniquely suggested by the SH model; the other one is to generate a relationship between the stars' maximum heights above the mid-plane and their mid-plane velocities as extrapolated from the galactocentric SH model.

2 DYNAMICS OF GALACTOCENTRIC SIMPLE HARMONIC MODEL

Different methods are used to analyze the galactic potential in the solar neighborhood, such as, measuring the Galactic escape speed of high velocity stars (Smith et. al. 2007 & Famaey et. al. 2007); the force perpendicular to the galactic plane (Oort 1960; Cr´ez´e et. al. 1998; Siebert et. al. 2003; Nipoti et. al. 2007; Holmberg & Flynn 2000); the velocity distribution (Bienaym´e 1999); or the stellar velocity ellipsoid tilt above the Galactic plane (Ollongren 1962; Hori & Liu 1962; Lynden-Bell 1962; Siebert 2008).

Here, galactocentripetal-centrifugal force made by the total mass enclosed by the stars galactic orbit is modeled as a simple pendulum that extends from the center of the galaxy to the star with the star acting as the pendulum bob and the galactocentrifugal force as the restoring force made by the "Machian" background force. To picture the analogy, imagine a pendulum hanging from the ceiling by a string. Now picture that the earth rotating around the sun with the ceiling is fixed toward the sun. The tension in the string would be the galactocentripital force and the earth gravitational force would be the restoring galactocentrifigual force.

Gravitationally supported disks possess angular momentum that allows the sun to rotate around the galactic center which introduces a centrifugal force from the background (Mack's principle). Stellar rotational motion about the galactic center is made by the galactocentral gravitational force of the total mass enclosed by the star's galactic orbit. Stellar oscillation about the galactic plane occurs, inspired by local perturbation, i.e., string-pendulum analogy is when you pull the pendulum up and let it swing about the equilibrium point.

3 ESTIMATE OF VERTICAL VELOCITY ELLIPSOID TILT AND MAXIMUM HEIGHT OF STARS FROM THE GALACTIC MID-PLANE

The velocity ellipsoid tilt can be derived from galactic mass models such as by Dehnen & Binney (1998) and its revised parameters provided by Binney & Tremaine (2008). Here, we are estimating the maximum height of stars vs. their mid-plane velocity as well as velocity ellipsoid tilt angles from (3-5) that were derived from the SH model (see Fig. 1).

Models that explain galactic disk dynamics exploit disk galactic potentials to determine the galactic velocity-distance relation. The inclination angle δ of the velocity ellipsoid with respect to the vertical (Siebert 2008) is commonly related to the gravitational potential and governed by,

$$\tan 2\delta = \frac{2\sigma_{UW}^2}{\sigma_U^2 - \sigma_W^2} \tag{1}$$

where σ_{UW}^2, σ_U^2, σ_W^2 are the second-order velocity distribution moments in the (U,W) velocity plane where U is stellar velocity in the galactic radial direction and W is the velocity in the vertical direction.

The common assumption to decouple vertical and radial motion in analyzing galactic dynamics has been demonstrated to be incorrect (Alex et. al. 2014). The authors showed that the different stellar subpopulations yield consistent results only when we allow the velocity ellipsoid in the disk to be tilted. The author also found that the velocity ellipsoid tilt angle increases with height $|z|$ from 5 ± 2^o at 0.5 kpc to 14 ± 3^o at 2.0 kpc, consistent with pointing toward the Galactic center at an angle $\tan(\alpha) \cong |z|/R$, which is close to alignment with a spherical coordinate system. This finding is confirmed by the proposed SH model, consistent with (3-5). For SH model, the tilt of the velocity ellipsoid is directly coupled to the shape of the gravitational potential which allows mass representation at the galactic center as point source and thus must be the same for all stellar subpopulation.

Here, we estimated the mid-plane velocity to maximum height produced by the galactocentric potential of the SH model. We also computed the velocity ellipsoid tilt vertical angles at different heights. We found that they agree with the estimated values from galactic dynamic models. In particular, we compared the velocity-distance of the stars in the solar neighborhood represented by the sun perpendicular to the mid-plane with those obtained by Kuijken & Gilmore (1989b) in the solar neighborhood, and the velocity ellipsoid tilt with those obtained by Zwitter (2008) and by Alex et. al. (2014).

