The emperor has no non-locality

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Abstract:

The Einstein-Podolsky-Rosen experiment and certain predictions of quantum mechanics in theoretical and experimental forms are sometimes described as exhibiting non-local action. We describe here an interpretation of one oft-discussed EPR experiment with a locally realistic model. We demonstrate a consistent description based on probabilistic measurement for Mermin and Aspect EPR setups, and show how Bell's theorem applies. Quantum non-locality is shown to be an interpretation dependent on deterministic measurement and vanishes when a treatment of probabilistic measurement and relevant information theory is included.

Résumé:
L'expérience d'Einstein-Podolsky-Rosen et certaines prédictions de la mécanique quantique sont parfois décrites comme présentant une action “non locale”.. Nous décrivons ici une interprétation localement réaliste d'une expérience EPR souvent discutée. Nous démontrons une description cohérente, basée sur la mesure probabiliste pour configurations EPR de Mermin et de Aspect, et nous montrons exactement comment le théorème de Bell s'applique. La non-localité quantique disparaît quand un traitement de mesure probabiliste et de la théorie de l'information pertinente sont entendu.

Keywords: quantum mechanics, epr, nonlocality, bell's theorem

1) Probabilistic Measurement

This term attempts to capture the essence of what is evident from experimental measurements: the outcome of a single external measurement of some physical system is not completely determined by internal variables of the object being measured. Rather, the outcome of a measurement of a system is determined by the physical interaction of internal variables in that system with external variables of the measurement device.

This is evident in the language and practical use of measurements. We are used to hearing measurements reported with error bars and standard deviations implying that measurements have at some level a probabilistic nature. When error bars are not given they are often assumed to exist at the level of the least significant digit of the reported measurement. Measuring instruments are calibrated (when they are calibrated properly) to some specified probable error, again implying our understanding of the probabilistic nature of the relevant measurement. The statistics of probabilistic measurement and the proper interpretation of measurement results form a vital area of study often lumped under the umbrella of statistics.

We can in one way see that this probabilistic nature of measurement is always present is by demonstrating equivalence with a communications channel in general. A measurement must be reported or communicated in some way as information. Information theory tells us quite clearly that a communications channel is a probability distribution function which maps some inputs to some outputs [for a good review see e.g. Cover & Thomas, 2006]. We can conclude without hesitation then that measurement, being communicative in nature, is probabilistic. It turns out that this behavior does not necessarily imply that anyone is "playing dice", as Einstein famously quipped. Rather, the probabilistic nature of measurement exists because an observer does not have an infinite amount of information available when making a measurement.

Consider for example a particle detector consisting of a photomultiplier tube and suitable pulse detection electronics (see Figure 1). The detector is active, and open to a certain direction. In that direction we consider a sphere of volume V just outside the detector but just adjacent to the active area. A simple model of this measuring device and detection volume would suggest that the reading of the device is determined exactly by the flux of particles at a time t entering the device in this volume V.

In fact, the behavior of a real particle detector in this scenario and its reading at a time $t+\delta$ is quantified with a response function which is not perfectly determined by what is specified in the volume *V* at the time *t*. There is a probability that any incoming particle in *V* will reach a dead area on the detector and will not register, as the detector is not perfectly efficient nor perfectly uniform. The reading on the detector could depend on factors not entirely contained in *V*, at a time *t*, including for example any background cosmic rays which might produce counts by entering detection areas from another direction. Electronic fluctuations, other noise, and positions and motions of individual atoms of detector components could all have some effect on the measurement. None of these effects were predictable by considering solely what went on in the volume *V* at the time *t*.

