A Proof of the Beal's Conjecture

Zhang Tianshu

Zhanjiang city, Guangdong province, China

Email: chinazhangtianshu@126.com

Introduction: The Beal's Conjecture was discovered by Andrew Beal in 1993. Later the conjecture was announced in December 1997 issue of the Notices of the American Mathematical Society. Yet it is still both unproved and un-negated a conjecture hitherto.

Abstract

In this article, first we classify A, B and C according to their respective odevity, and thereby ret rid of two kinds from $A^X+B^Y=C^Z$. Then affirmed $A^X+B^Y=C^Z$ in which case A, B and C have a common prime factor by concrete examples. After that, proved $A^X+B^Y\neq C^Z$ in which case A, B and C have not any common prime factor by the mathematical induction with the aid of the symmetric law of odd numbers after the decomposition of the inequality. Finally, we have proved that the Beal's conjecture does hold water after the comparison between $A^X+B^Y=C^Z$ and $A^X+B^Y\neq C^Z$ under the given requirements.

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The Proof

The Beal's Conjecture states that if $A^X+B^Y=C^Z$, where A, B, C, X, Y and Z are positive integers, and X, Y and Z are all greater than 2, then A, B and C must have a common prime factor.

We consider the limits of values of above-mentioned A, B, C, X, Y and Z as given requirements for hinder concerned equalities and inequalities.

First we classify A, B and C according to their respective odevity, and thereby remove following two kinds from $A^X+B^Y=C^Z$.

1. If A, B and C, all are positive odd numbers, then A^X+B^Y is an even number, yet C^Z is an odd number, so there is only $A^X+B^Y\neq C^Z$ according to an odd number ≠ an even number.

2. If any two in A, B and C are positive even numbers, and another is a positive odd number, then when A^X+B^Y is an even number, C^Z is an odd number, yet when A^X+B^Y is an odd number, C^Z is an even number, so there is only $A^X+B^Y\neq C^Z$ according to an odd number ≠ an even number.

Thus, we continue to have merely two kinds of $A^X+B^Y=C^Z$ under the given requirements, as listed below.

1. A, B and C, all are positive even numbers.

2. A, B and C are two positive odd numbers and a positive even number.

For indefinite equation $A^X+B^Y=C^Z$ under the given requirements plus aforementioned either qualification, in fact, it has many sets of solutions with A, B and C which are positive integers. Let us instance two concrete equations respectively to explain two such propositions below.

When A, B and C all are positive even numbers, if let A=B=C=2, X=Y=3, and Z=4, then indefinite equation $A^X+B^Y=C^Z$ is exactly equality $2^3+2^3=2^4$. Evidently, $A^X+B^Y=C^Z$ has here a set of solution with A, B and C which are positive integers 2, 2 and 2, and A, B and C have common even prime factor 2. In addition, if let A=B=162, C=54, X=Y=3, and Z=4, then indefinite equation $A^X+B^Y=C^Z$ is exactly equality $162^3+162^3=54^4$. Evidently, $A^X+B^Y=C^Z$ has here a set of solution with A, B and C which are positive integers 162, 162 and 54, and A, B and C have two common prime factors, i.e. even 2 and odd 3.

When A, B and C are two positive odd numbers and a positive even number, if let A=C=3, B=6, X=Y=3, and Z=5, then indefinite equation $A^X+B^Y=C^Z$ is exactly equality $3^3+6^3=3^5$. Manifestly $A^X+B^Y=C^Z$ has here a set of solution with A, B and C which are positive integers 3, 6 and 3, and A, B and C have common prime factor 3. In addition, if let A=B=7, C=98, X=6, Y=7, and Z=3, then indefinite equation $A^X+B^Y=C^Z$ is exactly equality $7^6+7^7=98^3$. Manifestly $A^X+B^Y=C^Z$ has here a set of solution with A, B and C which are positive integers 7, 7 and 98, and A, B and C have common prime factor 7.

Consequently, indefinite equation $A^X+B^Y=C^Z$ under the given requirements plus aforementioned either qualification is able to hold water, but A, B and C must have at least one common prime factor.

By now, if we can prove that there is only $A^X+B^Y \neq C^Z$ under the given requirements plus the qualification that A, B and C have not any common prime factor, then we proved completely the conjecture.

Since A, B and C have common prime factor 2 when A, B and C all are positive even numbers, so these circumstances that A, B and C have not any common prime factor can only occur under the prerequisite that A, B and C are two positive odd numbers and a positive even number.

If A, B and C have not any common prime factor, then any two of them have not any common prime factor either, because if any two have a common prime factor, namely A^X+B^Y or C^Z-A^X or C^Z-B^Y have a common prime factor, yet another has not the prime factor, then it would lead to $A^X+B^Y \neq C^Z$ or $C^Z-A^X \neq B^Y$ or $C^Z-B^Y \neq A^X$ surely according to the unique factorization theorem of natural number.

Since it is so, if we can prove $A^X+B^Y \neq C^Z$ under the given requirements plus the qualification that A, B and C have not any common prime factor, then the Beal's conjecture is surely tenable, otherwise it will be negated.

Unquestionably, let following two inequalities add together, are able to replace completely $A^X+B^Y \neq C^Z$ under the given requirements plus the qualification that A, B and C are two positive odd numbers and a positive even number without a common prime factor.

1. $A^X+B^Y \neq 2^Z G^Z$ under the given requirements plus the qualifications that A and B are two positive odd numbers, G is a positive integer, and A, B and 2G have not any common prime factor.

2. $A^X+2^YD^Y \neq C^Z$ under the given requirements plus the qualifications that A and C are two positive odd numbers, D is a positive integer, and A, C and 2D have not any common prime factor.

For $A^X+B^Y \neq 2^ZG^Z$, when G=1, it is exactly $A^X+B^Y \neq 2^Z$. When G>1: if G is a positive odd number, then the inequality changes not, namely it is still $A^X+B^Y \neq 2^ZG^Z$; if G is a positive even number, then the inequality is expressed by $A^X+B^Y \neq 2^W$ or $A^X+B^Y \neq 2^WH^Z$, where H is an odd number ≥3, and W > Z.

