# Deriving Newton's Second Law and Law of Gravity By Using Law of Conservation of Energy

Fu Yuhua

(CNOOC Research Institute, E-mail:fuyh1945@sina.com)

Abstract: According to the principle of the uniqueness of truth, there should be only one truth, namely law of conservation of energy, in the area of Newton Mechanics. Through the example of free falling body, this paper derives the original Newton's second law and the original law of gravity respectively by using the law of conservation of energy.

Key words: Uniqueness of truth, Newton Mechanics, law of conservation of energy, Newton's second law, law of gravity

## Introduction

Philosophers often say that, there should be a unique truth. According to this principle, and taking into account that the law of conservation of energy is the most important law in the natural sciences, therefore in the area of Newtonian mechanics, the law of conservation of energy should be the unique truth.

The law of conservation of energy states that the total energy of an isolated system remains constant.

As well-known, in Newton's classical mechanics, there were four main laws: the three laws of Newton and the law of gravity. If the law of conservation of energy is choosing as the unique truth (source law), then in principle, all the Newton's four laws can be derived according to the law of conservation of energy; after studying carefully we find that this conclusion may be correct. This paper discusses how to derive the original Newton's second law and the original law of gravity respectively by using the law of conservation of energy.

## 1 Deriving the original Newton's second law by using the law of conservation of energy

In this section, only Newton's second law can be derived, but we have to apply the law of gravity at the same time, so we present the general forms of Newton's second law and the law of gravity with undetermined constants firstly.

Assuming that for the law of gravity, the related exponent is unknown, and we only know the form of this formula is as follows

$$F = -\frac{GMm}{r^D} \qquad (1)$$

where: D is an undetermined constant, in the next section we will derive that its value is equal to 2.

Similarly, assuming that for Newton's second law, the related exponent is also unknown, and we only know the form of this formula is as follows

$$F = ma^{D'} \qquad (2)$$

where: D' is an undetermined constant, in this section we will derive that its value is equal to 1.

As shown in Figure 1, supposing that circle O' denotes the Earth, M denotes its mass; m denotes the mass of the small ball (treated as a mass point P), A O' is a plumb line, and coordinate y is parallel to AO'. The length of AC is equal to H, and O'C equals the radius R of the Earth.

We also assume that it does not take into account the motion of the Earth and only considering the free falling of the small ball in the gravitational field of the Earth (from point A to point C).

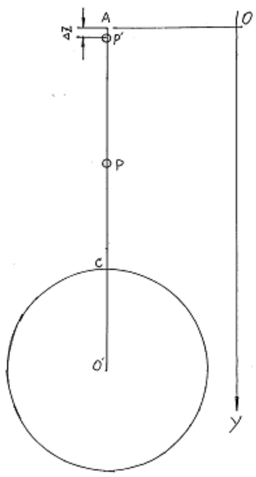


Figure 1 A small ball free falls in the gravitational field of the Earth

For this example, the value of $v_{\mathrm{P}}^2$ which is the square of the velocity for the small ball located at point $P$ will be investigated. To distinguish the quantities calculated by different methods, we denote the value given by the law of gravity and Newton's second law as $v_{\mathrm{P}}^2$, while $v'^2_{\mathrm{P}}$ denotes the value given by the law of conservation of energy.

Now we calculate the related quantities according to the law of conservation of energy.

From Eq.(1), the potential energy of the small ball located at point P is as follows

$$V=-\frac{GMm}{(D-1)r_{O'P}^{D-1}} \qquad (3)$$

According to the law of conservation of energy, we can get

$$-\frac{GMm}{(D-1)r_{O'A}^{D-1}}=\frac{1}{2}mv'^2_P-\frac{GMm}{(D-1)r_{O'P}^{D-1}} \qquad (4)$$

And therefore

$$v'^2_P=\frac{2GM}{D-1}\left[\frac{1}{r_{O'P}^{D-1}}-\frac{1}{(R+H)^{D-1}}\right] \qquad (5)$$

Now we calculate the related quantities according to the law of gravity and Newton's second law.

For the small ball located at any point P, we have

$$dv/dt=a \qquad (6)$$

We also have

$$dt=\frac{dy}{v}$$

Therefore

$$vdv=ady \qquad (7)$$

According to Eq.(1), along the plumb direction, the force acted on the small ball is as follows

$$F_a=\frac{GMm}{r_{O'P}^D} \qquad (8)$$

From Eq.（2）, it gives

$$a=(\frac{F_a}{m})^{1/D'}=(\frac{GM}{r_{O'P}^D})^{1/D'} \qquad (9)$$

According to Eq.(7), we have

$$vdv=\{\frac{GM}{(R+H-y)^D}\}^{1/D'}dy \qquad (10)$$

For the two sides of this expression, we run the integral operation from A to P, it gives

$$v_P^2=2(GM)^{1/D'}\int_0^{y_p}(R+H-y)^{-D/D'}dy$$

$$v_P^2=2(GM)^{1/D'}\{-\frac{1}{1-D/D'}[(R+H-y)^{1-D/D'}]\Big|_0^{y_p}\}$$

$$v_P^2=\frac{2(GM)^{1/D'}}{(D/D')-1}[\frac{1}{r_{O'P}^{(D/D')-1}}-\frac{1}{(R+H)^{(D/D')-1}}]$$

Let $v_P^2=v_P^{'2}$, then we should have: $1=1/D'$, and $D-1=(D/D')-1$; these two equations all give: $D'=1$, this means that for free falling problem, by using the law of conservation of energy, we strictly derive the original Newton's second law $F=ma$.

