Orbital Energy and Perturbing Potential.

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ABSTRACT. This article checks a perturbing gravitational potential, with the energy of elliptic orbits. This potential produces a permanent decrease of the eccentricity and the semi-major axis however it is a conservative angular momentum motion. Results have consistent accuracy with the detected magnitudes of the unexplained perturbations of the Astronomical Unit and also with eccentricity of the Moon, but with the opposite sign. Perturbing potential is also consistent with the relativistic precession of planets.

Keywords **: Quantum Gravitational potential; Orbital Energy; Astronomical Unit; Eccentricity.**

1. Introduction. A Target moving inside a quantum Gravitational Potential.

We will analyze the effects produced by a theoretical heuristic perturbing potential, on the elliptic gravitational trajectory of a target. First, set an inertial frame with the origin in the barycentre of a two-body system, as we are going to examine a target's orbit as a geodesic free-fall path, isolated from other gravitational interference.

The virtual energy emission linked with this potential, whatever could be the background transmission agents as particles (gravitons) and/or electromagnetic fields, should also be continuously emitted and updated from its central focus. This continuous update, must also shape the general relativity curve space-time framework were, gravitational effects are not forces but the outcome of the "*geometric*" structure of the universe. This heuristic potential, should also be consistent with the quantization of the gravitational field.

As proposed by F. Wilczek. [1], while many aspects of general relativity have been tested, and general principles of quantum dynamics demand its quantization, there is no direct evidence for that. However, the Cosmic Microwave Background due to a long wavelength stochastic background of gravitational waves from inflation in the early Universe, would firmly establish the quantization of gravity.

Consider a target with a radial speed Vr related to the inertial frame, moving in the same forward direction as the energy radial emission. The transit time of the emission crossing through the target, will be larger related with the transit time when the object is in a rest position and will decrease, if they are moving in opposite directions. The larger or reduced transit time between target and potential, is proportional to Vr/c. We assume potential's transmission velocity, equal to that of light (c).

Be t_1 the transit time of a potential crossing through an object. If the target moves in the same forward direction as the emission, the transit time t_2 will be larger than t_1 and will have the following expression, only acceptable if $Vr \ll$ c (leaving aside second order terms in magnitude, as radial acceleration) :

$$t_2 \cdot c = t_2 \cdot V_r + t_1 \cdot c \tag{1}$$

$$\frac{t_2 - t_1}{t_1} = \frac{Vr}{c}\,\frac{c}{c - Vr} \Rightarrow \frac{Vr}{c} \tag{2}$$

This coefficient $(t_2\text{-}t_1) / t_1$, is the dimensionless ratio of the new real disturbing time $(t_2\text{-}t_1)$ related to the unperturbed transit time (t_1).

Since the potential is an energy field with work characteristics, perturbance is proportional to the square of time as it is the product of acceleration by distance, product equivalent to energy. The disturbance is not lineal with time nor with the radial distance. As quantum electrodynamic iteration, the intensity is proportional to $[(t_2-t_1) / t_1]^2$.

We must also notice that motion of particles in an external gravitational field with a Maxwell framework, is in first order equivalent to a dynamic system linked with $(v/c)^2$. [2].

Heuristic quantum perturbing potential $S(\phi)$, is then define as :

$$S(\phi) = \mp \frac{GM}{r}\left(\frac{t_2 - t_1}{t_1}\right)^2 = \mp \frac{GM}{r}\left(\frac{Vr}{c}\right)^2 \tag{3}$$

$S(\phi) < 0$ (same sign as gravity) for $0 < \phi < \pi$ and

$S(\phi) > 0$ for $\pi < \phi < 2\pi$. (ϕ = true anomaly)

Final gravitational potential $P(\phi)$, will be the classic field, added with the perturbing potential $S(\phi)$, linked with the radial velocity of the target.

Potential $P(\phi)$ is defined as a slight perturbation to the newtonian gravitational potential:

$$P(\phi) = -\frac{GM}{r}\left[1 \mp \left(\frac{Vr}{c}\right)^2\right] = -\frac{GM}{r} + S(\phi) = Newtonian\ Potential + Perturbing\ Potential \tag{4}$$

There is not therefore a new potential but the same classic gravitational field, added with an infinitesimal perturbing action, that increases/decreases slightly the force of gravity when the target has a radial speed. As the newtonian field, potential $S(\phi)$ has a clear physical basis, consistent with the laws of impulse and momentum transfer, angular momentum conservation and the action/reaction effect of the usual mechanics.