Undoubtedly, $A^X+B^Y \neq 2^W$ can represent $A^X+B^Y \neq 2^Z$, and $A^X+B^Y \neq 2^WH^Z$ can represent $A^X+B^Y \neq 2^ZG^Z$, where H is an odd number ≥3, and W ≥ Z. So express $A^X+B^Y \neq 2^ZG^Z$ into two inequalities as the follows.

(1) $A^X+B^Y \neq 2^W$, where A and B are positive odd numbers without a common prime factor, and X, Y and W are integers ≥3.

(2) $A^X+B^Y \neq 2^W H^Z$, where A, B and H are positive odd numbers without a common prime factor, X, Y and Z are integers ≥3, W ≥ Z, and H ≥3.

For $A^X+2^YD^Y \neq C^Z$, when D=1, it is exactly $A^X+2^Y \neq C^Z$. When D>1: if D is a positive odd number, then the inequality changes not, namely it is still $A^X+2^YD^Y \neq C^Z$; if D is a positive even number, then the inequality is expressed by $A^X+2^W \neq C^Z$ or $A^X+2^WR^Y \neq C^Z$, where R is an odd number ≥3, and W >Y.

Undoubtedly, $A^X+2^W \neq C^Z$ can represent $A^X+2^Y \neq C^Z$, and $A^X+2^W R^Y \neq C^Z$ can represent $A^X+2^Y D^Y \neq C^Z$, where R is an odd number ≥3, and W ≥ Y. So express $A^X+2^Y D^Y \neq C^Z$ into two inequalities as the follows.

(3) $A^X+2^W \neq C^Z$, where A and C are positive odd numbers without a common prime factor, and X, W and Z are integers ≥3.

(4) $A^X+2^W R^Y \neq C^Z$, where A, R and C are positive odd numbers without a common prime factor, X, Y and Z are integers ≥3, W ≥Y, and R ≥3.

Hereinafter, we regard values of A, B, C, H, R, X, Y, Z and W in aforementioned four inequalities, added to their co-prime relation in each inequality, as known requirements for hinder concerned inequalities.

Thus it can be seen, proving $A^X+B^Y \neq C^Z$ under the given requirements plus the qualification that A, B and C have not any common prime factor is changed to prove the existence of the above-listed four inequalities under the known requirements. Such being the case, we shall first prove $A^X+B^Y \neq 2^W$ and $A^X+B^Y \neq 2^W H^Z$. For this purpose, we must expound certain circumstances relating to the first proof.

Let us divide all positive odd numbers into two kinds of ***A*** plus ***E,*** namely the form of ***A*** is 1+4n, and the form of ***E*** is 3+4n with n≥0. From small to large odd numbers of ***A*** & ***E*** arrange as the follows respectively.

A: 1, 5, 9, 13, 17, 21, 25, 29, 33, 37, 41, 45, 49, 53, 57, 61…1+4n …

E: 3, 7, 11, 15, 19, 23, 27, 31, 35, 39, 43, 47, 51, 55, 59, 63…3+4n …

We use ***A*** or ***E*** to denote each in sequence of two kinds' odd numbers too. Next, we list from small to large positive odd numbers and even numbers $2^W H^Z$ with H≥1 where W≥3 and Z≥ 3, also label a belongingness of each of odd numbers on the right side of itself.

