Quantum Gravitational Relativity – Part II

The theory presented in this paper is the second part of the quantum gravitational formulation of Einstein's special theory of relativity. This paper presents another plausible solution to the problem of length contraction introduced in Part I.

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1. The Quantum Gravitational Length Contraction

Let us assume that we have a rod of length l_0 fixed to a reference coordinate system S'. Let us also assume that the reference frame S' (which we shall call the *moving reference frame*), moves at a constant speed, *v*, in the *x* direction with respect to another reference system *S* (which we shall call the *fixed reference frame*). We also assume that there is no relative movement between S' and *S* in the other two directions (*y* direction and *z* direction). See Figure 1.

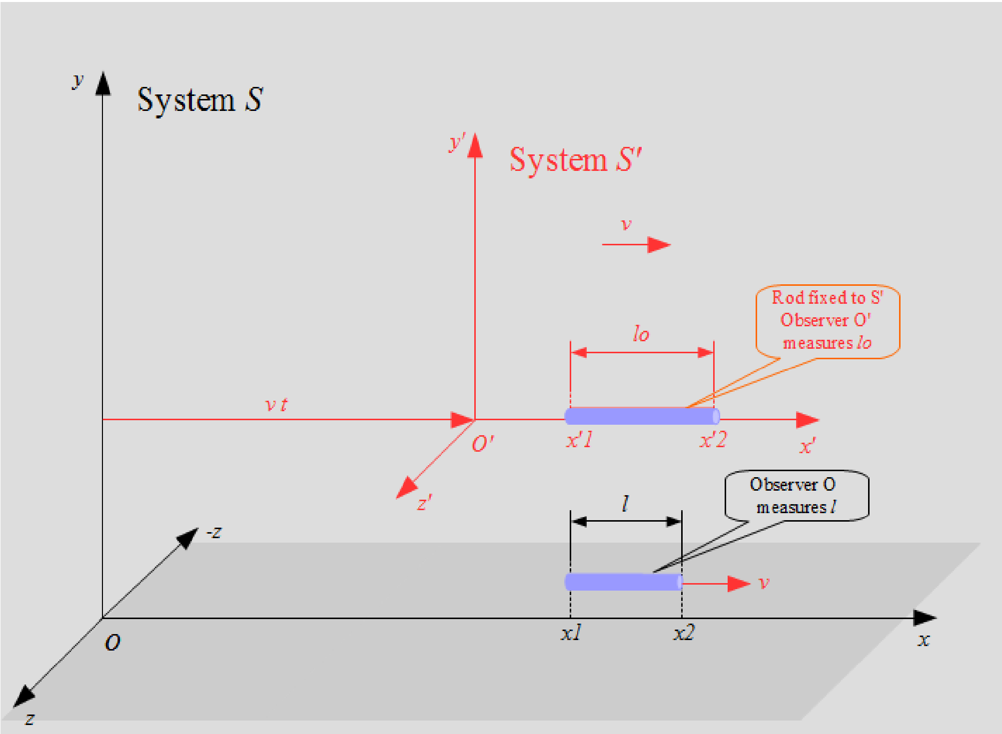


Figure 1. Reference frame *S′* moves at a constant speed, *v*, in the *x* direction with respect to reference system *S.*

We want to find the length of the rod, *l*, for an observer located on reference system *S*. To solve this problem we use the following equation of the Lorentz transformations:

$$x' = \frac{x - v\,t}{\sqrt{1-\beta^2}} \qquad (1.1)$$

where β is given by equation (1.3). Thus if we apply the coordinate transformation (1.1) to both ends of the rod we get the following two equations:

$$x'_1 = \frac{x_1 - v\,t_1}{\sqrt{1-\beta^2}} \qquad (1.2)$$

$$x'_2 = \frac{x_2 - v\,t_2}{\sqrt{1-\beta^2}} \qquad (1.3)$$

For an observer O' located on system S' the rod is at rest. Therefore this observer measures the following length:

$$l_0 = x'_2 - x'_1 \qquad (1.4)$$

Because l_0 is at rest with respect to observer O' this length is called *proper length*. Now we use the second side of equations (1.2), (1.3) to eliminate x'_2 and x'_1 form equation (1.4). This yields