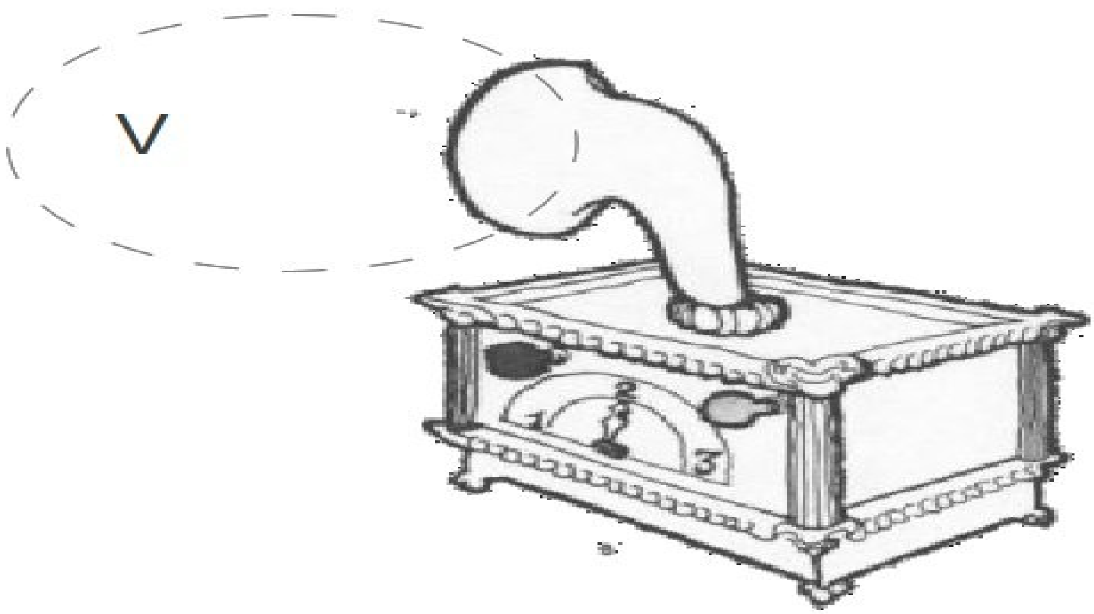

Figure 1. A detector is pictured as sampling particle flux from a volume *V* at a time *t*. The reading on the detector is the output of a probabilistic mapping of variables λ internal to the volume *V*. Adapted from [Mermin, 1985].

The detector efficiency can also have coupling of external variables with variables which are local to *V*. For example the probability of detection could vary with the incoming energy, spin, or other property of incoming particles in *V*.

2) The Mermin Gedanken

In the 1985 edition of Physics Today a remarkable article is found [Mermin, 1985]. In it is described the ethos and impact of the Einstein, Podolsky and Rosen's gedanken experiment and Bell's seminal 1964 paper, outlining debates among many well renowned physicists. Mermin describes a gedanken experiment as an example in which the contradiction laid out in the Bell paper is immediately accessible. Mermin's paper inspired many people struggling with understanding these phenomena and was further popularized in Roger Penrose's book The Emperor's New Mind [Penrose, 1989]. Although the setup described is not a perfect analogy with real measurements of spin ½ particles, it behooves us to consider it again here. Indeed it is answering Mermin's challenge here that forms the heart of this paper and our argument that non-local action is in no way required to explain these phenomena.

The Mermin EPR experiment consists of two detectors separated by some distance and a source directly between them which emits some objects towards the detectors. The detectors can each be set to any one of three settings. Each detector gives a binary measurement. In Mermin's paper, he describes this measurement as turning on either a red or a green light. In some runs of this experiment, two results are observed:

1) When the settings on the two detectors were identical (so that they were set in the same direction) the readings were identical (the colors of the lights matched).

2) When the directional settings of the detectors were randomized, the readings over time gave random results, such that one half the times the colors agreed.

These results model the behavior of certain objects in quantum mechanics. The objects created by the source and measured could be for example spin ½ particles, while the settings of the detectors are equi-angular co-planar orientations, with 120 degrees separation from one orientation to another. The remarkable conclusion often drawn from observations 1) and 2) is that "there is no local realistic model that can explain both these results simultaneously". To show this, the author claims that to be a locally realistic model, an object emitted by the source must have the information to predict the color that would appear on the detector for any orientation. He exhaustively lists all possible combinations of predicted colors that would yield result 1) and shows that these predicted colors cannot produce result 2). He urges us in the paper to "t*ry to invent some other explanation*" for these predicted readings of the detectors.

The Other Explanation

To accept this challenge in the spirit of the gedanken experiment, consider that the objects emitted by the source are paper envelopes containing inside a number, that is an angle which defines a unit vector $\vec{\sigma}$ (perpendicular to the line connecting source and detector). This vector is chosen by a pseudo-random number generator at the source and written down twice, sealed in two envelopes which are emitted simultaneously in opposite directions towards the detectors.