$1^Z \in \boldsymbol{A}$; $3 \in \boldsymbol{E}$; $5 \in \boldsymbol{A}$; $7 \in \boldsymbol{E}$; (2^3); $9 \in \boldsymbol{A}$; $11 \in \boldsymbol{E}$; $13 \in \boldsymbol{A}$; $15 \in \boldsymbol{E}$; (2^4); $17 \in \boldsymbol{A}$; $19 \in \boldsymbol{E}$; $21 \in \boldsymbol{A}$; $23 \in \boldsymbol{E}$; $25 \in \boldsymbol{A}$; $3^3 \in \boldsymbol{E}$; $29 \in \boldsymbol{A}$; $31 \in \boldsymbol{E}$; (2^5); $33 \in \boldsymbol{A}$; $35 \in \boldsymbol{E}$; $37 \in \boldsymbol{A}$; $39 \in \boldsymbol{E}$; $41 \in \boldsymbol{A}$; $43 \in \boldsymbol{E}$; $45 \in \boldsymbol{A}$; $47 \in \boldsymbol{E}$; $49 \in \boldsymbol{A}$; $51 \in \boldsymbol{E}$; $53 \in \boldsymbol{A}$; $55 \in \boldsymbol{E}$; $57 \in \boldsymbol{A}$; $59 \in \boldsymbol{E}$; $61 \in \boldsymbol{A}$; $63 \in \boldsymbol{E}$; (2^6); $65 \in \boldsymbol{A}$; $67 \in \boldsymbol{E}$; $69 \in \boldsymbol{A}$; $71 \in \boldsymbol{E}$; $73 \in \boldsymbol{A}$; $75 \in \boldsymbol{E}$; $77 \in \boldsymbol{A}$; $79 \in \boldsymbol{E}$; $3^4 \in \boldsymbol{A}$; $83 \in \boldsymbol{E}$; $85 \in \boldsymbol{A}$; $87 \in \boldsymbol{E}$; $89 \in \boldsymbol{A}$; $91 \in \boldsymbol{E}$; $93 \in \boldsymbol{A}$; $95 \in \boldsymbol{E}$; $97 \in \boldsymbol{A}$; $99 \in \boldsymbol{E}$; $101 \in \boldsymbol{A}$; $103 \in \boldsymbol{E}$; $105 \in \boldsymbol{A}$; $107 \in \boldsymbol{E}$; $109 \in \boldsymbol{A}$; $111 \in \boldsymbol{E}$; $113 \in \boldsymbol{A}$; $115 \in \boldsymbol{E}$; $117 \in \boldsymbol{A}$; $119 \in \boldsymbol{E}$; $121 \in \boldsymbol{A}$; $123 \in \boldsymbol{E}$; $5^3 \in \boldsymbol{A}$; $127 \in \boldsymbol{E}$; (2^7); $129 \in \boldsymbol{A}$; $131 \in \boldsymbol{E}$; $133 \in \boldsymbol{A}$; $135 \in \boldsymbol{E}$; $137 \in \boldsymbol{A}$; $139 \in \boldsymbol{E}$; $141 \in \boldsymbol{A}$; $143 \in \boldsymbol{E}$; $145 \in \boldsymbol{A}$; $147 \in \boldsymbol{E}$; $149 \in \boldsymbol{A}$; $151 \in \boldsymbol{E}$; $153 \in \boldsymbol{A}$; $155 \in \boldsymbol{E}$; $157 \in \boldsymbol{A}$; $159 \in \boldsymbol{E}$; $161 \in \boldsymbol{A}$; $163 \in \boldsymbol{E}$; $165 \in \boldsymbol{A}$; $167 \in \boldsymbol{E}$; $169 \in \boldsymbol{A}$; $171 \in \boldsymbol{E}$; $173 \in \boldsymbol{A}$; $175 \in \boldsymbol{E}$; $177 \in \boldsymbol{A}$; $179 \in \boldsymbol{E}$; $181 \in \boldsymbol{A}$; $183 \in \boldsymbol{E}$; $185 \in \boldsymbol{A}$; $187 \in \boldsymbol{E}$; $189 \in \boldsymbol{A}$; $191 \in \boldsymbol{E}$; $193 \in \boldsymbol{A}$; $195 \in \boldsymbol{E}$; $197 \in \boldsymbol{A}$; $199 \in \boldsymbol{E}$; $201 \in \boldsymbol{A}$; $203 \in \boldsymbol{E}$; $205 \in \boldsymbol{A}$; $207 \in \boldsymbol{E}$; $209 \in \boldsymbol{A}$; $211 \in \boldsymbol{E}$; $213 \in \boldsymbol{A}$; $215 \in \boldsymbol{E}$; $(2^3 \times 3^3)$; $217 \in \boldsymbol{A}$; $219 \in \boldsymbol{E}$; $221 \in \boldsymbol{A}$; $223 \in \boldsymbol{E}$; $225 \in \boldsymbol{A}$; $227 \in \boldsymbol{E}$; $229 \in \boldsymbol{A}$; $231 \in \boldsymbol{E}$; $233 \in \boldsymbol{A}$; $235 \in \boldsymbol{E}$; $237 \in \boldsymbol{A}$; $239 \in \boldsymbol{E}$; $241 \in \boldsymbol{A}$; $3^5 \in \boldsymbol{E}$; $245 \in \boldsymbol{A}$; $247 \in \boldsymbol{E}$; $249 \in \boldsymbol{A}$; $251 \in \boldsymbol{E}$; $253 \in \boldsymbol{A}$; $255 \in \boldsymbol{E}$; (2^8); $257 \in \boldsymbol{A}$; $259 \in \boldsymbol{E}$; $261 \in \boldsymbol{A}$; $263 \in \boldsymbol{E}$; $265 \in \boldsymbol{A}$; $267 \in \boldsymbol{E}$; $269 \in \boldsymbol{A}$; $271 \in \boldsymbol{E}$; …

Thus it can be seen, that permutations of from small to large seriate positive odd numbers are infinitely many cycles of ***A*** & ***E*,** to wit ***A, E, A, E, A, E, A, E, A, E, A, E, A, E, A, E, A, E, A, E, A, E, A, E*** ⋯

We list seriate kinds of odd numbers which have a common odd base number, and label a belongingness of each of them on the right of itself:

$1^1 \in \boldsymbol{A}$;	$3^1 \in \boldsymbol{E}$;	$5^1 \in \boldsymbol{A}$;	$7^1 \in \boldsymbol{E}$;	$9^1 \in \boldsymbol{A}$;	$11^1 \in \boldsymbol{E}$;
$1^2 \in \boldsymbol{A}$;	$3^2 \in \boldsymbol{A}$;	$5^2 \in \boldsymbol{A}$;	$7^2 \in \boldsymbol{A}$;	$9^2 \in \boldsymbol{A}$;	$11^2 \in \boldsymbol{A}$;
$1^3 \in \boldsymbol{A}$;	$3^3 \in \boldsymbol{E}$;	$5^3 \in \boldsymbol{A}$;	$7^3 \in \boldsymbol{E}$;	$9^3 \in \boldsymbol{A}$;	$11^3 \in \boldsymbol{E}$;
$1^4 \in \boldsymbol{A}$;	$3^4 \in \boldsymbol{A}$;	$5^4 \in \boldsymbol{A}$;	$7^4 \in \boldsymbol{A}$;	$9^4 \in \boldsymbol{A}$;	$11^4 \in \boldsymbol{A}$;
$1^5 \in \boldsymbol{A}$;	$3^5 \in \boldsymbol{E}$;	$5^5 \in \boldsymbol{A}$;	$7^5 \in \boldsymbol{E}$;	$9^5 \in \boldsymbol{A}$;	$11^5 \in \boldsymbol{E}$;
$1^6 \in \boldsymbol{A}$;	$3^6 \in \boldsymbol{A}$;	$5^6 \in \boldsymbol{A}$;	$7^6 \in \boldsymbol{A}$;	$9^6 \in \boldsymbol{A}$;	$11^6 \in \boldsymbol{A}$;
…	…	…	…	…	…