Point out that, if we apply potential $S(\phi)$ to any perfect sphere or any compact three-dimension target (instead of a single particle), the resultant ratio is three times $(Vr/c)^2$. [3]

Then, perturbing radial acceleration $Apr.$ produced by $S(\phi)$, would be :

$$Apr = \frac{d\,S(\phi)}{dr} = \frac{3GM}{r^2}\left(\frac{V_r}{c}\right)^2 \tag{5}$$

(positive, same sign as gravity) for $0 < \phi < \pi$ and negative for $\pi < \phi < 2\pi$.

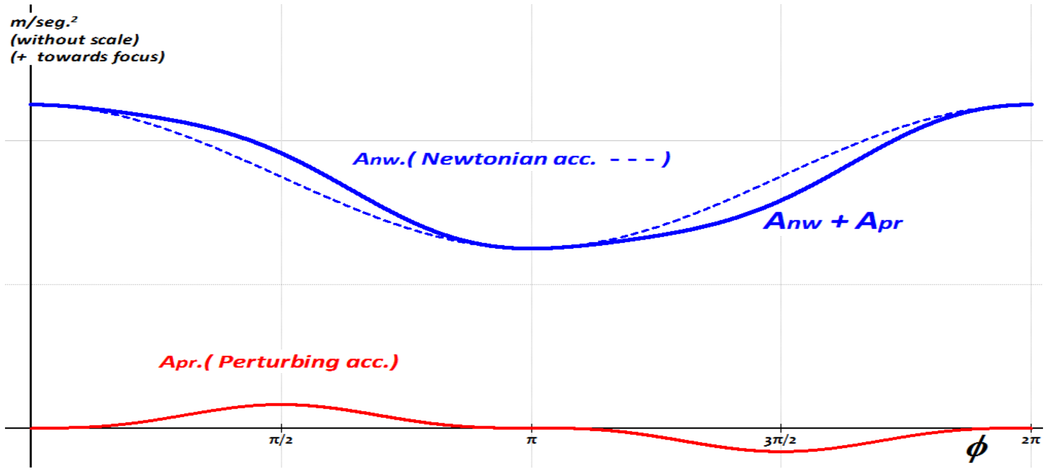

Figure-1

2. Orbital energy and Perturbing Potential S(ϕ).

A target following a closed elliptic trajectory, should be affected by this perturbing potential $S(\phi)$, linked with its own motion and velocity as any object embedded inside a gravitational potential.

As the target moves away from the Sun, the radial velocity has the same forward direction as the gravitational potential, so perturbing acceleration increases gravity. Perturbing acceleration is directed inward the orbit, so the target will move inward in relation with the position it should occupy in the expected track. As it comes closer to the Sun with a radial speed opposite to the gravitational potential, perturbing acceleration decreases gravity. The perturbing acceleration is then directed outside the orbit, so the target will move outward in relation with the position it should occupy in the keplerian trajectory. These inward and outward slight settings of the target, can be modeled as a real precession of the trajectory around the barycentre, turning a positive angle as the target´s motion. [4]

The elliptic trajectory has this geometric and gravitational parameters :

$$Vr = \dot{r} = \frac{e\, h\, \sin\phi}{p}; \quad r = \frac{p}{1 + e\cos\phi}; \quad r_i = \frac{p}{1+e} \quad ; \quad r_a = \frac{p}{1-e} \tag{6}$$

where e = eccentricity < 1; p = semi-latus; r_i = perigee; r_a = apogee; h = angular momentum per unit of mass

Then perturbing acceleration Apr, is :

$$Apr = \pm \frac{3(GM)^2}{c^2} \frac{1}{p^3} e^2 \left[\sin\phi\left(1 + e\cos\phi\right)\right]^2 \ \text{m/s}^2 \tag{7}$$