Building galaxy models using separable potentials is the basis of modern galactic dynamic models such as Eddington (1915). Such models are currently well developed and showed that physically motivated models of elliptical galaxies could be built from separable potentials (De Zeeuw 1985; Binney &Tremaine 1987; Chandraskehar 1840).

In Kuijken & Gilmore (1989b) model, the author describes galactic disk dynamics in the vertical direction by the following vertical force,

$$-K_z = 2\pi GK\frac{z}{\sqrt{z^2+D^2}} + 4\pi GFZ \tag{2}$$

where the K_z is the force field in the z direction and D is a disk scale-height. The first term in the r.h.s. is the contribution by the local disk with a surface density K and the second term is the contribution by a halo (including dark matter) of local F density. As discussed by Kuijken & Gilmore (1989b), the galactic rotation curve constrain the disk potentials and there must be a tradeoff between the disk parameter K and the halo parameter F, with halo contributes a significant part of the potential at high altitudes from the mid-plane (z~1 kpc); the lighter the disk, the heavier the halo has to be in order to make up for the missing radial acceleration. As has been the trend to model the disk potential, a local halo density is extrapolated. Comparing SH model to Kuijken & Gilmore (1989b), we combine both terms in (2) by approximating a galactocentric gravitational field (g_r) that is generated by a point-source mass enclosed by the star's galactic orbit. This is simply because the SH model includes the contributing disk mass enclosed by the Gaussian surface in the SH potential. We will proceed to calculate the disk stellar velocity-height and the velocity ellipsoid tilt angles.

In determining the parameter that describes the gravitational field g_r, a potential with no adjusting parameters, observational data is considered, i.e. at the location of the solar system, the field is taken as v_r^2/r, where v_r is the rotational velocity of the sun taken as $220\ km/s$ and r is the distance to the galactic center. Fig. 1 illustrates the mechanism of the sun vertical oscillation about the galactic mid-plane as described by the SH model.

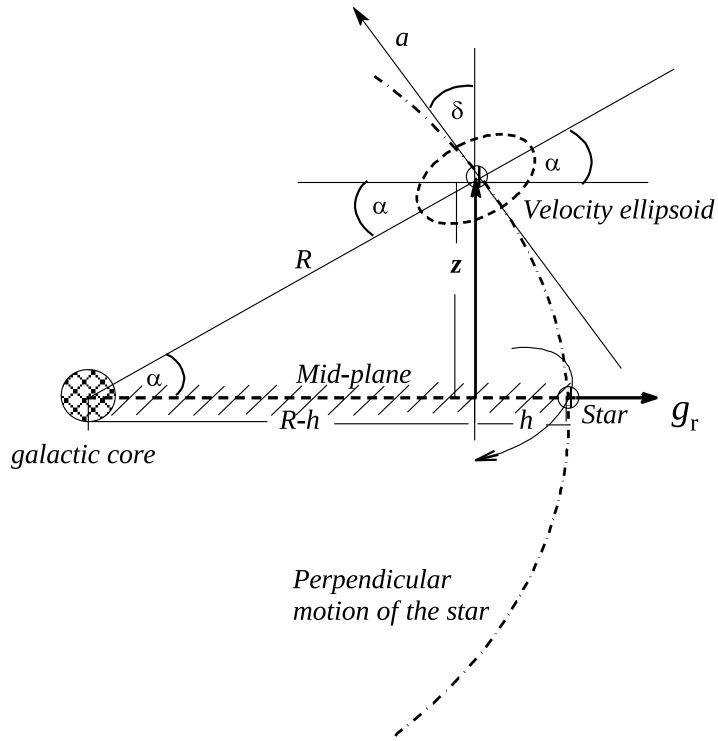

Figure 1. Vertical oscillation of the sun modeled as simple harmonic motion produces the velocity ellipsoid (dashed line) vertical tilt angle δ. Line a is tangential to the stellar path.

According to the SH model, the vertical tilt of the velocity ellipsoid simply occurs because the velocity in the direction of the stellar path (tangential line a) in the perpendicular plane defines the velocity in the direction of the minor axis of the velocity ellipsoid, while the radial velocity remains oriented along R , see Fig. 1. The symmetrical oscillation of the SH pendulum model about the galactic mid-plane explains why the velocity ellipsoid tilt pointing to the galactic center above and below the mid-plane.