Here, although the original law of gravity cannot be derived (the value of D may be any constant, certainly including the case that D=2), we already prove that the original law of gravity is not contradicted to the law of conservation of energy, or the original law of gravity is tenable accurately.

## 2 Deriving the original law of gravity by using the law of conservation of energy

In order to really derive the original law of gravity for the example of free falling problem, we should consider the case that a small ball free falls from point A to point P' (point P' is also shown in Figure1) through a very short distance $\Delta Z$ (the two endpoints of the interval $\Delta Z$ are point A and point P').

As deriving the original Newton's second law, we already reach

$$v'^2_{P'} = \frac{2GM}{D-1}[\frac{1}{(R+H-\Delta Z)^{D-1}} - \frac{1}{(R+H)^{D-1}}]$$

where： $R+H-\Delta Z = r_{O'P'}$

For the reason that the distance of $\Delta Z$ is very short, and in this interval the gravity can be considered as a linear function, therefore the work $W$ of gravity in this interval can be written as follows

$$W = F_{av}\Delta Z = \frac{GMm}{(R+H-\frac{1}{2}\Delta Z)^D}\Delta Z$$

where, $F_{av}$ is the average value of gravity in this interval $\Delta Z$, namely the value of gravity for the midpoint of interval $\Delta Z$.

Omitting the second order term of $\Delta Z$ ($\frac{1}{4}(\Delta Z)^2$), it gives

$$W = \frac{GMm\Delta Z}{(R^2+H^2+2RH-R\Delta Z-H\Delta Z)^{D/2}}$$

As the small ball free falls from point A to point P', its kinetic energy is as follows

$$\frac{1}{2}mv'^2_{P'} = \frac{GMm}{D-1}[\frac{(R+H)^{D-1}-(R+H-\Delta Z)^{D-1}}{(R^2+H^2+2RH-R\Delta Z-H\Delta Z)^{D-1}}]$$

According to the law of conservation of energy, we have

$$W = \frac{1}{2}mv'^2_{P'}$$

Substituting the related quantities into the above expression, it gives

$$\frac{GMm}{D-1}[\frac{(R+H)^{D-1}-(R+H-\Delta Z)^{D-1}}{(R^2+H^2+2RH-R\Delta Z-H\Delta Z)^{D-1}}]$$

$$= \frac{GMm\Delta Z}{(R^2+H^2+2RH-R\Delta Z-H\Delta Z)^{D/2}}$$

To compare the related terms, we can reach the following three equations

$$D-1=1$$
$$D/2=D-1$$
$$\Delta Z = (R+H)^{D-1}-(R+H-\Delta Z)^{D-1}$$

All of these three equations will give the following result

$$D=2$$

Thus, we already derive the original law of gravity by using the law of conservation of energy.

## 3 Conclusion and further topic

According to the above results it can be said that, for the free falling problem, we do not rely on any experiment, only apply law of conservation of energy to derive the original Newton's second law and the original law of gravity.

In references [1, 2], based on the equation given by Prof. Hu Ning according to general relativity and Binet's formula, we get the following improved Newton's formula of universal gravitation

$$F = -\frac{GMm}{r^2} - \frac{3G^2M^2mp}{c^2r^4} \qquad (11)$$

where: G is the gravitational constant, M and m are the masses of the two objects, r is the distance between the two objects, c is the speed of light, p is the half normal chord for the object m moving around the object M along with a curve, and the value of p is given by: p = a(1-$e^2$) (for ellipse), p = a ($e^2$-1) (for hyperbola), p = $y^2$/2x (for parabola).

This improved Newton's universal gravitation formula can give the same results as given by general relativity for the problem of planetary advance of perihelion and the problem of gravitational defection of a photon orbit around the Sun.

For the problem of planetary advance of perihelion, the improved Newton's universal gravitation formula reads

$$F = -\frac{GMm}{r^2} - \frac{3G^2M^2ma(1-e^2)}{c^2r^4} \qquad (12)$$

For the problem of gravitational defection of a photon orbit around the Sun, the improved Newton's universal gravitation formula reads

$$F = -\frac{GMm}{r^2} - \frac{1.5GMmr_0^2}{r^4} \qquad (13)$$

where： $r_0$ is the shortest distance between the light and the Sun, if the light and the Sun is tangent, it is equal to the radius of the Sun.

The funny thing is that, for this problem, the maximum gravitational force given by the improved Newton's universal gravitation formula is 2.5 times of that given by the original Newton's law of gravity.

The further topic is how to apply the law of conservation of energy to derive Eqs.(11), (12), (13), and the like.

References

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