$$l_0 = \frac{x_2 - x_1 - v(t_2 - t_1)}{\sqrt{1-\beta^2}} \qquad (1.5)$$

Normally, in Special Relativity, it is assumed that the two measurements of time made by the observer *O* of system *S* are simultaneous. As a consequence: $t_1 = t_2$. In this paper, however, I shall assume that the time difference $t_2 - t_1$ is not exactly zero but is equal to the minimum time T_{MIN} instead. I made this assumption because this seems to be the only other way to fixing the problem of length contraction (see Part I [1] for the other plausible solution). This non-zero time difference is imposed by nature, not by experimental error. Mathematically this means that:

$$t_2 - t_1 = T_{MIN} \qquad (1.6)$$

Now we can express equation (1.5) in terms of T_{MIN} as follows

$$l_0 = \frac{x_2 - x_1 - v\,T_{MIN}}{\sqrt{1-\beta^2}} \qquad (1.7)$$

Solving for $x_2 - x_1$ we get

$$x_2 - x_1 = l_0\sqrt{1-\beta^2} + v\,T_{MIN} \qquad (1.8)$$

But $x_2 - x_1$ is the length, *l*, of the rod measured by an observer located on system *S*.

$$x_2 - x_1 = l \tag{1.9}$$

Thus we can write

$$l = l_0 \sqrt{1-\beta^2} + v\, T_{MIN} \tag{1.10}$$

Now we assume that the minimum distance, L_{MIN} , and the minimum time, T_{MIN} , are related through the speed of light, *c,* through the equation:

$$c = \frac{L_{MIN}}{T_{MIN}} \tag{1.11}$$

This allows us to rewrite equation (1.10) as follows:

$$l = l_0 \sqrt{1-\beta^2} + \frac{v}{c} L_{MIN} \tag{1.12 a}$$

This is the formula for the quantum gravitational length contraction (QGR length contraction).
If we replace *v/c*, in this equation, by β , we get the formula for the length contraction in terms of this parameter:

$$l = l_0 \sqrt{1-\beta^2} + \beta\, L_{MIN} \tag{1.12 b}$$

Equivalently we can replace β , in equation (1.12 a) by *v/c* to obtain the formula for the quantum gravitational length contraction in terms of *v/c*:

$$l = l_0 \sqrt{1-\frac{v^2}{c^2}} + \frac{v}{c} L_{MIN} \tag{1.12 c}$$

This formula guaranties that the length, *l*, of a body will never appear to be smaller than the minimum length, L_{MIN} , regardless of the proper length, l_0 , of the body and its speed, *v*, with respect to the observer (of course l_0 must be greater or equal than L_{MIN}). This is reasonable since it doesn't make any sense that a body appears to be smaller than the minimum length imposed by nature (if there is one).

In other words nature does not have any length or distance smaller than L_{MIN} . So far no experiment could determine the exact value of L_{MIN} . However it is likely that this length to be equal to the Planck length, L_P . The quantum length contraction formula presented in **Table 1** assumes, precisely, that $L_{MIN} = L_P$. Having said that we have to keep in mind that L_{MIN} could be smaller than the Planck length (including a nil value in which case Special Relativity's length contraction formula will hold). We just don't know. Let us draw a table (**Table 1**) with the value of $\frac{l}{L_P} = \frac{l_0}{L_P}\sqrt{1-\beta^2} + \beta$ for different values of the parameter β and for $l_0 = n\, L_P$.

