Static Fields as Mass-Currents and Mercury's Perihelion Shift

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Abstract

The zero-mass limit for quantum-fields is performed. The resulting field-quanta consist of equal components of positive and negative frequency, which result in a conserved zero 4-vector of mass current. The transformation behaviour of a field's spin variables shows that only states of zero total twist survive this limit process. As a consequence neutrinos must have non-zero mass. Static fields are the classical interpretation of stationary mass-currents which consist of positive- and negative-frequency components of equal spatial k-vector (opposite for quanta!). Such objects, which require sources and sinks, do not occur in conventional quantum field theories, but are central to the case of gravitation. Consequently, static fields have a flow-nature with corresponding retardation effects, which provided a 19th century explanation of Mercury's perihelion shift.

Introduction

In a previous paper we have shown that Lorentz-transformations yield a two-valued “representation” in the Hilbert space of massive particles [1]. It was also shown that the Klein-Gordon operator can be generally factorized into two conjugated Dirac-type operators:

$$P^0 - \bar{I}\,\vec{P}\,\vec{\rho} - m\tau \quad \text{and} \quad P_0 + \bar{I}\,\vec{P}\,\vec{\rho} - m\tau \quad , \tag{1}$$

with

$$P^\mu = i\,\partial_\mu \quad , \tag{2}$$

the infinitesimal generators of 4-*d* translations and $\vec{\rho}$ the infinitesimal generators of 3-*d* rotation. Furthermore, $\bar{I}$ is the operation of complex conjugation, *m* the rest mass of the field under consideration, and τ the mass-sign operator.

There is an associated energy-momentum operator

$$sign(P^0)\,P^\mu \quad , \tag{3}$$

owing to the preservation of the time direction under orthochronous Lorentz-transformations. This distinction between mass (2) and energy (3) is a new aspect added by quantum mechanics to classical Special Relativity [2], and we will call here the whole of expression (2) the mass current operator.

Multiplication of either operator (1) with it's adjoint yields the Klein-Gordon operator

$$P^{\mu} P_{\mu} - m^2 \quad . \tag{4}$$

In the mentioned work [1] we considered the behaviour of a massive field under boosts. It was shown, that a field originally at rest develops a component of opposite mass under acceleration. The present note deals with the limit that the boost velocity approaches the speed of light with a simultaneous reduction of the rest mass of the field to arrive at the zero-mass case.

Zero-Mass Fields

When the rest-mass term is absent in the field equations (1,4) the field modes lie on the light cone, which signifies that their mass is exclusively of kinetic origin. The modes combine in different ways to produce the physically observable zero-mass objects: quanta and stationary fields.

Quanta

To arrive at a description of zero-mass quanta we start from a finite-mass field and perform a limit process in which a particle is moved at ever increasing speed, while its rest-mass is simultaneously reduced.

We have seen [1] that a pure matter state of mass m at rest $W(k) = W(m,0,0,0)$, $W(-m,0,0,0) = 0$, is boosted (in 1-direction for simplicity) to

$$\cosh(w) W(m\cosh(w), m\sinh(w), 0,0) + \sinh(w) W(-m\cosh(w), -m\sinh(w), 0,0) \quad , \tag{5}$$

where w is expressed by v, the speed of the boost, by

$$w = \frac{1}{2} \operatorname{artanh}(v) \quad . \tag{6}$$

In the limit process v approaches one and, thus, w goes to infinity. Simultaneously, m goes to zero such that the wave-number, k^0, stays finite. Remembering that we have

$$\cosh(w), \sinh(w) \rightarrow \frac{1}{2} e^{w} \quad , \tag{7}$$

for large w, we take

$$m = k^0 2 e^{-w} \quad . \tag{8}$$

Furthermore, in order to keep unit normalization we multiply expression (5) by

$$e^{-w} / \sqrt{2} \quad , \tag{9}$$

and end up with the field's state

$$\frac{1}{\sqrt{2}} [W(k^0, k^0, 0,0) + W(-k^0, -k^0, 0,0)] \quad . \tag{10}$$

In order to keep track of the transformation behavior under a Lorentz-transformations (again just in 1-direction) we add to w a finite quantity s. Using

$$\cosh(w+s), \sinh(w+s) \rightarrow \frac{1}{2} e^{w} e^{s} \quad , \tag{11}$$

and taking the same limit leads to

$$\frac{e^{s}}{\sqrt{2}}[W(k^{0} e^{s}, k^{0} e^{s}, 0,0) + W(-k^{0} e^{s}, -k^{0} e^{s}, 0,0)] \quad , \tag{12}$$

where, as in equation (6), s is connected to the speed u of the transformation by

$$s = \frac{1}{2} \operatorname{artanh}(u) \quad . \tag{13}$$

There are a few points to be emphasized here:

1. The boost transformation (12) becomes one-sided, as appropriate for a particle on the light cone. A zero-mass quantum has associated with it a direction. There is no transformation that would bring it to rest.
2. Negative- and positive-frequency components have a finite-mass, positive and negative, respectively, which are of purely kinematic origin. Since they have equal amplitude, the expectation value of the translation operator (2) is the zero 4-vector, a conserved quantity under Lorentz-transformations.
3. In contrast to finite-mass fields, which define a rest system, there is no obvious distinguished point on the s-axis. However, if a quantum is generated one would like to have a state normalized to a value of one in the inertial frame of the generating process, in which k^0 determines the energy of the quantum.
4. When going to a system in which we have $k' = ke^s$, the amplitude of the quantum is multiplied by a factor $e^s = k^{0\prime}/k^0$. Since e^{-2s} is precisely the Lorentz "contraction" of a wave packet moving at the speed of light, integration of the expectation value of the energy-momentum operator (3) over the packet yields its (light-type) energy-momentum 4-vector.
5. It must be remembered here that zero-mass wave-"functions" cannot be represented as simple functions because functions have always a positive density. Rather, they have to be understood as operators in Fock space, such that negative number-densities become possible when field-quantization is performed with zero vacuum-energy [2]. The same holds true for a massive particle for which, however, a representation as a single wave function is possible in its rest-system, at least in the limit of a very extended wave packet. This common representation requires quantization with a vacuum of infinite energy density, with the corresponding renormalization problems, and it blurs the close connection with their antiparticles, as different excitations of the same field.

A wave packet traveling in vacuum thus corresponds to simultaneous field excitations in opposite (conjugate) modes. The excitation in the mode pointing in the direction of the quantum's movement correspond to a positive number density (negative frequency). The oppositely directed mode has an excitation of negative number density (positive frequency). The two packets (which actually transport the energy) stay together because they move at group velocity

$$\vec{v}_g = \frac{\partial \omega(\vec{k})}{\partial \vec{k}} \quad , \tag{14}$$

which is parallel to $\vec{k}$ for the positive-mass cone (hyperboloid) and anti-parallel to $\vec{k}$ for the negative-mass one.

For the sake of simplicity we will refrain from treating the general case in which $\vec{u}$ and $\vec{k}$ are not collinear.

Non-Zero Spin

The internal degrees of freedom of a field transform according to (*j*,*k*)-representations of the homogeneous Lorentz group, where *j* and *k* characterize right- and left-twisted representations [3,4]. When performing the above $m \rightarrow 0$ limit, a convenient basis are the eigenstates of the spin-component along the boost direction, S_b, which transform according to [4]

$$e^{i\frac{w}{2}S_b^j} e^{-i\frac{w}{2}S_b^k} = e^{i\frac{w}{2}(m_j - m_k)} \quad , \tag{15}$$

since infinitesimal boost operators do not affect the spin variables. When *w* approaches infinity only states with m_j-m_k = 0 can approach a well defined limit. This excludes in particular the (½, 0) representation and its conjugate (0, ½), and shows that neutrinos must have a finite mass, a group-theoretical confirmation of a finding derived from the measured defect of solar electron-neutrinos via the mechanism of neutrino rotation [5].

Stationary Fields

A classical static field can be (3-dimensionally) Fourier transformed, i.e. be represented by the eigenfunctions of the translation operator. For a dynamical description we have to add frequency components. We can build up a zero-mass field by taking for a given 3-*d* mode negative- and positive-frequency components with equal absolute amplitude:

$$W(k^0, \vec{k}), W(-k^0, \vec{k}) \quad . \tag{16}$$

However, wave-packets built around these two modes travel in opposite directions. Consequently, we must assume a stationary situation in which a continuous process of refurnishing of the two modes takes place. Such a field must be generated by sources which provide the furnishing. Furthermore, by the same token, both modes (16) correspond to positive mass current in $\vec{k}$ -direction with norm k^0. Thus, the classical static field corresponds to a stationary mass-flow from sources of positive mass to sources of negative mass. (In idealized (standard) situations either one may be moved to infinity.) On the other hand, considering the energy-momentum operator we see that the stationary field (16) has a non-zero energy density but zero flow (momentum).

