PCT, Spin, Lagrangians, Part II

Hans Detlef Hüttenbach

They're building Church and University,
Deceiving the people continually.
Tell the children the truth!

Babylon System, Bob Marley

Abstract. I promised a second part. Here it is. I hesitated, because I don't like being destructive, and unfortunately, to deal with the current state of affairs in quantum theory means just doing this: I show you that the current gauge theory is fundamentally ill-designed, along with it fall Weinberg model and Wightman theory.

1. Continuing with the Lagrangian Formalism

Let me continue from where I left off in the former part: As we saw, the Lagrangian formalism for what is conceived to be the free quantum field amounts to evaluate the extremal for $\delta < \phi, A\phi >$ with $A := (1/2)(\Box\phi - m^2)$. Now, A is a self-adjoint operator, and for that operator it is plain vanilla that the kernel of A is the space of the extremals of $< \phi, A\phi >$, and in this case happens to be the space of solutions of the Klein-Gordon equation $\Box\phi - m^2\phi = 0$.
The problem in here is that from $\Box\phi - m^2\phi = 0$ it cannot be concluded that

$$\mathcal{L} := (-1/2)\big(< \partial_\mu\phi, \partial^\mu\phi > - < \phi, m^2\phi >\big)$$

or any of its equivalent reformulations is the correct Lagrangian: A can be anything, for example the square or n-th power of A have the same kernel, but they would define different Lagrangians. (In [6] I showed that there are two distinct Lagrangians solving the very same equation of motion. Now we see that there are arbitarily many.
Besides this: Does $m^2 x_\mu$ for $0 \leq \mu \leq 3$ possess the dimension of an action? No! But $m x_\mu$ does! - The Lagrangian Formalism is simply meaningless!

2. Gauge Fields

It is commonly held that the electromagnetic field was a $U(1)$ gauge field. Let me disprove it:
I refer to [8, Ch.7, p.125ff.] to discuss the things in a non-abstract mathematical way: Neuenschwander introduces a $U(1)$ gauge transformation as mapping $\phi(x) \mapsto e^{i\chi(x)}\phi(x)$ where χ is a real-valued function in [8, eq. 7.2] and defines gauge invariance as invariance of the equations of motion under that gauge transformation. He then shows that this gauge transformation transforms the partial derivatives ∂_μ to $D_\mu := \partial_\mu + iA_\mu$, where $A_\mu := \partial_\mu \chi$ (see: [8, eq. 7.1.7] and proves the gauge invariance of the Maxwell equations. Sofar, everything is fine. The detrimental error comes in [8, Sec. 7.4], where Neuenschwander argues that the equations of motions for a gauged transformed free field theory would be the same as the free Lagrange equation with minimal coupling $\phi \mapsto \phi - A$, where A is the 4-potential of an external electromagnetic field; so, both would be equivalent, i.e.: the theory of electromagnetism was a $U(1)$ gauge theory.
What is wrong is that the 4-potential $A_\mu = \partial_\mu \chi$ of a $U(1)$ transformation $\phi \mapsto e^{i\chi}\phi$ is integrable (within $\mathbb{R}^4$) to a scalar function function, which is χ. However, due to its non-zero rotation, a non-trivial electromagnetic 4-potential never is integrable within $\mathbb{R}^4$. So, whatever $U(1)$-gauge is chosen, no gauge field matches a non-zero electromagnetic 4-potential!

Therefore, whatever is described in a $SU(2) \times U(1)$ gauge theory, known as Salam-Weinberg model (see:[10]), it cannot be the unification of weak and electromagnetic theory!

