Duane – Hunt relation improved

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Abstract

In present paper the Duane-Hunt relation for direct measurement of the Planck constant is improved by including of relativistic corrections. New relation to determine the Planck constant, suggested in this paper contains Duane-Hunt relation as first term and can be applied in a wide range of energies.

The origin of quantization and nature of Planck constant are the most intrigue problems for more than century as of the origin of Quantum Theory. From the beginning the different attempts to explain the nature of the Planck constant were made. One interesting way to do this was that suggested by Boyer in 1978 [1] (see also [2, 3, 4] and references therein), within the framework of "Zero-Point Radiation". Another interesting idea that should be mentioned here was recently implemented in the framework of the electrodynamics model of the atom proposed by M. Percovac[5].

In our paper [6] we suggest more natural way to explain the origin of the Planck constant due to quantization of action. This way not only allow us to calculate correct value of the Planck constant from the first principles, (i.e. from cosmological parameters, or from the geometry of our Universe), but give us a theory to make unification of quantum and relativistic physics. In that paper the relation $E = h\nu + E_0(\nu)$, which may affect on the Planck constant measurements at different energy ranges, was obtained. Here, as well, we should mention work of V. Garcia Morales [7] in whish, starting from the thermodynamics, he argued that neither time nor space needs to be discrete but it is just action what is quantized. All this suggests the importance of accurate measurement of Planck's constant in all energy ranges for the possible detection of more subtle effects and, as a consequence, for experimental confirmation of the theory.

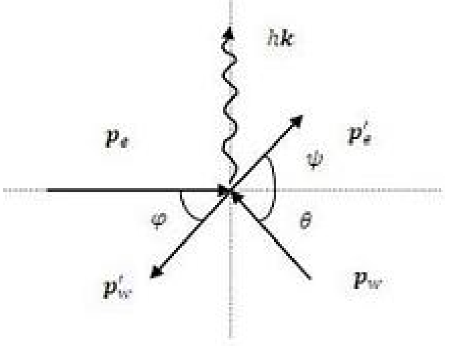
Recently there appears a compilation of experimental results on the Planck constant measurements for wide range of energy $[8-23]$ where not only experimental errors are discussed, but also supposed probable experimentally measurable dependence of the Planck constant on energy. So, more careful experiments over all energy ranges are needed.

At the moment there are a lot of measurements of the Planck constant at small energies from 0.001 to $1eV$ $[8-16]$. Also the measurements for $1MeV$ are available $[17-21]$, but in the range of energies from $1eV$ to $1MeV$ there are no precise data reported [8, 21]. In this case is of great importance not only elaborate new experimental technique, but also increase precision of actually available methods.

One such method of direct measurement of the Planck constant should be mentioned here is the technique based on the Duane-Hunt relation. This relation was written from classical point of view, without relativistic corrections. For this reason it shows dramatic discrepancy for the measured Planck constant in respect to the CODATA value [17 – 20] in second - third digit and cannot be used for precise measurements of the Planck constant in its original form [22 – 25].

In present paper we suggest precise relativistic relation between the incident electron energy and energy of produced X-ray photon, and also write out first terms of its expansion with accuracy up to 4-th term. This relation is more precise if compared with the Duane – Hunt expression and contain last one as zero term of expansion.

To begin with let's see the geometry of the emitting process under discussion.



$Fig1$. The figure schematically represents the momentum vectors of each of the entities involved, where the subscript (e) represents the electron and the subscript (w) corresponds to Wolfram, also primate superscript indicates values after collision, and hk is the momentum of the emitted photon.

In this case the conservation equations for temporal and spatial parts of the 4-momentum [24] are

$$E_e + E_w = E'_e + E'_w + E_\gamma \tag{1}$$

$$\boldsymbol{p}_e + \boldsymbol{p}_w = \boldsymbol{p}'_e + \mathbf{p}'_w + \boldsymbol{p}_\gamma \tag{2}$$

Or, $Eq(2)$ in the corresponding projections is

$$p_e - p_w \cos\theta = p'_e \cos\psi - p'_w \cos\varphi \tag{3}$$

$$p_w \sin\theta = p'_e \sin\psi - p'_w \sin\varphi + \hbar k \tag{4}$$

Here the subscript (e) and (w) distinguishes the electron and Wolfram respectively for each of the quantities involved in the interaction, where $\hbar$ is the reduced Planck's constant, k the wavenumber of the emitted photon, and $p's$ are the 3-moments of each constituent of the system before (p) and after (p') interaction.

