The General Robertson-Walker solution and new solution

Sangwha-Yi

Department of Math , Taejon University 300-716

ABSTRACT

In the general relativity theory, if we use Einstein's gravity field equation, we can discover the General Robertson-Walker solution and new solution in the cosmology.

PACS Number:**04,04.90.+e,98.80,98.80.E**

Key words:The general relativity theory,

The gravity field equation

The cosmology

The Robertson-Walker solution

New solution

e-mail address:sangwha1**@nate.com**

Tel:051-624-3953

1.Introduction

This theory's aim is that it discovers the General Robertson-Walker solution and new solution of the gravity field equation.

The Einstein equation is

$$R_{\mu\upsilon} = -\frac{8\pi G}{c^4}(T_{\mu\upsilon} - \frac{1}{2}g_{\mu\upsilon}T^{\lambda}{}_{\lambda}) \qquad (1)$$

In this time, the energy-momentum tensor $T_{\mu\upsilon}$ is

$$T_{\mu\upsilon} = pg_{\mu\upsilon} + (p/c^2 + \rho)U_{\mu}U_{\upsilon} \quad , \quad U_{\mu} = (c,0,0,0)$$

$$T_{00} = \rho(t)c^2 \quad , \quad T_{ij} = p(t)g_{ij} \qquad (2)$$

The metric tensor $g_{\mu\upsilon}$ is

$$g_{tt} = -1 \quad , g_{rr} = U(t,r) \quad , \quad g_{\theta\theta} = V(t,r), g_{\phi\phi} = V(t,r)\sin^2\theta \qquad (3)$$

In the cosmology, the proper time is

$$d\tau^2 = dt^2 - \frac{1}{c^2}[U(t,r)dr^2 + V(t,r)(d\theta^2 + \sin^2\theta d\phi^2)] \qquad (4)$$

$$\frac{1}{U}R_{rr} = \frac{1}{U}(\frac{V''}{V} - \frac{V'^2}{2V^2} - \frac{U'V'}{2UV}) - \frac{\ddot{U}}{2U} + \frac{\dot{U}^2}{4U^2} - \frac{\dot{U}\dot{V}}{2UV} = -\frac{4\pi G}{c^2}[\rho(t) - p(t)/c^2] \qquad (5)$$

$$\frac{1}{V}R_{\theta\theta} = -\frac{1}{V} + \frac{1}{U}(\frac{V''}{2V} - \frac{U'V'}{4UV}) - \frac{\ddot{V}}{2V} - \frac{\dot{V}\dot{U}}{4VU} = -\frac{4\pi G}{c^2}[\rho(t) - p(t)/c^2] \qquad (6)$$

If $U = \Omega^2(t)h(r)$, $V = \Omega^2(t)g(r)$, then Eq(5),Eq(6) is

$$[\ddot{\Omega}(t)\Omega(t) + 2\dot{\Omega}^2(t)] + [\frac{g''}{gh} - \frac{1}{2}\frac{g'^2}{g^2h} - \frac{g'h'}{2gh^2}] = \frac{4\pi G}{c^2}\Omega^2(t)[\rho(t) - p(t)/c^2] \qquad (7)$$

$$[\ddot{\Omega}(t)\Omega(t) + 2\dot{\Omega}^2(t)] + [\frac{g''}{2gh} - \frac{g'h'}{4gh^2} - \frac{1}{g}] = \frac{4\pi G}{c^2}\Omega^2(t)[\rho(t) - p(t)/c^2] \qquad (8)$$

Therefore,

$$\frac{g''}{gh} - \frac{1}{2}\frac{g'^2}{g^2h} - \frac{g'h'}{2gh^2} = -2k \quad , \qquad (9)$$

$$\frac{g''}{2gh}-\frac{g'h'}{4gh^2}-\frac{1}{g}=-2k \qquad (10)$$

2. The General Robertson-Walker solution

If Eq(10) $\times 2$ −Eq(9) is

$$\frac{1}{2}\frac{g'^2}{g^2h}-\frac{2}{g}=-2k$$

$$\rightarrow \frac{1}{2}\frac{g'^2}{g^2h}=\frac{2-2kg}{g}\rightarrow h=\frac{1}{4}\frac{g'^2}{g(1-kg)}$$

(11)

Hence, the proper time, Eq(4) is

$$d\tau^2=dt^2-\Omega^2(t)\frac{1}{c^2}[\frac{1}{4}\frac{g'^2}{g(1-kg)}dr^2+g(d\theta^2+\sin^2\theta d\phi^2)] \qquad (12)$$

If the condition of spherical coordinates, $g=r'^2$ is in Eq(12)

$$d\tau^2=dt^2-\Omega^2(t)\frac{1}{c^2}[\frac{1}{(1-kr'^2)}dr'^2+r'^2(d\theta^2+\sin^2\theta d\phi^2)] \qquad (13)$$

Eq(13) is the Robertson-Walker solution of the gravity equation.

Therefore, Eq(12) is the General Robertson-Walker solution of the gravity field equation.