$13^1 \in \boldsymbol{A}$;	$15^1 \in \boldsymbol{E}$;	$17^1 \in \boldsymbol{A}$;	$19^1 \in \boldsymbol{E}$;	$21^1 \in \boldsymbol{A}$;	$23^1 \in \boldsymbol{E}$; …
$13^2 \in \boldsymbol{A}$;	$15^2 \in \boldsymbol{A}$;	$17^2 \in \boldsymbol{A}$;	$19^2 \in \boldsymbol{A}$;	$21^2 \in \boldsymbol{A}$;	$23^2 \in \boldsymbol{A}$; …
$13^3 \in \boldsymbol{A}$;	$15^3 \in \boldsymbol{E}$;	$17^3 \in \boldsymbol{A}$;	$19^3 \in \boldsymbol{E}$;	$21^3 \in \boldsymbol{A}$;	$23^3 \in \boldsymbol{E}$; …
$13^4 \in \boldsymbol{A}$;	$15^4 \in \boldsymbol{A}$;	$17^4 \in \boldsymbol{A}$;	$19^4 \in \boldsymbol{A}$;	$21^4 \in \boldsymbol{A}$;	$23^4 \in \boldsymbol{A}$; …
$13^5 \in \boldsymbol{A}$;	$15^5 \in \boldsymbol{E}$;	$17^5 \in \boldsymbol{A}$;	$19^5 \in \boldsymbol{E}$;	$21^5 \in \boldsymbol{A}$;	$23^5 \in \boldsymbol{E}$; …
$13^6 \in \boldsymbol{A}$;	$15^6 \in \boldsymbol{A}$;	$17^6 \in \boldsymbol{A}$;	$19^6 \in \boldsymbol{A}$;	$21^6 \in \boldsymbol{A}$;	$23^6 \in \boldsymbol{A}$; …
…	…	…	…	…	…

From above-listed various kinds of odd numbers, we are not difficult to see, all odd numbers whereby each of $\boldsymbol{A}$ to act as a base number belong still within $\boldsymbol{A}$; odd numbers which even power of $\boldsymbol{E}$ forms belong $\boldsymbol{A}$; and odd numbers which odd power of $\boldsymbol{E}$ forms belong $\boldsymbol{E}$, i.e. $\boldsymbol{A}^{X} \in \boldsymbol{A}$, $\boldsymbol{E}^{2n} \in \boldsymbol{A}$ and $\boldsymbol{E}^{2n-1} \in \boldsymbol{E}$ where $n \geq 1$. Or all odd numbers whose exponents are even numbers belong $\boldsymbol{A}$, odd numbers which odd power of $\boldsymbol{A}$ forms belong $\boldsymbol{A}$, and odd numbers which odd power of ***E*** forms belong ***E*.**

Besides, two adjacent positive odd numbers which have either a common odd base number >1 or an identical exponent are an even number apart. But also such even numbers are getting greater and greater along which their base numbers or exponents are getting greater and greater.

Altogether, odd numbers which have an odd exponent and odd numbers which have an even exponent composed all odd numbers, i.e. ***A*** and ***E*.** Yet odd numbers whose exponents are greater than 2 are merely a part in them, and this part is included and dispersed within odd numbers of ***A*** and ***E***, thus they conform to the symmetric law of odd numbers we shall define as the follows.

We add even numbers $2^{W-1}H^Z$ among the sequence of odd numbers, and regard $2^{W-1}H^Z$ as a center of symmetry of odd numbers, where H is an odd number ≥ 1, $W\geq 3$, and $Z\geq 3$, similarly hereinafter. Then odd numbers on the left side of $2^{W-1}H^Z$ and odd numbers near $2^{W-1}H^Z$ on the right side of $2^{W-1}H^Z$ are one-to-one bilateral symmetries.

For example, if we regard 2^{W-1} as a symmetric center, then $2^{W-1}-1\in\boldsymbol{E}$ and $2^{W-1}+1\in\boldsymbol{A}$, $2^{W-1}-3\in\boldsymbol{A}$ and $2^{W-1}+3\in\boldsymbol{E}$, $2^{W-1}-5\in\boldsymbol{E}$ and $2^{W-1}+5\in\boldsymbol{A}$, $2^{W-1}-7\in\boldsymbol{A}$ and $2^{W-1}+7\in\boldsymbol{E}$ etc are one-to-one bilateral symmetry respectively.

We regard one-to-one bilateral symmetries between odd numbers of ***A*** and odd numbers of ***E*** for symmetric center $2^{W-1}H^Z$ as the symmetric law of odd numbers. At the number axis, it is exactly that one-to-one bilateral symmetries between odd points of ***A*** and odd points of ***E*** for symmetric center's point $2^{W-1}H^{Z}$.

Manifestly the symmetric law of odd numbers indicates that it can only symmetrize one of ***A*** and one of ***E*** for symmetric center $2^{W-1}H^{Z}$**,** yet can not symmetrize either two of ***A*** or two of ***E***.

After regard $2^{W-1}H^{Z}$ as a symmetric center, leave from $2^{W-1}H^{Z}$, there are both finitely many cycles of ***E*** & ***A*** leftwards until ***E***=3 plus ***A***=1, and infinitely many cycles of ***A*** & ***E*** rightwards.

According to the symmetric law of odd numbers, two distances from a symmetric center to bilateral symmetric ***A*** and ***E*** are possessed of the equal length at the number axis.

In addition, at the number axis, each and every integer's point expresses an integer, also the large or the small of an integer depends on the length of a line segment between zero and the integer's point.

Consequently, on the one hand, a sum of two each other's symmetric odd numbers ***A*** and ***E*** is equal to the double of even number which the symmetric center expresses. On the other hand, a sum of two non-symmetric odd numbers is absolutely unequal to the double of even number which the symmetric center expresses. In other words, let $2^{W-1}H^{Z}$ as a symmetric center, not only ***A*** and ***E*** whose sum equals $2^{W}H^{Z}$ are just the bilateral symmetry, but also $2^{W}H^{Z}$ as the sum of two odd numbers can only obtain from the addition of bilateral symmetric ***A*** and ***E***.

Before making the proof concerned, we give a stipulation that for an integer, if its exponent is greater than or equal to 3, then the integer is called an integer of the greater exponent; if its exponent is equal to 1 or 2, then the integer is called an integer of the smaller exponent.

Pursuant to preceding basic concepts, thereinafter, we set to prove the existence of aforementioned four inequalities, one by one.

Firstly, Prove $A^X+B^Y\neq 2^W$ under the known requirements.