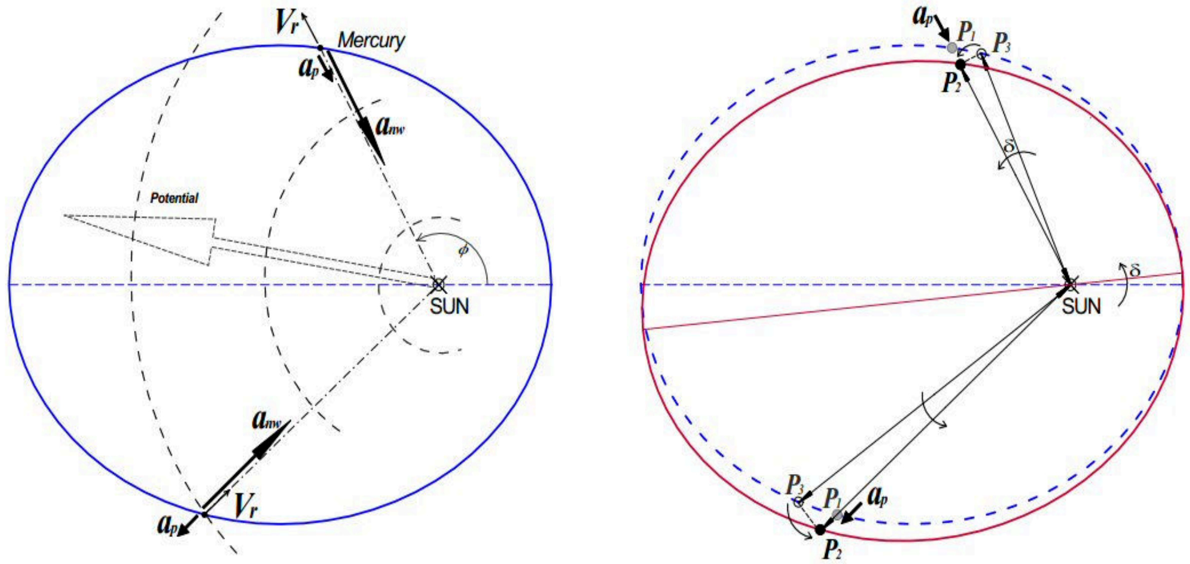

Figure-2 : Newtonian and radial perturbing acceleration. ***Vr*** = radial velocity. $\boldsymbol{a_{nw}}$ = newtonian acceleration. $\boldsymbol{a_p}$ = perturbing acceleration. $\boldsymbol{P_1}$= Position in the keplerian ellipse. $\boldsymbol{P_2}$= Position induced by perturbing acceleration. $\boldsymbol{P_3}$= Equivalent position of $\mathbf{P_2}$ in the keplerian ellipse. δ = Instantaneous Precession. ϕ = True anomaly.

As result of the increase/decrease of the transit time, perturbing acceleration *Apr.,* has the opposite direction of radial velocity and also against the radial motion of the target all along the orbit. These perturbing action means then, a continuous loss of kinetic energy, that is not recovered throughout the descending branch of the orbit, as it comes closer to the Sun. We must remember that a target under the classic Newtonian potential, also losses kinetic energy during the ascending branch, transforming it completely in potential energy, and recovers it totally during the descending branch. This is because gravitational acceleration has here, the same forward direction as radial velocity and motion. Newtonian gravitation is then an *energy* conservative potential but *S(ϕ)* is not however, both are *angular momentum* conservative motions, as they are only ruled by central forces. As a simple description, *S(ϕ)* should produce a gravitational "*drag*", but acting only against the radial velocity. The energy lost by unit of mass (*dE*) as result of the action of perturbing acceleration is :

$$dE = Apr \cdot dr \ ; \quad dr \ \text{ in a close elliptic orbit is } \quad dr = \frac{p}{(1+e\cos\phi)^2} e \sin\phi\, d\phi \tag{8}$$

Then,
$$dE = \frac{3GM}{r^2} \frac{V_r{}^2}{c^2} \frac{p}{(1+e\cos\phi)^2} e\sin\phi\, d\phi = 3\left(\frac{GM}{c\,p}\right)^2 e^3 \sin^3\phi\, d\phi \tag{9}$$

The total energy reduction by orbit should be :

$$\Delta E_{orbit} = 6\left(\frac{GM}{c\,p}\right)^2 e^3 \int_0^{\pi} \sin^3\phi\, d\phi = 8\left(\frac{GM}{c\,p}\right)^2 e^3 \quad jules\ /\ orbit \tag{10}$$

3. Energy and Orbital parameters.

The continue loss of energy along the orbit, will produce changes in its orbital parameters: a decrease of the semi-major axis (a) and eccentricity (e) . It will also produce a reduction of the orbital period (T).

Underline that the mentioned loss of energy, is only related to the theoretical newtonian one, as reference of this "perturbation method". Really, the trajectory should be an open geodesic as result of the target adapting itself instantaneously and in balanced with the energy obtained from the potential as it is, an energy emission altered by the own velocity of the target.