Specially, if $g=k'^2, k'$ is constant, Eq(12) is

$$d\tau^2=dt^2-\Omega^2(t)\frac{1}{c^2}[0\times dr^2+k'^2(d\theta^2+\sin^2\theta d\phi^2)]$$

$$=dt^2-\Omega^2(t)\frac{1}{c^2}k'^2(d\theta^2+\sin^2\theta d\phi^2)] \qquad (14)$$

In this time,

$$g_{tt}=-1\ ,\ g_{rr}=0\ ,\ g_{\theta\theta}=\Omega^2(t)k'^2\ ,g_{\phi\phi}=\Omega^2(t)k'^2\sin^2\theta \qquad (15)$$

Eq(15) is new solution of the gravity field equation in the cosmology.

3. New solution

Eq(14), new solution is occupied by the General Robertson-Walker solution.

Specially, the proper time is

$$d\tau^2=dt^2-\frac{1}{c^2}g_{ij}(t,\vec{x})dx^i dx^j$$

$$=dt^2-\frac{1}{c^2}\frac{a^2(t)}{|K|}[d\vec{x}\cdot d\vec{x}+\frac{k(\vec{x}\cdot d\vec{x})^2}{1-k\vec{x}\cdot\vec{x}}]$$

$$=dt^2-\Omega^2(t)\frac{1}{c^2}[\frac{1}{(1-kr^2)}dr^2+r^2(d\theta^2+\sin^2\theta d\phi^2)]$$

$$\Omega^2(t) = \frac{a^2(t)}{|K|},\ k = \frac{K}{|K|}\ ,\ K \text{ is constant}$$

$$a(t) = \frac{\Delta s}{\Delta s_0} = \frac{\sqrt{g_{ij}(t,\vec{x})dx^i dx^j}}{\sqrt{g_{ij}(t_0,\vec{x})dx^i dx^j}} \tag{16}$$

If $K > 0$,

$$ds^2 = \frac{1}{K}[d\vec{x}\cdot d\vec{x} + \frac{k(\vec{x}\cdot d\vec{x})^2}{1-k\vec{x}\cdot\vec{x}}], z^2 = 1-k\vec{x}\cdot\vec{x}, dz^2 = \frac{k(\vec{x}\cdot d\vec{x})^2}{z^2} = \frac{k(\vec{x}\cdot d\vec{x})^2}{1-k\vec{x}\cdot\vec{x}}$$

$$V = \int_{|\vec{x}|\le 1}\sqrt{g}d^3x(z>0) + \int_{|\vec{x}|\le 1}\sqrt{g}d^3x(z<0) = 2\int_{|\vec{x}|\le 1}\frac{K^{-3/2}d^3x}{\sqrt{1-\vec{x}\cdot\vec{x}}} = 2\pi^2 K^{-3/2} \text{ (if } k=1)$$

$$\frac{d^2\vec{x}}{ds^2} + K\vec{x} = 0$$

$$\vec{x} = \hat{n}\sin(s\sqrt{K}), |\hat{n}| = 1$$

If $\vec{x} = \vec{0}$, $s = 0$ ($z > 0$), $s = \frac{\pi}{\sqrt{K}}$ ($z < 0$),

Total distance $s = L = \frac{2\pi}{\sqrt{K}}$ ($z > 0$) (17)

4. Conclusion

Hence, if $g = L^2 = \frac{4\pi^2}{K}$, New solution, Eq(14) is

$$d\tau^2 = dt^2 - \frac{1}{c^2}\Omega^2(t)\frac{4\pi^2}{K}(d\theta^2 + \sin^2\theta d\phi^2) \tag{18}$$

In this time,

$$g_{tt} = -1\ ,\ g_{rr} = 0\ ,\ g_{\theta\theta} = \Omega^2(t)\frac{4\pi^2}{K}, g_{\phi\phi} = \Omega^2(t)\frac{4\pi^2}{K}\sin^2\theta \tag{19}$$

Reference

[1]S.Weinberg,Gravitation and Cosmology(John wiley & Sons,Inc,1972)

[2]P.Bergman,Introduction to the Theory of Relativity(Dover Pub. Co.,Inc., New York,1976),Chapter V

[3]C.Misner, K,Thorne and J. Wheeler, Gravitation(W.H.Freedman & Co.,1973)

[4]S.Hawking and G. Ellis,The Large Scale Structure of Space-Time(Cam-bridge University Press,1973)

[5]R.Adler,M.Bazin and M.Schiffer, Introduction to General Relativity(McGraw-Hill,Inc.,1965)

[6]A.Raychaudhuri, Theoretical Cosmology (Oxford University Press,1979)

[7]P.Peebles, Physical Cosmology (Princeton University,1971)

[8]E.Hubble, Proc.Nat. Acad. Sci. U.S. 15, 169(1929)

[9]A.Sandage, Astrophys. J.178, 1(1972)

[10]W.de Sitter,Pro. Kon. Ned. Akad. Wet. 19, 1217(1917), Pro. Kon. Ned. Akad. Wet. 20, 229(1917)