When an envelope arrives at a detector set to one of the three orientations $\dot{\lambda}$, the following procedure is used to set the color of the detector's light. First the envelope is is opened and the dot product $\dot{\lambda} \cdot \vec{\sigma}$ is determined. Because both $\dot{\lambda}$ and $\vec{\sigma}$ are unit vectors, this dot product is simply equal the cosine of θ, the angle between them. If this dot product is positive, we then have a chance to turn the green light on proportional to $\cos\theta$. If the dot product is negative, there is a chance to turn the red light on proportional to $\cos\theta$. A pseudorandom number generator is used at the detector to decide with these probabilities whether to turn on the appropriate light.

We consider as one 'run' of our experiment any time that the envelopes are emitted from the source, and a light is turned on at *both* observing platforms A and B. If either one or the other is not activated, we do not tally the state.

It should be evident that from construction above that the experimental apparatus will produce both our results 1) and 2) exactly! If the detectors are in the same orientation as each other, they will always share the same sign of the dot product to whichever vector is emitted from the source. This means that there are only three possible results: a) both lights remain off, b) one light on one light off, and c) both lights on and agreeing. For the purposes of our experiment, with coincidence detection, only those results of type c will contribute to our experiment as a run and we immediately see that all runs with the two observers choosing the same orientation will show equality of light choice. Result 1) will be satisfied.

If several runs of the experiment are carried out and the orientations are set at random, the resulting probability of matching lights will be ½. A computer program simulating this arrangement is included in the appendix. With our system of envelopes and random number generators at the detectors, we have exactly duplicated the so called quantum calculation which predicts result 2) above. The behavior of EPR systems can be explained with ordinary local measurement.

Real Spin Coupled Systems

While Mermin's gedanken experiment is useful to illustrate the supposed paradox of non-locality in quantum mechanics, it is important to realize we are not capturing all the physics of the interaction of spin ½ particles with specific detector constructions and geometries. In particular, the result 1) will not hold precisely in any real experiment. If two spin ½ particles are generated by source with zero net angular momentum initially, they will be oppositely oriented and so a better analogy would be to have considered lights of opposite colors to be lit when the detectors have the same orientation. No matter, the numbers emitted by the source could sum to zero rather than being equal (conservation of angular momentum).

No detector is either 100% efficient nor devoid of noise or background counts so the result 1) could not hold even if appropriate spin coupled particles could be generated. In practice, coincidence electronics are used in real EPR type experiments such as [Aspect et al., 1984] and [MODERN XP] to minimize single detections, very much like our simulation does. Our solution to Mermin's challenge is also clearly not capturing all the real physics of spin ½ particle detection, but is an ad-hoc construction which satisfies Mermin's criteria. In reality, a quantized binary measurement of a spin in some direction will have a probability proportional in some way to the angle between the detector orientation and the particle spin.

3) Bell's Inequality Revisited

In [Bell, 1964], something like a proof by contradiction is given. An initial assumption is made and labeled as “an assumption of local realism”, and then it is shown that a contradiction is arrived at. The assumption begins in his equation 1:

$$A(\vec{a},\vec{\lambda})=\pm 1, B(\vec{b},\vec{\lambda})=\pm 1$$

Here A and B are the results of measurements of particle spin components in directions *a* and *b*, and lambda represents any set of hidden variables which are physical and local to the particles in question after they are created at the source.

Rather than being an assumption of local realism, this is an assumption of deterministic measurement, for it suggests that later measurements A,B at arbitrary accuracy in a distant location are completely determined by the finite final local variables lambda. Finite local variables in an emitted particle will not always be able to predict a later measurement at arbitrary accuracy, rather they can only affect the measurements probabilistically. In chaotic systems the uncertainty of a later measurement can even increase exponentially in the uncertainty of earlier hidden variables. Bell's conclusions which derive from this formalism are therefore mistaken, in that the settings on one measurement device must not in any way influence another far off device to explain these statistics.