The equation of energy in a closed elliptic orbit is :

$$E = -\frac{GM}{2\,a} \quad ; \text{ then,} \quad dE = \frac{GM}{2}\,\frac{1}{a^2}\,da \tag{11}$$

Then, from equation (9), :

$$da = \mp\, 6\,\frac{GM}{c^2}\,\frac{e^3}{(1-e^2)^2}\,sin^3\,\phi\; d\phi \tag{12}$$

Point out that this result is exactly the same as derive from Gauss or Lagrange planetary equations or Burn equations [5] for an *Apr* perturbing acceleration. It is always negative as it is the continuous reduction of the orbital energy under the action of perturbing potential *S(ϕ).* Eccentricity is also influenced by this permanent cut down of the orbital kinetic energy. As result of Gauss planetary equation :

$$de = \mp\frac{3GM}{c^2 p}\,e^2\,\sin^3(\phi)\,d\phi\,; \text{ and in a complete orbit, } \quad \Delta e_{orbit} = -\,8\,\frac{GM}{c^2 p}\,e^2 \tag{13}$$

As deduction from equations (12) and (13),

$$\frac{da}{de} = \frac{2\,p\,e}{(1-e^2)^2} \tag{14}$$

which confirms that the semi-latus (p) is constant related with a and e, which is consistent with a conservative motion of the angular momentum (h) , however non conservative with the orbital energy.

As result of the major axis and eccentricity reduction and a constant semi-latus, the evolution of any elliptic orbit, means a continuous decrease of the aphelion distance and the opposite increase of the perihelion.

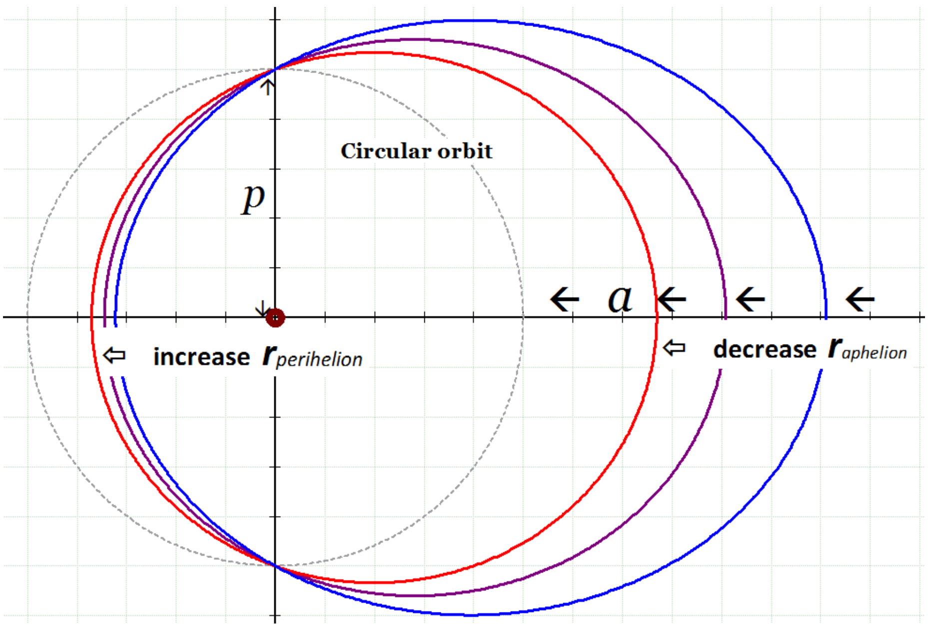

Figure-3

And then :

$$\Delta a_{orbit} = -\,16\,\frac{GM}{c^2}\,\frac{e^3}{(1-e^2)^2} \quad ; \quad \Delta r_i = -\,a\Delta e + \Delta a_{orbit}\cdot(1-e)\,; \quad \Delta r_a = a\Delta e + \Delta a_{orbit}\cdot(1+e) \tag{15}$$

First of all point out that the variations of the semi-major axis, only depends on the eccentricity. Underline also that as result of continuous decrease of eccentricity, elliptic orbits will progress to a nearly circular one, however with an extraordinary low rate of change . This also means that the reduction ratio of the major axis (Δa) is not constant along time and then (a), will converge to the semi-latus (p), after an "infinite" extension of time.