When proceeding to an inertial system which moves at speed $\vec{u}$ towards the resting source of a stationary field we obtain from expression (16) the pair:

$$\begin{aligned} &\cosh(s)\, W(k^0 e^s, \vec{k}\, e^s) + \sinh(s)\, W(-k^0 e^s, -\vec{k}\, e^s) \quad , \\ &\cosh(-s)\, W(-k^0 e^{-s}, \vec{k}\, e^{-s}) + \sinh(-s)\, W(k^0 e^{-s}, -\vec{k}\, e^{-s}) \quad , \end{aligned} \tag{17}$$

where we reused the fact that under a boost a field develops a conjugate component, as shown in [1]. For quanta, as seen above (equations 10,12), the two components are always present with equal weight, whence the hyperbolic functions add up to the exponential.

From expression (17) we see that the mass-current vector develops a zero-component ($k^0 sinh(s)$), while the norm of the mass-flow 4-vector keeps its constant value k^0.

Stationary fields are absent in traditional quantum field theory as separate entities. They are commonly treated as classical “static” field, and whenever they play a role, they are lumped together with massive fields. An example is the hydrogen atom, which is treated as an electron moving in the proton's classical electric field. A full treatment would consider the electromagnetic field as a separate entity with a stationary component providing the mass-flow from the proton as the source to the electron as the sink. Similarly, the treatment of Bremsstrahlung lumps the scattering static field together with it's nucleus. In Quantum Electrodynamics the electromagnetic field occurs as dynamic entity exclusively in the form of quanta. Since their mass-current is identically zero, mass conservation is not an issue at vertices, and photon lines need not be directed.

The situation is completely opposite for the gravitational field. Because gravitation attracts sources of equal sign (a property of scalar fields, see Reference [6]) there are huge sources (stars) and the stationary fields are all dominant. To the best of our knowledge a corresponding quantum-field theoretical treatment has not been worked out, even in the case of electrodynamics. The corresponding field propagators must account for the mass flow, i.e. the field lines need a direction (double arrows, which compensate to zero in the case of quanta). However, to work this out in detail is a separate task, which would exceed the scope of this short note.

Gravitation

We have proposed that the gravitational field is a scalar [2]. The interpretation of the Klein-Gordon operator as a product of a mass spending and a mass receiving current immediately suggests how interaction with any other field χ takes place, namely by a mutual mass exchange through a bilinear coupling of the two fields

$$L_I \sim {}_{\chi}J^{\mu}{}_{\psi}J_{\mu} + {}_{\psi}J^{\mu}{}_{\chi}J_{\mu} = \hat{\chi}\,\partial^{\mu}\chi\,\hat{\psi}\,\partial_{\mu}\psi + \hat{\psi}\,\partial^{\mu}\psi\,\hat{\chi}\,\partial_{\mu}\chi \quad , \tag{18}$$

which compensates to zero in an equilibrium situation. As an aside, the propagation part of any free field can be interpreted as a self-interaction via such a term, because the combination $\chi\,\hat{\psi}$ goes over into the unit operator in that case.

The interaction of (massive) particles, χ, with the field is in principle of the Bremsstrahlung type. As explained above the conventional treatment of this process, via an instantaneously acting static field, is perfectly acceptable in the electromagnetic case which is effectively short-range by saturation. For the non-saturating gravitational interaction, dealing with with up to astronomical distances, such a treatment is certainly inadequate. The source must not be included in the process. How a correspondingly modified graph of the Bremsstrahlung type might look like is shown in Figure 1.

For a (isolated) bounded body at rest (e.g. the Sun), mass-current coupling (18) ensures that the scattering processes of the body's particles with the field lead to an average stationary field associated with the body, reflecting it's shape and possibly symmetry. If the mutual mass-exchange through the interaction (18) is not in equilibrium, the body is accelerated by processes of the type illustrated in Figure 1, which imply the emission of field quanta. (A single-vertex graph involving just the stationary-field lines is not allowed because it violates energy-momentum conservation.) Acceleration and emission of field quanta must always go hand in hand. For example, when the body finds itself in an external stationary field a net flow (18) results not only from that field, but also because the body is driven out of equilibrium with its own stationary field. It is this part of the process, that is not accounted for in the setup of General Relativity via the Strong Equivalence Principle [7].