Let's dig deeper and work out, what could be missing to fix this:
The space X of all complex linear combinations of the four Dirac matrices $\gamma_0, \dots, \gamma_3$ is a four-dimensional vector space with a (continuous) inner product, defined by:

$$< \sum_\mu \lambda_\mu \gamma_\mu, \sum_\nu \kappa_\nu \gamma_\nu >:= \sum_{\mu,\nu} \gamma_\mu^* \bar{\lambda}_\mu \kappa_\nu \gamma_\nu = \sum_\mu \bar{\lambda}_\mu \kappa_\mu .$$

This space X then is isometric with $\mathbb{C}^4$, although the notion of orthonomality is lost within X, because $\gamma_\mu^* \gamma_\nu \neq 0$ for $\mu \neq \nu$. So, a unitary mapping on $\mathbb{C}^4$ maps into a unitary mapping on X. Similarly, the restriction of $\iota : \mathbb{C}^4 \ni (\lambda_0, \dots, \lambda_3) \mapsto \sum_\mu \lambda_\mu \gamma_\mu \in X$ to real values of λ_μ is an isometry of $\mathbb{R}^4$ to some (yet still complex) subspace $Y \subset X$, say. Both spaces $\mathbb{R}^4$ and Y as well as $\mathbb{C}^4$ and X induce a different concept of differentiation: The differentials $dx_0, \dots, dx_3$ on $\mathbb{R}^4$ or $\mathbb{C}^4$ become $dx_0\gamma_0, \dots, dx_3\gamma_3$ on X and Y. Now, considering an electromagnetic for potential $A = (A_0, \dots, A_3)$, then by gauge invariance, as a function on $\mathbb{R}^4$, we can ensure that $A_\mu dx_\nu = -A_\nu dx_\mu$ for $\mu \neq \nu$ (which is not integrable). However, if I map this over to X (or Y), A becomes $\iota A = \gamma_0 A_0 + \cdots + \gamma_3 A_3$, and path integration along the μ-th coordinate comes with an extra factor γ_μ. That means, we can now integrate ιA along paths $\omega : [0, 1] \ni \xi \mapsto X$ to some scalar function χ, which is real-valued if all the A_μ are, so $e^{i\chi}$ will be the desired gauge transformation.

So, the trick played is to integrate $\iota A = \sum_\mu \gamma_\mu A_\mu$ within X, but not A within $\mathbb{R}^4$. But, where can we get the extra factors γ_μ from? - Let's have a look:

A current is described in electrodynamics as $j = \rho(u_0, \ldots, u_3)$, where u is the 4-velocity. If that was to read instead $j = \sum_\mu j_\mu \gamma_\mu$, not only we would get the invariant $j^2 = j_0^2 - \cdots - j_3^2$ for its square, also, by leaving out the Dirac matrices, we would erroneously turn the spinor into a scalar. Now j comes as a factor in the resulting force on a test charge (density) in motion, j', say. Since we dropped the Dirac matrices in j, integrating the force in $\mathbb{R}^4$ will run us in problems: What would come out as a scalar energy function after path integration the spinor force will become a non-integrable, complicated vector-potential of scalar components. And to enforce the integrability of that vector-potential, it would need the artificial insertion of the Dirac matrices again, which will give us a scalar action function.

But watch out: Beginning with a spinor j, the force (Per spinor test charge) becomes itself a spinor force field F, path integration within Y will result in a scalar potential, and another integration will then give the action as a spinor field, and not a scalar field.

That implicitly means that as a gauge theory, electromagnetism cannot be a $U(1)$-gauge theory: instead, the gauge transformation comes out as $\phi \mapsto e^{i \sum_\mu \gamma_\mu \chi_\mu}$, which, according to the metrics chosen in isometry with $\mathbb{C}^4$ is is a unitary group, albeit one of dimension four, and not one! And there is only one unitary group (up to isomorphism) of this dimension, which is $U(2)$.

Again, this is satisfactory: In a charge symmetric world there is no absolute means to tell what charge is positive and what negative: all we know is that one observer may look at an electron as being negatively charged, while another observer may determine its positive charge. A $U(2)$ model fits perfectly (and again advocates that it might be a good idea to accept the co-existence of positrons with electrons).