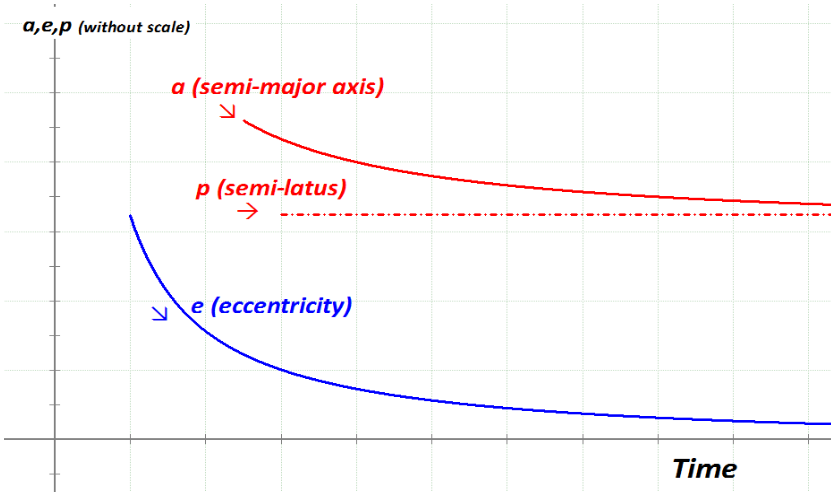

Figure-4

The orbital period will also decrease as it does the axis of the elliptic orbit :

$$T = 2\pi \sqrt{\frac{a^3}{GM}} \quad ; \qquad dT = 3\pi \sqrt{\frac{p}{GM(1-e^2)}}\, da \tag{16}$$

and in one orbit,

$$\Delta T_{orbit} = -\frac{48\,\pi}{c^2} \sqrt{\frac{GM\;p}{(1-e^2)^5}}\; e^3 \;\; sec. \tag{17}$$

If we apply this equations to the planets in the Solar System, we obtain the next results:

Solar System						/Orbit		/Year		/Orbit	/Year
Planets	e	a (m)	p (m)	Dec. a/Orbit(m)	Dec. e/Orbit	Dec. Apg.(m)	Inc. Peg.(m)	Dec. Apg.(m)	Inc. Peg.(m)	Dec. T (seg.)	Dec. T (seg.)
Mercury	0,2056	6,0450E+10	5,7894E+10	-224,0	-8,63E-09	-791,6	343,7	-3.080	1.337	0,045	0,18
Venus	0,0068	1,0821E+11	1,0820E+11	-0,01	-5,05E-12	-0,6	0,5	-0,90	0,88	2,00E-06	3,25E-06
Earth	0,0167	1,4964E+11	1,4960E+11	-0,11	-2,20E-11	-3,4	3,2	-3,40	3,18	3,48E-05	3,48E-05
Mars	0,0934	2,2993E+11	2,2793E+11	-19,6	-4,52E-10	-125,4	86,2	-65,8	45,2	0,008	4,03E-03
Júpiter	0,0484	7,8023E+11	7,7840E+11	-2,7	-3,56E-11	-30,6	25,2	-2,56	2,11	0,002	1,63E-04
Saturn	0,0541	1,4309E+12	1,4267E+12	-3,8	-2,42E-11	-38,6	31,1	-1,31	1,05	0,004	1,24E-04
Urano	0,0472	2,8774E+12	2,8710E+12	-2,5	-9,17E-12	-29,0	24,0	-0,34	0,28	0,003	4,10E-05
Neptun	0,0086	4,4984E+12	4,4981E+12	-0,02	-1,94E-13	-0,9	0,9	-5,39E-03	5,21E-03	2,607E-05	1,58E-07
Pluton	0,2488	5,7830E+12	5,4250E+12	-413,51	-1,35E-10	-1296,0	468,9	-5,3887	1,9499	8,131E-01	3,38E-03

Table-1

The most significant result is the reduction of 224 m per orbit in the semi- major axis of Mercury that means a decrease of 3,080 m in the apogee and an increase of 1,337 m in the perigee per year. This large magnitudes, should have been detected before if those effects where really produced, so we should conclude that $S(\phi)$ perturbing potential is not present in the motion of planets in the Solar System. However, we must consider that the location of Mercury in the theoretical Newtonian elliptic orbit, is altered by the relativistic precession and also the secular gravitational action of the rest of the Planets. That means a large real transversal shift of its perihelion, about 300 Km per orbit related with the theoretical keplerian one. Mention also that the apogee distance has periodic oscillations of about 1,000 Km each 6 years, following half of the period of planet Jupiter. Although it is highly improbable, may be those perturbations mentioned before, be "hidden" behind this real, and also not regular movements.