Let us regard 2^{W-1} as the symmetric center of odd numbers to prove $A^X+B^Y\neq 2^W$ under the known requirements by the mathematical induction with the aid of the symmetric law of odd numbers.

(1) When W-1=3, each other's symmetric odd numbers on two sides of symmetric center 2^3 are listed below.

1^3, 3, 5, 7, (2^3), 9, 11, 13, 15

To wit: ***A, E, A, E,*** (2^3), ***A, E, A, E***

It is clear at a glance, that there are not two odd numbers of the greater exponents altogether on two odd places of every bilateral symmetry for symmetric center 2^3. So we get $A^X + B^Y \neq 2^4$.

When W-1=4, each other's symmetric odd numbers on two sides of symmetric center 2^4 are listed below.

1^4, 3, 5, 7, 9, 11, 13, 15, (2^4) 17, 19, 21, 23, 25, 3^3, 29, 31

To wit: $\boldsymbol{A^4}$***, E, A, E, A, E, A, E,*** (2^4), ***A, E, A, E, A,*** $\boldsymbol{E^3}$***, A, E***

Evidently, there are not two odd numbers of the greater exponents altogether on two odd places of every bilateral symmetry for symmetric center 2^4. So we get $A^X + B^Y \neq 2^5$.

When W-1=5 and W-1=6, each other's symmetric odd numbers on two sides of symmetric center 2^6 including 2^5 are listed below.

1^6, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 3^3, 29, 31, (2^5), 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, (2^6), 65, 67, 69, 71, 73, 75, 77, 79, 3^4, 83, 85, 87, 89, 91, 93, 95, 97, 99, 101, 103, 105, 107, 109, 111, 113, 115, 117, 119, 121, 123, 5^3, 127

To wit: ***A^6*, *E*, *A*, *E*, *A*, *E*, *A*, *E*, *A*, *E*, *A*, *E*, *A*, *E^3*, *A*, *E*,** (2^5), ***A*, *E*, *A*, *E*, *A*, *E*, *A*, *E*, *A*, *E*, *A*, *E*, *A*, *E*, *A*, *E*, *A*, *E*,** (2^6), ***A*, *E*, *A*, *E*, *A*, *E*, *A*, *E*, *A^4*, *E*, *A*, *E*, *A*, *E*, *A*, *E*, *A*, *E*, *A*, *E*, *A*, *E*, *A*, *E*, *A*, *E*, *A*, *E*, *A*, *E*, *A*, *A^3*, *E***

Likewise there are not two odd numbers of the greater exponents altogether on two odd places of every bilateral symmetry for symmetric center 2^6 or 2^5. So we get $A^X + B^Y \neq 2^7$ and $A^X + B^Y \neq 2^6$.

(2) Suppose that when W-1=K with K $\geq$ 6, there are not two odd numbers of the greater exponents altogether on two odd places of every bilateral symmetry for symmetric center 2^K. So we get $A^X+B^Y\neq 2^{K+1}$ under the known requirements, where K $\geq$ 6.

(3) Prove that when W-1=K+1, there are not two odd numbers of the greater exponents altogether either on two odd places of every bilateral symmetry for symmetric center 2^{K+1}. Namely it needs us to prove $A^X+B^Y \neq 2^{K+2}$ under the known requirements.

Proof * We known that permutations of odd numbers on two sides of 2^{W-1} including 2^K plus 2^{K+1} conform to the symmetric law of odd numbers, also odd numbers on two sides of 2^K and of 2^{K+1} arrange as the follows.

A, E, A, E, …***A, E, A, E,*** (2^K), ***A, E, A, E,*** …***A, E, A, E,*** …***A, E, A, E,*** (2^{K+1}), ***A, E, A, E,*** …***A, E, A, E,*** …***A, E, A, E, A, E, A, E,***…***A, E, A, E.***

Now that one-to-one bilateral symmetric odd numbers for symmetric center 2^{W-1} are only ***A*** and ***E***, then two distances that bilateral symmetric ***A*** and ***E*** away from 2^{W-1} are equivalent to the length of an odd number.

Actually, all odd numbers of bilateral symmetries for symmetric center 2^K are exactly all odd numbers on the left side of 2^{K+1}. Thus for odd numbers of bilateral symmetries for symmetric center 2^{K+1}, their a half on the left side of 2^{K+1} retained still original places, while another half on the right side is formed from 2^{K+1} plus each of odd numbers on two sides of 2^K.

Suppose that A^X and B^Y are any pair of bilateral symmetric odd numbers for symmetric center 2^K, then we have $A^X+B^Y=2^{K+1}$. In $A^X+B^Y=2^{K+1}$, we regard A as one of ***A***, then A^X is one of ***A*** too, yet B^Y can only be one of ***E*** according to the preceding conclusion drawn.

Since there are not two odd numbers of the greater exponents altogether on two odd places of every bilateral symmetry for symmetric center 2^K according to second step of the mathematical induction, so let A^X as an odd number of the greater exponent, and let B^Y as an odd number of the smaller exponent, i.e. let $X \geq 3$ and $Y < 3$.

By now, let B^Y plus 2^{K+1} makes B^Y+2^{K+1}, then B^Y+2^{K+1} is still one of $\boldsymbol{E}$.

Please, see a simple illustration at the number axis as follows.

					A^X+2^{K+1}		B^Y+2^{K+1}	
1, 3...	A^X	2^K	B^Y	2^{K+1}	$2^{K+2}-B^Y$	3×2^K	$2^{K+2}-A^X$	2^{K+2}

Since where $X \geq 3$ and $Y<3$, there is $A^X+B^Y=2^{K+1}$, then $B^Y+2^{K+1}=A^X+2B^Y$, and $A^X+2B^Y=2^{K+1}+B^Y=2^{K+1}+(2^{K+1}-A^X)=2^{K+2}-A^X$, so there is $B^Y+2^{K+1}= A^X+2B^Y=2^{K+2}-A^X$.

Manifestly, when $X \geq 3$ and $Y < 3$, A^X and $2^{K+2}-A^X$ (or B^Y+2^{K+1}, A^X+2B^Y) are bilateral symmetric odd numbers for symmetric center 2^{K+1}, so there is $A^X+2^{K+2}-A^X= A^X+(B^Y+2^{K+1})= A^X+(A^X+2B^Y) =2^{K+2}$.