I have one further point to make: The gauge invariance allows us to get rid of the symmetric part of A, i.e. that part, that can be integrated into a scalar function within the Euclidean metrics. In the first part I showed that there is this antisymmetric part A, and that there is also this symmetric part, which I denoted as the "neutral mass". Because the group of unitary mappings on $\mathbb{C}^4$ is the group $U(4)$, that would mean that, given all fields were gauge theories, all neutral fields would go into a $U(4)/U(2)$ gauge field, where the divisor denotes the quotient class. It would also then imply that we would only have two long-ranged forces, one for charges, and the rest for neutral matter. I am rather sceptical as this being ultima ratio, and I would expect either progress and change as to this.

3. Wightman Axioms

In the mid 60's Arthur Wightman condensed what were thought to be the basic assumptions and ingrediences of a Quantum Field Theory in a set of

axioms (see e.g.: [9] for a good online source other than the original book [11]).
In it, he states under the (W0) axiom, that the energy-momentum operator is to have its spectrum contained in the forward light cone.
That seems fair - at first sight, because of causality reasons. But, it it detrimental:

Not only does it disallow time (or energy) inversion, which is so important in classical physics, it disallows partity onversion either: Parity inversion is the transformation of the location coordinates $x \mapsto -x$. It is an error to think that parity inversion maps the forward light cone into itself: it does not: when you flip the time axis upside down, then the parity also gets inverted! That means, you can't have parity symmetry without the symmetry of time inversion. So, not only would you loose the principle of time invariance, you also would loose momentum conservation!
Sorry: That's not what it's supposed to be! It is just the old Platonic error, be it Dirac, be it Wightman: A symmetry does not ask for exclusion, but demands coexistence!

(When you theoretically cut off three of a baby's limbs, would you expect this theoretical child to behave like a real child?)

Neither is it comprehensible to exclude the interchange of time and location coordinates for the sake of a dogma of positivity of the Minkowski metrics, whereas in classical mechanics the replacement of any generalized location coordinate with the time coordinate is allowed, nor is it comprehensible to better speak of virtual. positive energetic particles moving backwards in time, rather than frankly of real particles with negative energy, moving forward in time!

There is actually more going wrong with the Wightman theory:
In axiom 1, the existence of a unitary representation of the Poincaré group on a Hilbertspace $\mathcal{H}$ is demanded, where the the phase symmetric classes $\{e^{i\lambda}\Psi|\lambda \in \mathbb{R}\}$ of the unit vectors $\Psi \in \mathcal{H}$ are the states of the theory. Now, Wightman does not explicitly demand the existence of self-adjoint, commuting location operators $Q_1, \ldots, Q_n$ with $\mathbb{R}$ as their spectrum, each. However, this is quantum-theoretically needed, in order to be able to deal with fields as operator-valued distributions of $x_1, \ldots, x_n$ (and time t), as Wightman does later on: all dimensional coordinates must be eigenvalues of observables, which are self-adjoint operators on that Hilbertspace $\mathcal{H}$. But that means that no state of $\mathcal{H}$ can ever be translationally invariant along any of its location coordinates: For, if $Q_k = \int_{-\infty}^{+\infty} \lambda dE_\lambda$ is the spectral representation of Q_k and $\Psi \in \mathcal{H}$ is a unit vector, then $\int_{-\infty}^{+\infty} \|\Psi(\lambda)\|^2 d\lambda = 1$, where $\Psi(\lambda) := E_\lambda \Psi$, whereas translational invariance just demands $\Psi(\lambda) = \Psi(\lambda + \kappa)$ for all $\kappa \in \mathbb{R}$. Now, in a subsequent axiom, Wightman demands the existence of a unique Poincaré invariant state Ψ_0, which is what is called the vacuum state. Because Poincaré invariance includes translational invariance, Ψ_0 cannot be a phase symmetric class of unit vectors of $\mathcal{H}$. In all, that shows that if Ψ_0 was to exist, then $Q_1, \ldots, Q_n$ are undefined as observables on $\mathcal{H}$. But where

do the location coordinates then come from? Above that: as an extension of quantum theory, quantum field theory is in need of the location observables.

