A Plain Proof of Fermat's Last Theorem[1]

ABSTRACT. This paper offers a plain proof of Fermat's Last Theorem using the cosine rule.

1 Introduction

Fermat's Last Theorem states that no positive integers x, y, z satisfy $x^n + y^n = z^n$ for any integer $n > 2$.(cf. [1]) This paper will offer a plain proof of Fermat's Last Theorem using the cosine rule.

2 Proof

$$x^n + y^n = z^n \ (2 < n \in \mathbb{Z}^+; x, y, z\text{: pairwise coprime; } \mathbb{Z}^+\text{: positive integer}) \tag{1}$$

2.1 For the case a, b, c : odd prime

Let a, b, c: odd prime p, then

$$x^p + y^p = z^p \ (p\text{: odd prime; } x, y, z\text{: pairwise coprime}). \tag{2}$$

If there exist x, y, z satisfying (2), then from (2) it follows that $(x+y)^p > z^p$, $x+y > z$, $z+x > y$, $z+y > x$. Accordingly, x, y, z always form a triangle. Thus, x, y, z satisfy

$$x^2 + y^2 - 2xy\cos\zeta = z^2; \angle\zeta\text{: opposite of } z. \tag{3}$$

Moreover, from (2) and (3) it follows that

$$(x^p + y^p)^2 = (x^2 + y^2 - 2xy\cos\zeta)^p. \tag{4}$$

Then, let z be a constant, the graphs of (2) and (3) must meet each other at least at one point (x, y). Thus, there is no need for (4) to be an identity. However, x, y of the point (x, y) must satisfy $x + y \mid x^p + y^p = z^p$, i.e. $(x+y)^2 \mid (x^p + y^p)^2$, hence x, y of the point (x, y) must satisfy

$$(x+y)^2 \mid (x^2 + y^2 - 2xy\cos\zeta)^p. \tag{5}$$

Then, $x^2 + y^2 - 2xy\cos\zeta = (x+y)^2 - 2xy(1+\cos\zeta)$, and $(x+y)^2 > 2xy(1+\cos\zeta)$ because $(x-y)^2 < (x+y)^2 - 2xy(1+\cos\zeta) < (x+y)^2$. Hence, $(x+y)^2 \mid x^2 + y^2 - 2xy\cos\zeta$ is possible only when $1 + \cos\zeta = 0$, i.e. $p = 1$. Hence, (4) cannot be satisfied when $(x+y)^2 \mid x^2 + y^2 - 2xy\cos\zeta$.
Moreover, (5) cannot be satisfied, when $x^2 + y^2 - 2xy\cos\zeta$ is divisible not by $(x+y)^2$ but only by $x+y$. It is because $2 \nmid p$. This means that in this case (4) cannot be satisfied.
Accordingly, no pairwise coprimes x, y, z satisfy (1) when n: odd prime. This means that according to the laws of exponents no pairwise coprimes x, y, z satisfy (1), even when $p \mid n$.
Hence, no pairwise coprimes x, y, z satisfy (1) for $2 < n \in \mathbb{Z}^+$, unless $n = 2^m$, where $2 \le m \in \mathbb{Z}^+$.

2.2 For the case $n = 2^m$

$$x^4 + y^4 = z^4 \tag{6}$$

That no positive integers x, y, z satisfy (6) was proven by Fermat.([2]) Hence, according to the laws of exponents no positive integers x, y, z satisfy (1) for $a = 2^m$.

3 Conclusion

In conclusion, no positive integers x, y, z satisfy $x^n + y^n = z^n$, when n is a multiple of an odd prime or a multiple of 4, i.e., for any positive integer $n > 2$. QED.

References

[1] Wiles, A., Modular elliptic curves and Fermat's Last Theorem, *Ann. Math.* **142**(1995), 443–551.
[2] Freeman, L., Fermat's One Proof, http://fermatslasttheorem.blogspot.kr/, Retrieved 2015-04-18.

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