But then, when $X \geq 3$ and $Y \geq 3$, there is $A^X+B^Y \neq 2^{K+1}$ according to second step of the preceding supposition, so has $A^X+ [A^X+2B^Y] =2[A^X+B^Y] \neq 2^{K+2}$.

That is to say, when $X \geq 3$ and $Y \geq 3$, A^X and B^Y+2^{K+1} (or A^X+2B^Y, $2^{K+2}-A^X$) are not bilateral symmetric odd numbers for symmetric center 2^{K+1}.

Thus it can be seen, when A^X and B change not, A^X+2B^Y expresses two each other's- disparate odd numbers due to $Y<3$ or $Y \geq 3$.

So let $A^X+2B^Y=\boldsymbol{E}^\varepsilon$ with $Y<3$ plus $\varepsilon<3$, and let $A^X+2B^Y=F^P$ with $Y \geq 3$, where $X \geq 3$, F is an odd number ≥ 1 and P is an integer ≥ 1. After that, get $A^X+ \boldsymbol{E}^\varepsilon =2^{K+2}$ and $A^X+F^P \neq 2^{K+2}$ according to aforesaid got $A^X+(A^X+2B^Y) =2^{K+2}$ with $Y<3$ and $A^X+(A^X+2B^Y) \neq 2^{K+2}$ with $Y \geq 3$, where $X \geq 3$.

After change F of F^P as $\boldsymbol{E}$ like the base number of $\boldsymbol{E}^\varepsilon$, the both exponents are not alike surely. So let $F^P=\boldsymbol{E}^M$, then M of $\boldsymbol{E}^M$ is greater than ε of $\boldsymbol{E}^\varepsilon$, i.e. $M \geq 3$, and M is a real number. After that, we get $A^X+\boldsymbol{E}^M >2^{K+2}$, of course has $A^X+\boldsymbol{E}^M \neq 2^{K+2}$, in addition, has got $A^X+\boldsymbol{E}^\varepsilon=2^{K+2}$.

If further change M of $\boldsymbol{E}^M$ and ε of $\boldsymbol{E}^\varepsilon$ as an identical exponent Y, then there are $A^X+\boldsymbol{E}^Y=2^{K+2}$ with Y<3 and $A^X+\boldsymbol{E}^Y \neq 2^{K+2}$ with $Y \geq 3$, such being the case values which A^X and $\boldsymbol{E}$ in the equality and the inequality express are just the same respectively.

Then you should discover that $\boldsymbol{E}^Y$ in $A^X+\boldsymbol{E}^Y=2^{K+2}$ is one of ***E,*** yet for $\boldsymbol{E}^Y$ in $A^X+\boldsymbol{E}^Y \neq 2^{K+2}$, we can only define what it is a positive odd number.

Since $\boldsymbol{E}$ of $\boldsymbol{E}^Y$ in $A^X+\boldsymbol{E}^Y=2^{K+2}$ with Y<3 and B of B^Y in $A^X+B^Y=2^{K+2}$ with Y<3 are one and the same, also $\boldsymbol{E}$ of $\boldsymbol{E}^Y$ in $A^X+\boldsymbol{E}^Y=2^{K+2}$ with Y<3 and $\boldsymbol{E}$ of $\boldsymbol{E}^Y$ in $A^X+\boldsymbol{E}^Y \neq 2^{K+2}$ with Y≥3 are one and the same, therefore can substitute B for $\boldsymbol{E}$ in $A^X+\boldsymbol{E}^Y=2^{K+2}$ plus $A^X+\boldsymbol{E}^Y \neq 2^{K+2}$.

Thus we can substitute $A^X+B^Y=2^{K+2}$ for $A^X+\boldsymbol{E}^Y=2^{K+2}$ with Y<3, and substitute $A^X+B^Y \neq 2^{K+2}$ for $A^X+\boldsymbol{E}^Y \neq 2^{K+2}$ with Y≥3, where X≥3. This shows that we have proven $A^X+B^Y \neq 2^{K+2}$ such being the case A^X alone is an odd number of the greater exponent.

In preceding proof, if let B^Y as an odd number of the greater exponent, then A^X is surely an odd number of the smaller exponent. From this, concluded a conclusion via the inference like the above is one and the same with the preceding conclusion.

If A^X and B^Y are two odd numbers of the smaller exponents, after either A^X or B^Y plus 2^{K+1} makes a greater odd number, then the greater odd number and un-incremental one in A^X plus B^Y are bilateral symmetry for symmetric center 2^{K+1} too, but at least one in both of them is not possessed of the greater exponent.

To sum up, we have proven that when W-1=K+1 and K≥6, there is only $A^X+B^Y\neq 2^{K+2}$ under the known requirements. In other words, there are not two odd numbers of the greater exponents altogether on two odd places of every bilateral symmetry for symmetric center 2^{K+1}.

Apply the preceding way of doing, we can continue to prove that when W-1=K+2, K+3, … up to every integer ≥2, there are entirely $A^X+B^Y\neq 2^{K+3}$, $A^X+B^Y\neq 2^{K+4}$, … up to $A^X+B^Y\neq 2^W$ under the known requirements.

Secondly, Let us successively prove $A^X+B^Y\neq 2^WH^Z$ under the known requirements, and point out H ≥3 in them at here emphatically.

We shall set to prove $A^X+B^Y\neq 2^WH^Z$ under the known requirements by the mathematical induction, thereinafter.

(1) When H=1, $2^{W-1}H^Z$ to wit 2^{W-1}, we have proven $A^X+B^Y\neq 2^W$ under the known requirements in the preceding section. Namely there are not two odd numbers of the greater exponents altogether on two odd places of every bilateral symmetry whereby 2^{W-1} to act as the center of the symmetry.