Let's assume that the initial electron transfers all of its energy and momentum to the photon (this is the case in the framework of a Duane – Hunt relation). i.e. $p'_e \ll p_e$ and $p'_e \approx 0$. Thus $Eq.$(3) and (4) take the form,

$$p_e - p_w \cos\theta = -p'_w \cos\varphi \tag{5}$$

$$p_w \sin\theta = \hbar k - p'_w \sin\varphi \tag{6}$$

We square $Eq.(5)$ takes the form

$$p'^2_w \cos^2\varphi = \left(p_w \cos\theta - p_e\right)^2 \tag{7}$$

and rewrite $Eq.(6)$ to have

$$\hbar k = p_w \sin\theta + p'_w \sin\varphi \tag{8}$$

from $Eq.(7)$ through a trigonometric identity, we obtain,

$$p'^2_w \cos^2\varphi = p'^2_w - \left(\hbar k - p_w \sin\theta\right)^2 \tag{9}$$

and from $Eqs.(7)$ and (9), we have

$$\left(p_w \cos\theta - p_e\right)^2 = p'^2_w - \left(\hbar k - p_w \sin\theta\right)^2 \tag{10}$$

Relativistic equations for energy are also valid, they are $E^2 = m^2c^2 + p^2c^2$ and $E = K + mc^2$, where E is the total energy, K kinetic energy, and p is the 3-momentum. From this relation and $Eq.(1)$ we have the following expression for energy:

$$K_e + m_ec^2 + K_w + Mc^2 = K'_e + m_ec^2 + K'_w + Mc^2 + h\nu \tag{11}$$

where by assumption $v'_e \ll v_e$, it implies that $K'_e \approx 0$, therefore

$$K_e + K_w = K'_e + K'_w + h\nu \tag{12}$$

When it reaches the anode, the electron has a kinetic energy of about $K_e = eU$, and K_w, K'_w are nonrelativistic, namely can be substituted by classical expression $\frac{p^2}{2m}$, then

$$eU + \frac{p_w^2}{2M} = h\nu + \frac{p'^2_w}{2M} \tag{13}$$

Hence, the square of momentum of the Wolfram nucleus after the interaction is

$$p'^2_w = 2M\left(eU + \frac{p_w^2}{2M} - h\nu\right) \tag{14}$$

the momentum of the electron p_e in terms of its total energy is

$$p_e c = \sqrt{\left(eU\right)^2 + 2m_ec^2eU} \tag{15}$$

Substituting $Eqs.(14)$ and (15) into $Eq.(10)$ and solving it for $h\nu$, one can arrive to the following expression,

$$h\nu = \left(Mc^2 - p_wc\sin\theta\right)\left[-1 + \sqrt{1 + \frac{2Mc^2eU + 2p_wc\cos\theta\sqrt{(eU)^2 + 2m_ec^2eU} - (eU)^2 - 2m_ec^2eU}{\left(Mc^2 - p_wc\sin\theta\right)^2}}\right] \tag{16}$$

Finally leaving the first order terms in the expansion, we get

$$h\nu = eU + \frac{p_w \cos\theta \sqrt{(eU)^2 + 2m_e c^2 eU}}{Mc} - \frac{(eU)^2}{2Mc^2} - \frac{m_e}{M}(eU) \tag{17}$$

As one can see, the first term is precisely the Duane-Hunt law $h\nu = eU$, and the second term corresponds to the relativistic correction to the kinetic energy of the nucleus of Wolfram.

The aim of the experiment is to determine the critical or threshold voltage at which x-rays of a given wavelength just begin to appear as the shortest wavelengths emitted in the continuous spectrum. The frequency ν corresponding to this wavelength is believed to be related to the threshold voltage U at which it just begins to be excited by the well-known relation, bremsstrahlung radiation represents an important x-ray production mechanism in ion-atom collisions and for very heavy projectiles contains information about the coupling of electromagnetic field with matter in the presence of strong fields [26, 27]. Such experiments are planned to be performed in the nearest future and are expected to reveal information about the short–wavelength limit of the electron–nucleus bremsstrahlung. In addition to the characteristic values for the configuration in experimental arrangements provide data to calculate the Planck constant h, and compare with actual experiment $[9 - 21]$.

We have shown that, with conventional quantum considerations, the relativistic corrections to classical quantum law led to greater accuracy in method based on the Duane-Hunt law, and allow us to determine the Planck constant more precisely (with appropriate experimental data), in addition corroborate the dependence on energy in its determination, as celebrated for many years by means of the law of Duane-Hunt.

On the basis of detailed relativistic consideration of electron collisions with target atoms, an exact relation between the potential difference (the tube high voltage which correspond to the initial energy of the electron), and the threshold energy of the emitted photon were obtained. This ratio, which is a generalization of Duane-Hunt relation, allows carrying out accurate measurement of the Planck's constant in a wide range of energies.

To conclude it should be mentioned again there exist a large number of physical theories, particularly based on the geometric approach $[6 - 7, 28 - 31]$. Thus, the problem of accurate experimental determination of Planck's constant is of crucial importance, since such measurements allow us to effectively discriminate from the large variety of theories, electing those that correspond to reality.

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