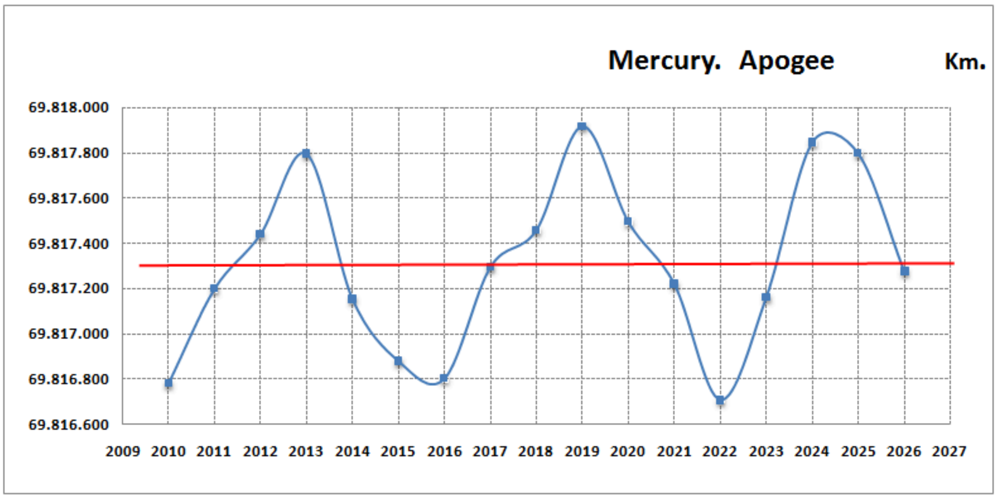

Figure-5. Apogee distance oscillations between years 2010 - 2026. (HORIZONS and INPOP13c)

Something similar takes place with the oscillations of the relativistic precession along the orbit of Mercury. There are few theoretical articles about this issue, neither the deduction of an accurate observational draw of the relativistic trajectory of Mercury, as an open geodesic free-fall path, isolated from other planets gravitational interference. It is supposed a lineal constant and gradual progression of precession, but without any observational radiometric deduction evidence yet.

Now that we are close to reach the centenary of the formulation and first success of General Relativity, in november 2015, Messenger spacecraft radiometric data, should provide an excellent opportunity to perform and update this classic test. It is certainly a difficult and complex duty but clearly available with the current development of our technology, and as far as I know, it has never been done before. Next INPOP planetary ephemerides and other works, should have the chance to develop it. [6], [7], [8].

The variations of eccentricity, major axis and period of some comets and asteroids should be :

COMETS	T (Years)	a (AU)	e	Dec. a /orbit (Km)	Dec. T/orbit (sec.)
2P Encke	3,31	2,22	0,847	-180	-84,71
45P Honda-Mrkos	5,25	3,02	0,824	-129	-71,02
73P Schwassmann	5,36	3,06	0,693	-29	-16,14
46P Wirtanen	5,44	3,09	0,658	-21	-11,66
81P Wild 2	6,40	3,45	0,539	-7	-4,31
76P West-Kohoutek	6,46	3,45	0,541	-7	-4,39
67P Churyumov	6,57	3,51	0,632	-17	-9,80
27P Crommelin	27,90	9,21	0,919	-759	-728,78
55P Tempel-Tuttle	32,90	10,33	0,906	-547	-556,64
1P Halley	75,98	17,94	0,967	-5.157	-6.911,46

Table-2

The reductions produced by the theoretical perturbing potential $S(\phi)$, are difficult to detect related to the unperturbed axis and the complete period of the orbit. It would be necessary to dedicate a future astrophysics observational programme, properly designed just to detect that so small magnitudes. The best candidates to study are Comet Encke, Honda-Mrkos and Comet Halley.

4. The Astronomical Unit.

The traditional definition of the astronomical unit has been to consider it as the mean distance between the Earth and the Sun. However the International Astronomical Union 2012 Assembly, (as adopted in IAU 2009), proposed the re-definition as a conventional unit of length equal to 149,597,870,700 m ± 3 m.