However, Bell's inequality is still applicable. The situation is well described in a publication from Arnold Reinhold [1987], in which he shows how Bell's inequality applies equally to macroscopic phenomena (modified from a version by Jay Sulzberger). This short article is strongly recommended for those interested in interpreting Bell's inequality. His conclusion:

> *"Notice there is nothing in this story about quantum mechanics, determinism, action at a distance or any of that stuff. Bell's inequalities are a simple theorem in Probability 101, which gives conditions on when a set of marginal probability distributions could have been derived from a single joint distribution."*

The nonintuitive results of some quantum physics experiments exist because some probabilities are not always intuitive. The "birthday problem" and the "Monty Hall problem" are two examples of macroscopic systems that obey surprising probabilities. In this spirit of the macroscopic example, I offer my own EPR paradox, aka Schrödinger's Mother:

> *Erwin arrives home and hears that there is a person in the room on his right, and another person in the room on his left. He knows one is his mother and the other is his brother. However, there is not enough information to determine which is which. Each person has a 50% probability of being the mother or the brother. Erwin decides to look to the room on the right. He sees: his brother. He immediately knows where his mother is. At that instant, a non-local phenomenon occurred. The information that he observed his brother traveled superluminally and arrived in the opposite room. The superposed wavefunction there instantly collapsed and became Schrodinger's mother.*

4) Further Considerations

We support the conclusions of authors such as Emilio Santos [2012] that the behavior of quantum systems is well described by local realistic models. Although we describe here only the Mermin setup, this discussion might be traced back to the projection postulate of Von Neumann. We have explicitly shown that a local realistic model does exist that explains the predictions of EPR experiments, in particular for the Mermin setup. In contrast to this conclusion, the Mermin analysis of the EPR experiment and also some of the Bell formalism enumerates the possible outcomes of a measurement by assuming that we can tag an emitted particle with the value of a later measurement. After thus enumerating the possibilities, we are then left with no local model to describe the observed probabilities.

Such enumeration before the fact of measurement is inherently flawed, because measurement is inherently probabilistic. Instead, each measurement apparatus is a consistent local physical device, with a probabilistic map to the states of the system to be measured. Any emergence of statistics showing apparatus to be correlated will be solely due to the fact that the devices are indeed physically correlated. For example, if they are measuring particles from the same source that obey certain conservation relations and therefore have related or entangled probabilities. Such correlation does not require nonlocality nor action at a distance to be explained.

Our solution is devilishly simple. By allowing a random process to occur at measurement, as it does in real life, one can thus explain the predictions of quantum mechanics – using quantum mechanics. A physical theory of local hidden variables coupled with a probabilistic measurement apparatus *can* reproduce the predictions of quantum mechanics.

There is certain similarity between this interpretation of the Mermin EPR and the so-called "detection loophole" described by Pearle [1970]. However the detection loophole as usually described is a way in which an experimental system could still have a deterministic future embedded in local hidden variables as is done in Bell's equation 1 above. Such a loophole also takes the route of coincidence selection that we take in describing the Mermin system. However, what is presented here is not a "loophole" at all. We are not finding a way that hidden variables alone can still determine future measurements in a way consistent with quantum mechanics. Rather, we are pointing out that hidden variables coupled with a probabilistic measurement apparatus can *always* reproduce the predictions of quantum mechanics. There is no need for, nor evidence of nonlocality.

References

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Appendix: Simulation of Mermin EPR