$\beta \equiv \frac{v}{c}$	$\frac{l}{L_P} = \sqrt{1-\beta^2}+\beta$ where $l_0 = L_P$	$\frac{l}{L_P} = 5\sqrt{1-\beta^2}+\beta$ where $l_0 = 5\,L_P$	$\frac{l}{L_P} = 10\sqrt{1-\beta^2}+\beta$ where $l_0 = 10\,L_P$
0	1	5	10
0.000 001	1.000 001	5.000 001	10.000 001
0.099 504	1.094 538	5.074 688	10.049 876 (maximum)
0.1	1.094 987	5.074 937	10.049 874
0.196 116	1.176 700	5.099 020 (maximum)	10.001 919
0.2	1.179 796	5.098 980	9.997 959
0.3	1.253 939	5.069 696	9.839 392
0.4	1.316 515	4.982 576	9.565 151
0.5	1.366 025	4.830 127	9.160 254
0.6	1.4	4.6	8.6
0.7	1.414 143	4.270 714	7.841 428
$\sqrt{2}/2$	1.414 214 (maximum)	4.242 641	7.778 175
0.8	1.4	3.8	6.8
0.9	1.335 890	3.079 450	5.258 899
0.99	1.131 067	1.695 337	2.400 674
0.999	1.043 710	1.222 551	1.446 102
0.9999	1.014 042	1.070 609	1.141 318
0.99999	1.004 462	1.022 351	1.044 711
0.999999	1.001 413	1.007 070	1.014 141
1.0	1	1	1

Table 1: The quantum formulas for the Lorentz length contraction. Note that L_{MIN} has been replaced by L_P (Planck length). The maximum values for these curves are shown in colour.

2. Graphics and Analysis

Let us graph the length contraction formulas from Special Relativity:

$$l = l_0 \sqrt{1 - \frac{v^2}{c^2}} \qquad (2.1)$$

and from Quantum Gravitational Relativity (QGR). In this case we shall assume that:

$$L_{MIN} = L_P \qquad (2.2)$$

This means that formula (1.12 c) becomes

$$l = l_0 \sqrt{1 - \frac{v^2}{c^2}} + \frac{v}{c} L_P \qquad (2.3)$$

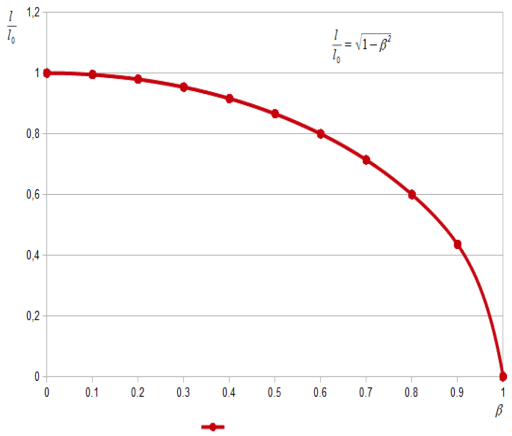
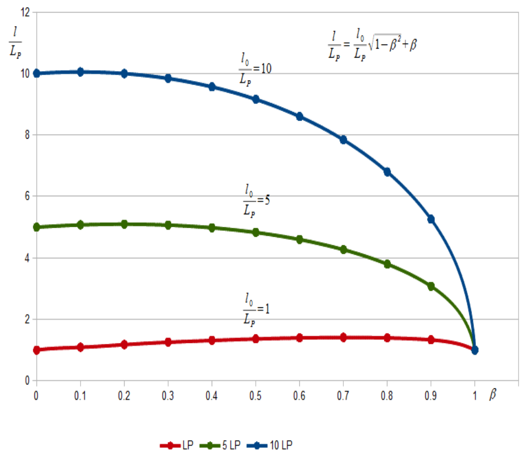
	Special Relativity (SR)	Quantum Gravitational Relativity (QGR)
Length contraction formula	$l = l_0 \sqrt{1 - \frac{v^2}{c^2}}$ (2.1)	$l = l_0 \sqrt{1 - \frac{v^2}{c^2}} + \frac{v}{c} L_P$ (2.3)
Graphics	Graph 1 	Graph 2 

Figure 2: The graph on the left (graph 1) corresponds to the relativistic length contraction formula. The graphs on the right (graph 2) corresponds to the quantum gravitational formula. Note that the red curve of graph 2 is always greater than 1 for all values of v in the interval: $0 < v < c$. This means that, in this case, there is no *length contraction* but *length dilation* only.