In table 1, equations (3-5) were used to compute the velocity-distance and the vertical velocity ellipsoid tilt angles of the sun at all elevations following the illustration in Fig. 1,

$$v^2 = 2g_r h \tag{3}$$

where v is the tangential velocity of the sun along its path in the (U,W) plane in the local velocity coordinates, where U is the velocity component in the radial direction, W is the vertical velocity and g_r is taken as $2\times10^{-10}\,ms^{-2}$ computed from the rotational velocity of the sun, h is the height of the sun above the radial lowest point of the pendulum along the mid-plane. Equation (3) is derived from conservation of mechanical energy of the pendulum. Equations (4,5) are derived from geometrical configuration of Fig. 1.

$$R^2 = z^2 + (R - h)^2 \tag{4}$$

$$\alpha = \sin^{-1}\frac{z}{R};\ \delta = \alpha \tag{5}$$

where R is the distance from the sun to the center of the galaxy taken as 27700 *ly*, δ is the vertical velocity ellipsoid tilt which is the angle between the minor axis of the ellipsoid and the vertical *z*-axis, and z is the maximum height of the star.

4 VERTICAL TILT ANGLES, VERTICAL MAXIMUM HEIGHT

We used the data by Kuijken & Gilmore (1989b) for the mid-plane stellar velocities versus their vertical heights in the neighborhood of the sun, and using equations (3,4) we evaluate maximum stellar height versus mid-plane velocity for the solar neighborhood and equation (5) to generate vertical ellipsoid tilts corresponding to maximum stellar heights. This is done in Table 1 computed from the SH model and compared with those of Kuijken & Gilmore (1989b), which were computed from distribution function modeling technique from two separable components of the potential. Table 2 compares ellipsoid vertical tilt angles obtained by SH model exploiting equation (5) with those obtained by Alex et. al. (2014) exploiting equation (1) and using a well-characterized sample of >16,000 G-type dwarf stars from the SEGUE survey and fit their discrete kinematic data using a likelihood method that accounts for halo star contaminants. In combination with a Markov Chain Monte Carlo (MCMC) sampling, the authors robustly measured the velocity ellipsoid components as function of height away from the Galactic mid-plane.

Table 1 Solar mid-plane velocities vs maximum heights

Velocity (m/s) Kuijken & Gilmore (1989b) At Z=0	Z_{max} (Pc) Kuijken & Gilmore (1989b)	Z_{max} (Pc) SH
0	0	0
8400	100	149
15700	200	279
21900	300	390
27300	400	486
32100	500	571
36500	600	650
40600	700	722
44500	800	792

48300	900	859
51800	1000	921
58600	1200	1042
68300	1500	1214
83200	2000	1477

Table 2 The tilt angles of the velocity ellipsoid in the Milky Way disk at the solar radius.

Z (Pc) Alex et. al. (2014)	α *in degrees* Alex et. al. (2014)	α in degrees SH Equation (5)
425	4.92 ± 1.83	3.05
522	4.21 ± 1.91	3.74
589	5.05 ± 1.92	4.22
653	4.23 ± 2.06	4.68
715	6.71 ± 2.12	5.13
777	7.93 ± 1.87	5.58
841	6.88 ± 2.23	6.04
1064	7.44 ± 2.31	7.65
1156	9.50 ± 2.32	8.31
1263	9.12 ± 2.84	9.09
1392	9.26 ± 2.65	10.03
1546	10.88 ± 3.21	11.15
1724	9.67 ± 2.91	12.45
1949	13.62 ± 2.69	14.12

It is noted that the SH column in Table 1 represents the contribution from the total enclosed mass by the solar orbit. In the SH model, the height and the velocity ellipsoid tilt are solely determined by the proposed Gaussian mass-potential. By comparing the tilt angle of 6.75^0 at 1 kpc obtained by simple harmonic model by Zwitter (2008) of $7.3^0 \pm 1.8^0$ and by Alex et. al. (2014) we find a close estimate.