In 2004 Krasinsky and Brumberg [9] reported an increase of the astronomical unit (as a distance) of 15 ± 4 cm /year, secular variation within the covariant analysis of the standard error in AU. The data was obtained by astronomical detection through the analysis of radiometric measurements of distances between the Earth and the major planets including observations from Martian orbiters since 1.971. In 2005, Standish detected a secular rate of 7 ± 2 cm/year [10]. Highlight a recent comprehensive review about this anomalous issue and also a collection of unexplained phenomena within our Solar system [11], [12], [13]

There have been some theoretical proposal to explain it [14], [15], [16], [17], however it would be necessary to determine with better accuracy those data, after removing other non gravitational effects and clear up some controversial status about the issue.

As showed in Table-1, the increase of the distance between the Earth and the Sun produced by perturbing potential $S(\phi)$, is of 3,19 m /year in the perigee , and a decrease 3,41 m of in the aphelion, that means a rate reduction of the semi-major axis of 11,0 cm/year.

This result is perfectly consistent with the reference data. However, the increase only happens in the perihelion as consequence of a general reduction of the major axis and eccentricity, produced by the continuous and infinitesimal loss of energy mentioned before.

This effect of $S(\phi)$ perturbing potential perfectly consistent with the magnitude of the data detected, should need to be better analysed and also needs to confirm the reference points of the orbit where the data was collected.

Finally, we must also mention that $S(\phi)$, has also a perfect accuracy with the relativistic precession of planets.[3]

4. The Eccentricity of the Moon.

The anomalous increase of the eccentricity of the Moon has been presented in [18], collecting the data extracted by the Lunar Laser Ranging along 39 years since its deployment in the Moon by the Apollo missions. It has also deserve many articles searching a scientific explanation of this singularity of the motion of our satellite [19], [20]. Although, the motion is extremely complex because the influence of the Sun, the tidal and dissipation effects and the actions in a gravitational two body system.

The exact quotation of the reference document mentioned before [18] is :

…Accounting for the difference in de/dt from the simple LLR integration model and the more complete Earth model, the unexplained eccentricity rate is (0.9±0.3) x 10^{-11} / yr, equivalent to an extra 3.5 mm/yr in the perigee rate.

These means a decrease in the perigee, whose magnitude has been reduced to 2mm/year, as result of an improvement of the model of tides on the earth. The unexplained eccentricity rate increase can be determine to 0,5 x 10^{-11} /year [19].

If we apply equations (13) and (15) to the Earth-Moon system:

$$GM = 3.986 \times 10^{14} \ m^3 s^{-2} \ ; \ e = 0.0549 \ ; \ a = 3.84399 \times 10^8 \ m.$$

Then, the decrease of eccentricity in one orbit of the Moon is then : $e_{orbit} = 0{,}279 \cdot 10^{-12}$ (18)

and referred to a year :

$$e_{year} = \frac{365}{27{,}3} \cdot 0{,}279 \cdot 10^{-12} = 3{,}73 \cdot 10^{-12} \quad (19)$$

That means an increase rate of the perigee distance of 1.28 mm/year and a decrease rate in the apogee of 1.60 mm/year.

This effect of $S(\phi)$, is consistent with the magnitude of the reference data, however with the opposite sign as result of a non conservative perturbing potential that reduces the total energy of the system related with the newtonian orbit.

5. Conclusions and open comments.

S(ϕ) perturbing potential is a conservative angular momentum motion as it is only ruled by central forces. However produces a continuous dissipation of kinetic energy and then a permanent progress of elliptic orbit to a nearly circular one.

S(ϕ) perturbing potential, is accuracy consistent with the reported increase of the Astronomical Unit but only in the perigee. Produces a permanent increase of the perigee of the Moon, consistent with the collected data only in magnitude but with the opposite sign, equivalent to a continuous decrease of the eccentricity of the orbit.

*S(ϕ)*perturbing potential, is perfectly consistent with the relativistic precession of planets and produces a similar effect as the observed flat rotation curves of spiral galaxies It would be appropriate to complete the studies related with spiral galaxies, but now, with these new potential proposals.

Now that we are close to reach the centenary of the formulation and first success of General Relativity, Messenger spacecraft, should provide an excellent opportunity to perform and update this classic test and check the relativistic gradual progression of precession along a complete orbit and other variations if there are any. Near 100 years since then, the scientific community has not develop yet the deduction of an accurate observational draw of the relativistic and complete trajectory of Mercury, as an open geodesic free-fall path, isolated from other planets gravitational interference.

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