Graph 1 of **Figure 2** indicates that, according to Special Relativity, for an observer, *O*, fixed to a reference system *S*, the length of a body will always appear to be contracted in the direction of movement. If the body's proper length is equal to the Planck length, then this observer will always measure a length smaller than the Planck length. This violates the second principle of this theory (see Part I). However, graph 2 of **Figure 2** tell us a completely different story. According to QGR the observed length of a body for the same observer will never appear to be smaller than the Planck length. Thus a particle of length $l_0 = L_P$, for example, will always appears to be longer for any observer for which the particle is not at rest. This result does not violate the second postulate of this theory. Another point to observe in graph 2 is that there is an interval of velocities for which there is no *length contraction* in the direction of movement but the opposite: *length dilation.* In particular, for $l_0 = L_P$, there is no length contraction at all (see red curve of graph 2, **Figure 2**). Thus a body whose proper length is the Planck length, L_P , will always appear to be longer for an observer that is not at rest with respect to the body.

We shall now find the derivative of function (2.3) with respect to the velocity, *v*. The result is

$$\frac{dl}{dv} = -\frac{l_o}{c}\frac{\beta}{\sqrt{1-\beta^2}} + \frac{L_P}{c} \qquad (2.4\text{ a})$$

or equivalently:

$$\frac{dl}{dv} = -\frac{v}{c^2}\frac{l_0}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{L_P}{c} \qquad (2.4\text{ b})$$

Now we shall find the value of β (which we shall call β_{max}) for which function (2.4 a) has a maximum. This means that we must find the value of β that satisfies the equation:

$$\frac{dl}{dv} = 0 \qquad (2.5)$$

The solution to this equation is:

$$\beta_{max} = \frac{1}{\sqrt{1+\left(\frac{l_0}{L_P}\right)^2}} \qquad (2.6)$$

Thus the value of β_{max} for the three cases considered in **Table 1** are:

Case 1: $l_0 = L_P$

$$\beta_{max}(l_0 = L_P) = \frac{\sqrt{2}}{2} \approx 0.707\,107$$

Case 2: $l_0 = 5\,L_P$

$$\beta_{max}(l_0 = 5\, L_P) = \frac{\sqrt{26}}{26} \approx 0.196\,116$$

Case 3: $l_0 = 10\, L_P$

$$\beta_{max}(l_o = 10\, L_P) \approx 0.099\,504$$

3. Conclusions

Special Relativity predicts that a particle whose proper length is the Planck length will always appear to be contracted in the direction of movement. This behaviour violates one of the postulates of this theory: the *length quantization postulate*. This formulation, on the other hand, predicts that a particle whose proper length, l_0, is equal to the Planck length, L_P, will not appear to be contracted in the direction of movement. On the contrary, this particle will always appear to be longer than its proper length for any observer that is not at rest with respect to the particle. The problem with the solution to the length contraction problem presented in this paper is that, unless we invoke the Heisenberg uncertainty principle, we cannot justified the time difference equation: $t_2 - t_1 = T_{MIN}$. This means that we were forced to assume that nature does not allow simultaneous measurements. In other words we have assumed that we cannot perform two different measurement at the same time. The minimum time difference between any two measurements must be equal to the minimum time, T_{MIN}, imposed by nature. Finally we have assumed that this time equals the Planck time, T_P.

Appendix 1
Nomenclature

The following are the symbols used in this paper

$h =$ Planck's constant
$c =$ speed of light in vacuum
$G =$ Newton's gravitational constant
$l_0 =$ proper length
$l =$ "contracted" length (or "dilated" length according to QGR)
$t_0 =$ proper time
$t =$ "dilated" time
$T_{MIN} =$ Minimum time with physical meaning
$L_{MIN} =$ Minimum length with physical meaning
$T_P =$ Planck time
$L_P =$ Planck length
$v =$ speed of a massive body with respect to certain observer
$\beta =$ ratio between the speed of a massive body, v, to the speed of light, c.
$\beta_{max} =$ value of β for which function (2.4 a) has got a maximum.

REFERENCES

[1] R. A. Frino, *Quantum Gravitational Relativity – Part I*, vixra.org: viXra 1505.0160, (2015).