(code also archived at: http://github.com/lukassaul/mermin-sim)

```
import java.util.Random;
import java.util.Date; // for seeding prng

/**
*  Demonstrate local behavior with Mermin-EPR characteristics
*  by simulation of D. Mermin's Gedanken
*
*  Dr. Lukas Saul, Oxford, April. 2015
*/
public class Mermin {
     Random r;
     int n = 10000;  // number of tests to perform

     public final static void main(String[] args) {
          Mermin m = new Mermin();
     }

     /**
     *  Construct a simulation
     */
     public Mermin() {

          int answers1[] = new int[n]; // for case where settings agree
          int answers2[] = new int[n]; // for case where settings are random

          r=new Random();   // init quasirandom generator to current time

          for (int i=0; i<n; i++) {

               // source: choose a random angle (number) from 0 to 2pi for the spin of emitted particles
               double angle = r.nextDouble()*Math.PI*2.0;

               // set the first detector to one of three positions randomly
               double pos1 = r.nextDouble()*3.0;
               double pos1angle = 0.0;
               if (pos1>0.0 & pos1<1.0) pos1angle = 0.0;
               if (pos1>1.0 & pos1<2.0) pos1angle = Math.PI*2.0/3.0;
               if (pos1>2.0 & pos1<3.0) pos1angle = Math.PI*4.0/3.0;

               // for first test run lets take the second detector to be the same as 1st always
               double pos2angle = pos1angle;

               // for second test run we take second detector to be also random
               double pos3 = r.nextDouble()*3.0;
               double pos3angle = 0.0;
               if (pos3>0.0 & pos3<1.0) pos3angle = 0.0;
               if (pos3>1.0 & pos3<2.0) pos3angle = Math.PI*2.0/3.0;
               if (pos3>2.0 & pos3<3.0) pos3angle = Math.PI*4.0/3.0;


               // ok lets see what the results of this run are
               // getresult needs to return 0 (one or both off), 1 (disagree), or 2(agree)


               answers1[i] = getResult(pos1angle, pos2angle, angle);
               answers2[i] = getResult(pos1angle, pos3angle, angle);
          }

          // Output our result of the simulation to stdout
          int numTrue2 = 0; int numFalse2 = 0;
          for (int i=0; i<n; i++) {
               if (answers1[i]==2) numTrue1++;
               if (answers1[i]==1) numFalse1++;
               if (answers2[i]==2) numTrue2++;
```

```
            if (answers2[i]==1) numFalse2++;
        }
        System.out.println("numTrue 1,  numFalse1 " + numTrue1 + " " + numFalse1);
        System.out.println("numTrue 2,  numFalse2 " + numTrue2 + " " + numFalse2);

        // calculate probabilities based on total cases of double coincidence
        double prob1 = (double)numTrue1/(double)(numTrue1+numFalse1);
        double prob2 = (double)numTrue2/(double)(numTrue2+numFalse2);
        System.out.println("1st test: " + prob1 + " 2nd test: " + prob2);
    }

    /**
    *  This returns 0 if one or both still unlit, 1 if disagree, and 2 if agree
    *
    *   SettingA is detector 1 orientation, SettingB is detecter 2 orientation,
    *   spin is particle spin (angle)
    *   settings are 0, 2*pi/3, and 4*pi/3
    */
    public int getResult(double settingA, double settingB, double spin) {
        // set up our random number generator at each detector
        Date d = new Date();

        // current status of light at each detector (0=off, 1=red, 2=green)
        int lightA = 0;
        int lightB = 0;

        // we calculate the difference between
        //the detector angle and particle angle
        double diffA = Math.abs(settingA-spin);
        double diffB = Math.abs(settingB-spin);

        // maximum angle between the two is 180 degrees ..
        // if it's more use the equivalent between 0 and PI
        if (diffA > Math.PI) diffA = 2.0*Math.PI - diffA;
        if (diffB > Math.PI) diffB = 2.0*Math.PI - diffB;


        // There are two choices.
        //  If we are within 90 degrees we go for green.
        //  Outside 90 degrees, we go red.
        if (diffA > Math.PI/2.0) {
            // A has a chance to light up green
            if (r.nextDouble() < 0.0-Math.cos(diffA)) lightA = 2;
        }
        if (diffA < Math.PI/2.0) {
            // A has a chance to light up red
            if (r.nextDouble() < Math.cos(diffA)) lightA = 1;
        }
        if (diffB > Math.PI/2.0) {
            // B has a chance to light up green
            if (r.nextDouble() < 0.0-Math.cos(diffB)) lightB = 2;
        }
        if (diffB < Math.PI/2.0) {
            // A has a chance to light up red
            if (r.nextDouble() < Math.cos(diffB)) lightB = 1;
        }

        // if the lights are the same return true.  else return false.
        if (lightA==0 | lightB==0) return 0;
        else if (lightA==lightB) return 2;
        else if (lightA!=lightB) return 1;
        return 3; // (never)
    }
}
```