In spite of not having a fully worked out theory, we can still draw an important conclusion. Since a static field is actually a stationary mass flow, it becomes clear, that there must be a retardation effect for any, in particular gravitational, interaction, due to the limited-speed nature of Special Relativity. A calculation along these lines has already been performed at the end of the 19th century for the case of Mercury in the field of the Sun [8]. Starting from the perihelion shift of Mercury it arrived at the correct speed of propagation for the gravitational field. Although the form of retardation may have appeared somewhat arbitrary, the basic idea seems to have been floating around at the time. For a historical account see Reference [9]. Needless to say, that further dynamical aspects of the gravitational field are absent in this calculation as well.

Conclusion

In summarizing, we make the following points:

1. Performing the limit of mass-less fields yields quanta built out of a combination of positive- and negative-frequency components, which show a one-sided transformation behaviour under boosts.
2. Not all spin-components of a field survive this limit process. This affects in particular the states of neutrinos and implies that they must have a finite mass.
3. It is argued, that classical static fields are not static but consist of a stationary mass current that flows between sources of either sign (expressions 16,17). This view is fundamentally different from General Relativity's understanding!
4. The stationary nature of the gravitational field implies that the planets experience a retarded potential of the sun. This is precisely the view presented in Reference [8], where a correct value for the perihelion shift of Mercury was calculated.

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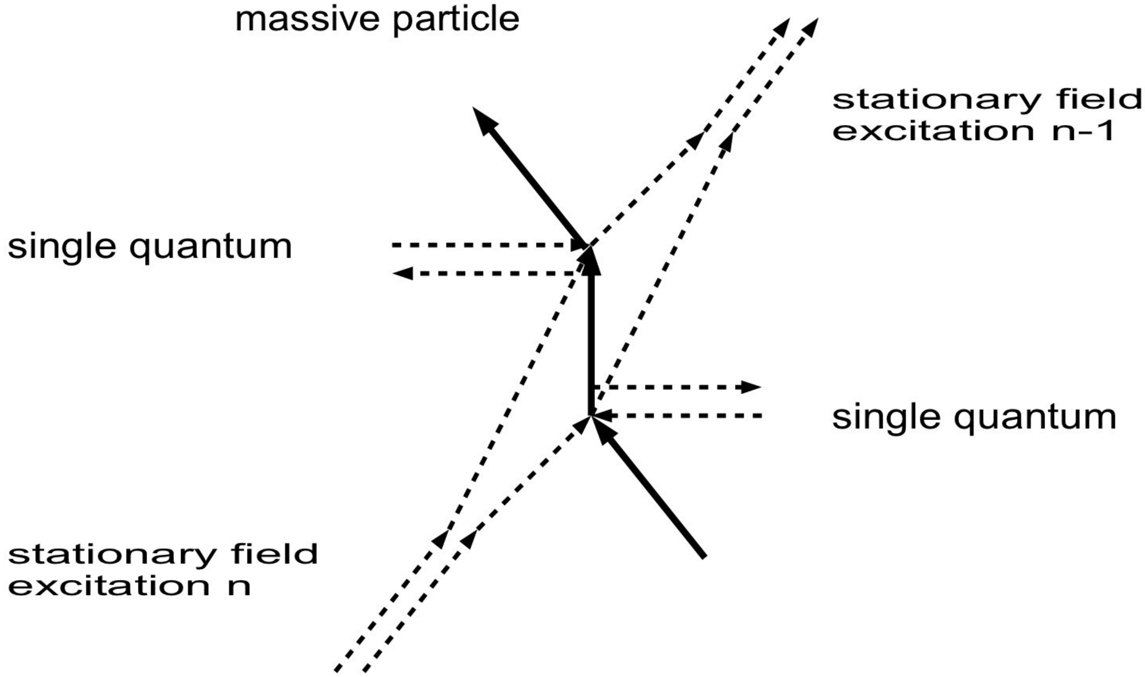

Figure 1. Graph of Bremsstrahlung type in a static field. The massive particle (solid arrows) interacts with the mass-less field (dashed arrows), which must occur as double-arrows in external lines. For quanta the two arrows are opposite (equation 10), for the stationary (static) field they are parallel (equation 16). In the absence of the stationary field (Compton-type scattering), each vertex process involves two step operators (mass current, equation 18) of the mass-less field which (de-)excite a quantum, a pair of modes of opposite wave vectors. In the presence of the stationary field one of these step operators modifies one of the stationary field modes.