Linear, Radial & Scalar Magnitudes

Agustín A. Tobla

(2015) Buenos Aires
Argentina

In classical mechanics, this paper presents the definitions and the relations of the linear, radial and scalar magnitudes of a pair of particles ij.

Introduction

i) The definitions of the linear, radial and scalar magnitudes of a pair of particles ij, where $\vec{r}_i$ and $\vec{r}_j$ are the positions of the particles i and j, are as follows:

§ The linear position $\vec{r}_{ij}$, the linear velocity $\vec{v}_{ij}$ and the linear acceleration $\vec{a}_{ij}$, are given by:

$$\vec{r}_{ij} \doteq (\vec{r}_i - \vec{r}_j)$$

$$\vec{v}_{ij} \doteq d(\vec{r}_{ij})/dt = (\vec{v}_i - \vec{v}_j)$$

$$\vec{a}_{ij} \doteq d^2(\vec{r}_{ij})/dt^2 = (\vec{a}_i - \vec{a}_j)$$

§ The radial position r_{ij}, the radial velocity $\dot{r}_{ij}$ and the radial acceleration $\ddot{r}_{ij}$, are given by:

$$r_{ij} \doteq |\,\vec{r}_i - \vec{r}_j\,|$$

$$\dot{r}_{ij} \doteq d(r_{ij})/dt = [\,(\vec{v}_i - \vec{v}_j) \cdot (\vec{r}_i - \vec{r}_j)\,] \,/\, |\,\vec{r}_i - \vec{r}_j\,|$$

$$\ddot{r}_{ij} \doteq d^2(r_{ij})/dt^2 = [\,(\vec{a}_i-\vec{a}_j)\cdot(\vec{r}_i-\vec{r}_j)+(\vec{v}_i-\vec{v}_j)\cdot(\vec{v}_i-\vec{v}_j)-[(\vec{v}_i-\vec{v}_j)\cdot(\vec{r}_i-\vec{r}_j)]^2/(\vec{r}_i-\vec{r}_j)^2\,] \,/\, |\,\vec{r}_i-\vec{r}_j\,|$$

§ The scalar position τ_{ij}, the scalar velocity $\dot{\tau}_{ij}$ and the scalar acceleration $\ddot{\tau}_{ij}$, are given by:

$$\tau_{ij} \doteq {}^1\!/_2\,(\vec{r}_i - \vec{r}_j) \cdot (\vec{r}_i - \vec{r}_j)$$

$$\dot{\tau}_{ij} \doteq d(\tau_{ij})/dt = (\vec{v}_i - \vec{v}_j) \cdot (\vec{r}_i - \vec{r}_j)$$

$$\ddot{\tau}_{ij} \doteq d^2(\tau_{ij})/dt^2 = (\vec{a}_i - \vec{a}_j) \cdot (\vec{r}_i - \vec{r}_j) + (\vec{v}_i - \vec{v}_j) \cdot (\vec{v}_i - \vec{v}_j)$$

ii) The relations between the linear, radial and scalar magnitudes of a pair of particles ij, they can be obtained from the above definitions, are as follows:

$$\tau_{ij} = {}^1\!/_2\, r_{ij}\, r_{ij} = {}^1\!/_2\, \vec{r}_{ij} \cdot \vec{r}_{ij}$$

$$\dot{\tau}_{ij} = \dot{r}_{ij}\, r_{ij} = \vec{v}_{ij} \cdot \vec{r}_{ij}$$

$$\ddot{\tau}_{ij} = \ddot{r}_{ij}\, r_{ij} + \dot{r}_{ij}\, \dot{r}_{ij} = \vec{a}_{ij} \cdot \vec{r}_{ij} + \vec{v}_{ij} \cdot \vec{v}_{ij}$$

iii) The magnitudes [$\vec{r}_{ij}, r_{ij}, \dot{r}_{ij}, \ddot{r}_{ij}, \tau_{ij}, \dot{\tau}_{ij}$ and $\ddot{\tau}_{ij}$] are invariant under transformations between inertial and non-inertial reference frames.