(2) When H=J, $2^{W-1}H^Z$ to wit $2^{W-1}J^Z$, suppose $A^X+B^Y\neq 2^WJ^Z$ under the known requirements, where J is an odd number ≥1. Namely suppose that there are not two odd numbers of the greater exponents altogether on two odd places of every bilateral symmetry whereby $2^{W-1}J^Z$ to act as the center of the symmetry, where J is an odd number ≥1.

(3) When H=K, $2^{W-1}H^Z$ to wit $2^{W-1}K^Z$, prove $A^X+B^Y\neq 2^WK^Z$ under the known requirements, where K=J+2. Namely prove that there are not two odd numbers of the greater exponents altogether on two odd places of every bilateral symmetry whereby $2^{W-1}K^Z$ to act as the center of the symmetry, where K=J+2.

Proof * Since after regard $2^{W-1}J^Z$ as a symmetric center, a sum of every two bilateral symmetric odd numbers is equal to 2^WJ^Z, yet a sum of any two odd numbers of no symmetry is unequal to 2^WJ^Z absolutely.

In addition, there are not two odd numbers of the greater exponents altogether on two odd places of every bilateral symmetry for symmetric center $2^{W-1}J^Z$ according to step 2 of the preceding mathematical induction.

Thus we suppose that A^X and B^Y are bilateral symmetric odd numbers for symmetric center $2^{W-1}J^Z$, and let X<3 and Y≥3, then there is $A^X+B^Y=2^WJ^Z$.

Regard $2^{W-1}K^Z$ as a symmetric center, then 0 and 2^WK^Z, B^Y and $2^WK^Z-B^Y$ are the bilateral symmetry respectively, and there is $B^Y+(2^WK^Z-B^Y)=2^WK^Z$.

By now, let A^X plus $2^W(K^Z-J^Z)$ makes $A^X+2^W(K^Z-J^Z)$, then $A^X+2^W(K^Z-J^Z)$ $=A^X+2^WK^Z-2^WJ^Z=2^WK^Z-(2^WJ^Z-A^X)=2^WK^Z-B^Y$ due to $A^X+B^Y=2^WJ^Z$, where X<3 and Y≥3.

Now that $A^X+2^W(K^Z-J^Z)=2^WK^Z-B^Y$, also for symmetric center $2^{W-1}K^Z$, B^Y and $2^WK^Z-B^Y$ are bilateral symmetry, then B^Y and $A^X+2^W(K^Z-J^Z)$ are bilateral symmetry too. So we get $B^Y+[A^X+2^W(K^Z-J^Z)]=2^WK^Z$, where X<3 and Y≥3.

Since $B^Y+[A^X+2^W(K^Z-J^Z)]=[A^X+B^Y]+2^W(K^Z-J^Z)$ & supposed $A^X+B^Y\neq 2^WJ^Z$, so get $B^Y+[A^X+2^W(K^Z-J^Z)]=[A^X+B^Y]+2^WK^Z-2^WJ^Z\neq 2^WK^Z$, where $X\geq 3$, $Y\geq 3$. But then, there is $A^X+B^Y\neq 2^WJ^Z$, where $X\geq 3$ and $Y\geq 3$. Now that for symmetric center $2^{W-1}K^Z$, B^Y and $A^X+2^W(K^Z-J^Z)$ with $X<3$ are bilateral symmetry, then B^Y and $A^X+2^W(K^Z-J^Z)$ with $X\geq 3$ are not bilateral symmetry, where $Y\geq 3$, so there is $B^Y+[A^X+2^W(K^Z-J^Z)]\neq 2^WK^Z$, where $X\geq 3$ and $Y\geq 3$, according to the preceding conclusion got.

Thus it can be seen, $A^X+2^W(K^Z-J^Z)$ expresses two each other's- disparate odd numbers due to $X<3$ or $X\geq 3$.

So let $A^X+2^W(K^Z-J^Z)=\boldsymbol{A}^\varepsilon$ with $\varepsilon < 3$, and let $A^X+2^W(K^Z-J^Z)=\boldsymbol{A}^M$ with $M\geq 3$. After that, get $B^Y+\boldsymbol{A}^\varepsilon=2^WK^Z$ and $B^Y+\boldsymbol{A}^M\neq 2^WK^Z$ according to aforesaid got $B^Y+[A^X+2^W(K^Z-J^Z)]=2^WK^Z$ with $X<3$ and $B^Y+[A^X+2^W(K^Z-J^Z)]\neq 2^WK^Z$ with $X\geq 3$, where $Y\geq 3$.

Let us change ε of $\boldsymbol{A}^\varepsilon$ and M of $\boldsymbol{A}^M$ as an identical exponent X, then there are $B^Y+\boldsymbol{A}^X=2^WK^Z$ with $X<3$ and $B^Y+\boldsymbol{A}^X\neq 2^WK^Z$ with $X\geq 3$, where $Y\geq 3$.

Evidently, $\boldsymbol{A}^X$ in $B^Y+\boldsymbol{A}^X=2^WK^Z$ and A^X in $A^X+B^Y=2^WK^Z$ are one and the same, where $X<3$ and $Y\geq 3$.

Like that, $\boldsymbol{A}^X$ in $B^Y+\boldsymbol{A}^X\neq 2^WK^Z$ and A^X in $A^X+B^Y\neq 2^WK^Z$ are one and the same, where $X\geq 3$ and $Y\geq 3$.

Thus we can substitute $A^X+B^Y=2^WK^Z$ for $B^Y+\boldsymbol{A}^X=2^WK^Z$ with $X<3$, and substitute $A^X+B^Y\neq 2^WK^Z$ for $B^Y+\boldsymbol{A}^X\neq 2^WK^Z$ with $X\geq 3$, where $Y\geq 3$. This shows that we have proven $A^X+B^Y\neq 2^WK^Z$ such being the case B^Y alone is an odd number of the greater exponent.

In preceding proof, if let A^X as an odd number of the greater exponent, then B^Y is surely an odd number of the smaller exponent. From this, concluded a conclusion via the inference like the above is one and the same with the preceding conclusion.