5 IS IT A MONDIAN POTENTIAL OR EXTENDED DARK MATTER HALO ONE?

Modified Newtonian Dynamics (MOND) is a theory that successfully describes flat rotation curves by modifying Newtonian dynamics without resorting to mysterious, yet to be detected, Dark Mater (DM) halo (Milgrom 1983). MOND, described by rotational velocity of $v = \sqrt[4]{GMa_0}$, describes gravitational field as proportional to r^{-1}at weak fields with critical acceleration of $a_0 = 1.2 \times 10^{-10}\, m/s^2$ as compared to Newton's force law which is assumed as proportional to r^{-2} at all field strengths. Stellar accelerations in galactic disks fall within the weak field defined by MOND. The theory remarkably describes flat stellar rotational velocity correctly. SH model considers a galactocentric potential as the sole parameter that describes disk dynamics including stellar dynamics perpendicular to the disk; an indication of a preference for MONDin potential over a DM one.

DM models that describe disk dynamics so far have been focusing on extended global potentials with contribution from local disk. In that line of reasoning, galactic models describe DM extended halos such that it produces flat rotational curves. SH model successfully includes the local contribution of the disk in a one-global potential.

DM halo profiles are normally described as a function of the altitude from the galactic center with a scaling radius. For example, a DM profile that is extensively used to simulate cold DM halos is the NFW profile (Navarro et. al. 1996) which defines the potential in terms of a characteristic density and a scale radius of the distribution. Another popular halo profile used to describe RCs of galaxies and the kinematics of individual spirals (Salucci et. al. 2007) points to dark halos with a central constant-density core, in particular, to the Burkert halo profile (Burkert 1995; Salucci & Burkert 2000), where the relative density distribution is described in terms of central density, the core radius and another two free parameters. The RC then can be modeled in terms of the contributions from the stellar disk, the bulge, the gaseous disk and the dark matter halo (Karukes et. al. 2015). So, galactic mass models define stellar rotational velocities and rotational velocities in turn should in principle define disk kinematics perpendicular to the disk plane as well. This is what the SH model is set to achieve.

To ensure good approximation with a point-like galactocentric global potential, it should include DM halo that must conspire with baryonic mass to produce flat rotation curves. Disk velocity ellipsoid tilt as well as maximum-height to mid-plane velocity must follow suit. Whether it is a DM extended halo conspiring with normal matter or a MOND potential it is yet to be conclusively determined. A MOND potential can describe a global one term-potential well while a DM halo/baryonic composite potential is a difficult compromise to describe a solo-galactic potential that describes the whole of galactic dynamics with no adjusting parameters. Such a compromise might need new physics as of how dark matter conspires with baryonic matter precisely to obtain flat rotation curves. Such an attempt was made by the same author of this paper by which DM and baryonic matter gravitationally self-repel but mutually attract (Barghout 2014; Barghout 2015).

We conclude that, separable (unrelated) local and global potentials to makeup the governing galactic potential (as has been the trend to separate the potentials) may not be an accurate approach since both potentials are indeed overlapping locally, i.e., local disk mass is part of the global radial mass as described by SH dynamics. Therefore, it seems that a working global potential at all radial distances is an indicator of a self-sustaining dynamical force law as that of MOND and a DM-controlled universe by which an unexplained DM-baryonic tuning is indeed an approach of which an alternative DM-baryon interaction is highly needed.

6 CONCLUSION

It has been shown that a single galactocentric gravitational potential, with no adjusting parameters, obtained from a simple harmonic model describes well galactic disk dynamics. Stellar vertical velocity ellipsoid tilt angles and mid-plane velocities versus vertical heights were computed for the solar radius up to 2000 pc above the mid-plane and found to agree with theoretical models that describe galactic dynamics in the solar neighborhood. It was concluded that the total mass, including a dark matter halo, enclosed by the orbit of the star is the primary cause of both stellar velocity ellipsoid tilt and stellar mid-plane velocities versus their perpendicular maximum heights. The SH model successfully includes the local contribution of the disk in a one-global galactocentric potential that describes both stellar rotational velocities and perpendicular to plane disk kinematics. By considering a global potential of the simple harmonic type to describe disk dynamics we conclude that dark matter halo must conspire with baryonic mass to produce flat rotation curves and disk velocity ellipsoid tilt must follow suit.

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