Further, accepting the nonexistency of the location observables, another axiom states the cyclicity of the vacuum state. Because the Wightman fields are operator valued distributions with support in the forward time-like light cone, that implies that the vacuum state cannot be unique: There must be three other ones, one for the negative timelike cone, and two for the two spacelike regions of either sign of time: these are needed to conform with the symmetries of space, time (, and charge) inversions.

You might wonder, why I included charge inversion above, for, isn't scalar Wightman theory a neutral theory?
The answer is that a neutral particle theory is not neutral, because it does not contain any charges, but because it contains an equal amount of opposite charges: opposite charges simply don't just add up to nothing! So, a theory is neutral if and only if it is invariant w.r.t. charge inversion. And invariance is not equal to symmetry! To get it clear: We conceive an atom as a system consisting of a positively charged nucleus and an oppositely charged shell of electrons: that way, the atom is overall neutral, but it is not charge inversion symmetric, because the nucleus is heavier than the electron shell. A symmetric situation would predict negatively charged nucleusses surrounded by positrons at an equal rate. What's going on? Let's drill into it:
We know, we can deflect the electrons by light: light enters, scatters with the electrons and exits. Now let's revert the time direction: light enters, scatters with the electrons, and exits - just as before. However, as we invert time, everything that went in one spatial direction before, now goes the opposite way: parity changed upon time inversion. So, the whole system undergoes a $\mathcal{TP}$ inversion. But $\mathcal{PCT} \equiv \mathbb{1}$, which means that the system is charge invariant: that means that as long as $\mathcal{P}$ and $\mathcal{T}$ are symmetries, $\mathcal{C}$ also is, and physically, no charged particle can be distinguished from its $\mathcal{PT}$ inversion. (This is mirrored by the fact that the electromagnetic field is phase invariant.) There is hence no physical reason to expect that all atoms have in common a positively charged nucleus and negatively charged electrons: as to the laws of electrodynamics, this is just a gauging convention.

4. Mass Field

By the many years of its existence, classical mechanics is commonly held to be a homogenous, self-consistent theory. Inside, however, it separates into different concepts: one is the motional, another one a dynamical theory. As to the motional theory, the system under consideration is a time curve $t \mapsto (q_1, \ldots, q_N)$ of the N generalized location coordinates in space and time. In the dynamical part, then, that's not enough: we also need their canonically conjugated N momentum coordinates plus the total energy (which is the canonical conjugate of time). This then leads straight into the Hamiltonian

mechanics, in which 2N independent momentum and location parameters are coupled by N Hamiltonian equations (of first order).
Interestingly, a different concept emerged even before that: The dynamical system could be replaced by a mass-valued function $m(\vec{x}(t)) = m(t, \vec{x}))$ in space and time. Since the masses of all bodies are positive, they all add up, so all positons, their speed, their momenta, and their energy can be identified, just through a mass-valued time curve in a three dimensional Euclidean space. That means that a dynamical mechanical system of n objects would be completely determined through the mass density curve $t \mapsto \rho(\vec{x}(t))$, and the famous Poisson equation, $\nabla^2\Phi = \rho$, would then state the equivalence of the gravitational field with the mechanical system. (And it was then really understood that way by former physics.) Now, given the continuity of ρ, I can integrate this scalar field along its three spatial ccordinates to a vector field, and, replacing the differential dx_k, for $1 \leq k \leq 3$ by $\sigma_k dx_k$, where the σ_k are the Pauli matrices, I can sum the vector components up to a spinor field $S(\vec{x}(t))$. Then $\not\nabla S(\vec{x}(t)) = m(\vec{x}(t))$ with $\not\nabla := \sum_{1\leq k\leq 3} \sigma_k \partial_k$. I can now enforce the Poisson equation by multiplying ρ to the left by the derivative $-\not\nabla\rho'(\vec{x}(t))$ of a test mass distribution ρ' and integration over $\mathbb{R}^3$, where I assume that both ρ and ρ' have a compact support in $\mathbb{R}^3$ for each t:

$$< -\not\nabla\rho', \not\nabla S >:= \int_{\mathbb{R}^3} -(\not\nabla\rho(\vec{x}(t)))\not\nabla S(\vec{x}(t))d^3x = \int \rho'(\vec{x}(t))\nabla^2 S(\vec{x}(t))d^3x.$$