If A^X and B^Y are two odd numbers of the smaller exponents, after either A^X or B^Y plus $2^W(K^Z-J^Z)$ makes a greater odd number, then the greater odd number and un-incremental one in A^X plus B^Y are bilateral symmetry for symmetric center $2^{W-1}K^Z$ too, but at least one in both of them is not possessed of the greater exponent.

To sum up, we have proven $A^X+B^Y\neq 2^W K^Z$ under the known requirements, where K=J+2. Namely when H=J+2, there are not two odd numbers of the greater exponents altogether on two odd places of every bilateral symmetry whereby $2^{W-1}(J+2)^Z$ to act as the center of the symmetry.

Apply the above-mentioned way of doing, we can continue to prove that when H=J+4, J+6 … up to every odd number ≥1, there are $A^X+B^Y\neq 2^W(J+4)^Z$, $A^X+B^Y\neq 2^W(J+6)^Z$… up to $A^X+B^Y\neq 2^W H^Z$ under the known requirements, and point out H≥3 in them at here emphatically.

Thirdly, we shall proceed to prove $A^X+2^W\neq C^Z$ under the known requirements below.

Proof* Since we have proven $A^X+B^Y\neq 2^W$ under the known requirements, thereby can affirm $E^P+C^Z\neq 2^M$, where E and C are positive odd numbers without a common prime factor, P, Z and M are integers ≥3.

Since E and C have not a common prime factor, then get $E^P \neq C^Z$ according to the unique factorization theorem of natural number, so let $C^Z > E^P$.

Since there is $2^M = 2^{M-1} + 2^{M-1}$, then we deduce $E^P + C^Z > 2^{M-1} + 2^{M-1}$ or $E^P + C^Z < 2^{M-1} + 2^{M-1}$ from $E^P + C^Z \neq 2^M$.

Namely there is $C^Z - 2^{M-1} > 2^{M-1} - E^P$ or $C^Z - 2^{M-1} < 2^{M-1} - E^P$.

Besides, $A^X + E^P \neq 2^{M-1}$ exists objectively according to proven $A^X + B^Y \neq 2^W$ under the known requirements, where A and E are positive odd numbers without a common prime factor, and X, P and M-1 are integers ≥3.

Thus we deduce $2^{M-1} - E^P > A^X$ or $2^{M-1} - E^P < A^X$ from $A^X + E^P \neq 2^{M-1}$.

Therefore there is $C^Z - 2^{M-1} > 2^{M-1} - E^P > A^X$ or $C^Z - 2^{M-1} < 2^{M-1} - E^P < A^X$.

Consequently there is $C^Z - 2^{M-1} > A^X$ or $C^Z - 2^{M-1} < A^X$.

In a word, there is $C^Z - 2^{M-1} \neq A^X$, i.e. $A^X + 2^{M-1} \neq C^Z$.

For $A^X + 2^{M-1} \neq C^Z$, let $2^{M-1} = 2^W$, we obtain $A^X + 2^W \neq C^Z$ under the known requirements.

Fourthly, let us last prove $A^X + 2^W R^Y \neq C^Z$ under the known requirements, and point out R≥3 in them at here emphatically.

Proof* Since we have proven $A^X + B^Y \neq 2^W H^Z$ under the known requirements, of course can get $F^S + C^Z \neq 2^N R^Y$ too, where F, C and R are positive odd numbers without a common prime factor, S, Z and Y are integers ≥3, N=Y+PY, P≥0, and R≥3.

Since F and C have not any common prime factor, so get $F^S \neq C^Z$ according to the unique factorization theorem of natural number, and let $C^Z > F^S$.

Since $2^N R^Y = 2^{N-1} R^Y + 2^{N-1} R^Y$, then deduce $F^S + C^Z > 2^{N-1} R^Y + 2^{N-1} R^Y$ or $F^S + C^Z < 2^{N-1} R^Y + 2^{N-1} R^Y$ from $F^S + C^Z \neq 2^N R^Y$.

Namely there is $C^Z - 2^{N-1} R^Y > 2^{N-1} R^Y - F^S$ or $C^Z - 2^{N-1} R^Y < 2^{N-1} R^Y - F^S$.

In addition, according to proven $A^X + B^Y \neq 2^W H^Z$ under the known requirements, we can get $A^X + F^S \neq 2^{N-1} R^Y$, where A, F and R are positive odd numbers without a common prime factor, X, S and Y are integers ≥ 3, $N-1 = Y + DY$, $D \geq 0$, and $R \geq 3$.

So we deduce $2^{N-1} R^Y - F^S > A^X$ or $2^{N-1} R^Y - F^S < A^X$ from $A^X + F^S \neq 2^{N-1} R^Y$.

Thus there is $C^Z - 2^{N-1} R^Y > 2^{N-1} R^Y - F^S > A^X$ or $C^Z - 2^{N-1} R^Y < 2^{N-1} R^Y - F^S < A^X$.

Consequently there is $C^Z - 2^{N-1} R^Y > A^X$ or $C^Z - 2^{N-1} R^Y < A^X$.

In a word, there is $C^Z - 2^{N-1} R^Y \neq A^X$, i.e. $A^X + 2^{N-1} R^Y \neq C^Z$.

For $A^X + 2^{N-1} R^Y \neq C^Z$, let $2^{N-1} = 2^W$, we obtain $A^X + 2^W R^Y \neq C^Z$ under the known requirements, and point out $R \geq 3$ in them at here emphatically.

To sun up, we have proven every kind of $A^X + B^Y \neq C^Z$ under the given requirements plus the qualification that A, B and C have not a common prime factor.

In addition, previously we have proven that $A^X + B^Y = C^Z$ under the given requirements plus the qualification that A, B and C have at least a common prime factor has certain sets of solutions with A, B and C which are positive integers.

After pass the comparison between $A^X + B^Y = C^Z$ and $A^X + B^Y \neq C^Z$ under the given requirements, we have reached inevitably the conclusion that an indispensable prerequisite of the existence of $A^X+B^Y=C^Z$ under the given requirements is that A, B and C must have a common prime factor.

The proof was thus brought to a close. As a consequence, the Beal conjecture does hold water.