Because this is an unsymmetric, non-quadratic bilinear form, I prefer multiplication to the left not by the derivative $\not\nabla\rho'$, but by ρ' itself, which will turn the above equation into

$$\int \rho'(\vec{x}(t))\rho(\vec{x}(t))d^3x = -\int S'(\vec{x}(t))\nabla^2 S(\vec{x}(t))d^3x.$$

Now, mass is energy (divided by the square of the speed of light), so S is the action. Once again: The knowledge of the action of a mechanical system in space and time completely determines the mechanical system itself (and vice versa).

Remark 4.1. If the non-relativistic gravitational field is a gauge theory at all, then it must be an $SU(2)$-field, because its unitary group is spanned by the three Pauli spinors, therefore is a three-dimensional group, which determines it to be $SU(2)$. And it looks like to become a gauge theory, if only one accepts the formal relation $1/(\sum_k \lambda_k\sigma_k) = d(ln(\sum_k \lambda_k\sigma_k)/d(\sum_k \lambda_k\sigma_k)$ for $\lambda_k \neq 0$, $(1 \leq k \leq 3)$.

Then, when it comes to relativistics, we can do the analogous thing as above, we just need to replace the Pauli spinors by the Dirac spinors, and the closed loops in space become closed loops in space-time, albeit not intersecting the light cone. Maxwell's equations become the natural extension of Poisson's equation to the relativistic theory, in which spinors represent a unified concept of matter, including charges (see: [5] for further details).

References

[1] Clay Institute (A. Jaffe, E. Witten), *First Millenium Problem: Yang-Mills Fields And Mass Gap*, http://www.claymath.org/sites/default/files/yangmills.pdf, 2000

[2] A. Einstein, *Ist die Trägheit eines Körpers von seinem Energieinhalt abhängig?*, http://www.physik.uni-augsburg.de/annalen/history/Einstein-in-AdP.htm, 1905.

[3] R. P. Feynman, *Lectures on Physics*, Vol. I-III, Addison Wesley, 1977.

[4] H. D. Hüttenbach, *Analytic Functions For Clifford Algebras*, http://vixra.org/pdf/1408.0084v3.pdf, 2014.

[5] H. D. Hüttenbach, *The Action Function of Adiabatic Systems*, http://vixra.org/pdf/1404.0441v2.pdf, 2014.

[6] H. D. Hüttenbach, *In/Equivalence of Klein-Gordon and Dirac Equation*, http://vixra.org/pdf/1410.0154v2.pdf, 2014.

[7] H. Kleinert, *Particles and Quantum Fields*, http://users.physik.fu-berlin.de/~kleinert/b6/, book publishing in preparation, 2014.

[8] D. E. Neuenschwander, *Emmy Noether's Wonderful Theorem*, John Hopkins University Press, 2011.

[9] NLab, n.n. *Wightman Axioms*, http://ncatlab.org/nlab/show/Wightman+axioms, 2015

[10] V. Kaplunowski, *Glashow-Weinberg-Salam-Theory*, http://bolvan.ph.utexas.edu/~vadim/classes/2012f/GWS.pdf, 2012.

[11] R.F. Streater, A. Wightman, *PCT, Spin and Statistics and All that*, Princeton University Press, 1978.

[12] S. Weinberg, *Cosmology*, Oxford University Press, 2008.

[13] Wikipedia, *Wheeler-Feynman Absorber Theory*, http://en.m.wikipedia.org/wiki/Wheeler-Feynman_absorber_theory, 2014.

[14] Wikipedia, *Creation and annihilation operators*, http://en.m.wikipedia.org/wiki/Creation_and_annihilation_operators, 2014.

Hans Detlef Hüttenbach
e-mail: detlef.huettenbach@computacenter.com