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Black Holes without Singularity?

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Abstract

We propose a model scheme of microscopic black holes. We assume that at the center of the hole there is a spin $\frac{1}{2}$ core field. The core is proposed to replace the singularity of the hole. Possible frameworks for non-singular models are discussed briefly.

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1 Introduction and Summary

The motivation behind the model described here is to find a way to go beyond the Standard Model (BSM), including gravity. Gravity would mean energies of the Planck scale, which is far beyond any accelerator experiment. This work is hoped to be a small step forward in exploring the role on gravity in particle physics while any complete theory of quantum gravity is still in an early developmental phase, and certainly beyond the scope of this note.

In particular we pay attention to the nature of microscopic quantum black holes at zero temperature. We make a gedanken experiment of what might happen when exploring a microscopic black hole deep inside with a probe. In [1] we made two assumptions

(i) when probed with a very high energy $E \gg E_{\text{Planck}}$ point particle a microscopic black hole is seen as a fermion core field in Kerr, and ultimately Minkowski, metric. The point-like core particle of the hole may have a high mass, something like the Planck mass. However, in the Minkowski metric limit the mass should approach zero. The core field is called here gravon. It should have a position in the standard model together with other fermions.

(ii) the core may be the stable (or decaying) remnant of the hole, and the black hole singularity is replaced by the core field.

The core is introduced to illustrate the case of singularity free black hole, but it may also be a possible remnant with its own interactions. It may be a candidate for dark matter. In this note we try to discuss theoretical models available in the literature to get support, or deprecation, for the above assumptions.

With the Planck scale having its the conventional value 10^{19} GeV finding a gravon is hard. Gamma-ray signals from the sky may be a promising way. A gamma-ray, or particle, with energy half the Planck mass would be a clear signal of the models of this type.

There are models which may bring the relevant energy scale down within reach of the LHC. Provided the Planck scale is down at TeV scale black holes with mass in the TeV region may be formed. The basic idea discussed here does not depend on the value of the Planck scale.

In sections below we discuss briefly some possible scenarios of microscopic black holes, renormalization group improved gravity, bouncing universes, their roles in cosmology and testing the model. The nature of the present note is a concise preliminary (potpourri) survey of literature with a limited scope.

2 Black Hole Finals

2.1 Discrete Emission Lines

Bekentein's formula [2] for black hole horizon area quantization reads [3] in gravitational units G = c = 1 $A_n = \gamma\hbar n$ where the fudge factor γ is $\gamma = 4\ln(k)$, k is an integer, yielding

$$A_n = 4n\hbar\ln 3, n = 1, 2, 3... \tag{1}$$

A Schwarzschild black hole has a discrete energy (mass) spectrum of the form

$$M_n = \sqrt{\frac{\hbar\ln 3}{4\pi}n} \tag{2}$$

The quantized black hole emits gravitational radiation with frequencies which are integer multiplet of the fundamental black hole frequency

$$\omega_0 = (M_{n+1} - M_n)/\hbar = \frac{\ln 3}{8\pi M} \tag{3}$$

In a quantum theory of gravity, the black-hole spacetime is expected to possess a set of zero-point quantum gravity fluctuations. It has been suggested in [4] and [5] that these zero-point fluctuations of the black hole spacetime and, in particular, the quantum gravity fluctuations of the black hole horizon may enable quanta to tunnel out of the black hole. These black hole spacetime fluctuations are characterized by the fundamental resonance frequency ω_0 of the black hole spacetime, and multiplets of it $2\omega_0, 3\omega_0, ...$) to tunnel out of the quantum black hole. Therefore a quantum black hole is expected to show discrete line emission best measureable when the black hole mass approaches the Planck mass.

2.2 Black Hole Remnants

Giddings has discussed black hole remnants in detail [6]. We cite his work below briefly although we think it likely that information is lost in gradually by entanglement during Hawking radiation, see eg. [7]

In [6] it is assumed there is an upper bound on the information content within a given volume, determined by the Planck length. The entropy in a black hole formed from an initial mass M requires a volume V_M that grows with M. A natural guess is that the entropy density is bounded by the Planck density, which gives the estimate $V_M \sim l^3_{\mathrm{Planck}}(M/M_{\mathrm{Planck}})^2$. A black hole of mass M should have a finite-sized core in which information is distributed (on the surface) instead of being concentrated at the singularity. One might expect that this has volume of order V_M, and a naive estimate of the radius of the core region is therefore

$$r_M \sim l_{\mathrm{Planck}}(M/M_{\mathrm{Planck}})^{2/3} \tag{4}$$

There are two different logical possibilities for how a core with size of order V_M can be accommodated within the black hole. The first is that the core extends out to some radius that grows with the mass, for example as given by the entropy estimate above.

A second possibility is suggested by the work of refs. [8] and [9]: in curved space an arbitrarily large volume can be hidden within a fixed radius. Thus the volume of the core may grow as its radius stays fixed; the core can be thought of as a large internal geometry attached to the outside geometry through a fixed-size neck.

Now consider what happens to the core as the black hole evaporates. With the arguments of [10] and [11] one assumes that the information does not escape in the Hawking radiation or through topology change. If one supposes the curvature bound for the core radius, then the core radius shrinks with the mass of the evaporating black hole. But since the core itself must act as a repository of information one expects it not to shrink. Indeed, in the model of [10] it grows, albeit in a highly cutoff-dependent way. If the radius of the horizon has shrunk down to the Planck size, the horizon and neck meet. Now there are two possibilities. One is that the neck can pinch off; the core then becomes a child universe. Another possibility is that the core stays connected to our Universe through the neck. In that case, one views it as a Planck-sized remnant from the outside. The large interior of the core contains the missing information but is not visible without passing through the Planck-sized neck.

If such remnants in fact exist, then it should in principle be possible to find them through direct observation. One looks for massive high density objects that nonetheless do not have horizons and therefore may emit or scatter radiation from their surfaces. Furthermore, such objects could clearly have astrophysical consequences, although determining these consequences would depend both on the initial mass distribution of black holes from which they formed as well as on their dynamics (e.g. stability). The latter in particular is uncertain due to the lack of detailed knowledge about the short distance physics responsible for their existence. Possible implications for, e.g., dark matter remain to be investigated.

One can conclude that black holes have finite-sized cores and that these cores could become massive remnants after Hawking emission. Such remnants do not have the black hole information problem.

3 Einstein-Dirac Cosmology

The work of Finster and Hainzl [12] gives indication of singularity avoidance in Friedmann-Robertson-Walker (FRW) cosmology. The authors study Einstein-Dirac (ED) equations

$$R^i_j - \frac{1}{2}R\delta^i_j = 8\pi\kappa T^i_j \tag{5}$$

$$(\mathcal{D} - m)\Psi = 0 \tag{6}$$

where T^i_j is the energy-momentum tensor of the Dirac particles, κ is the gravitational constant, $\mathcal{D}$ is the Dirac operator and Ψ the wave function. For metric the closed Friedmann-Robertson-Walker is chosen

$$ds^2 = dt^2 - R^2(t)d\sigma^2 \tag{7}$$

where R is the scale function and $d\sigma^2$ is the line element on the unit 3-sphere

$$d\sigma^2 = \frac{dr^2}{1-r^2} + r^2 d\theta^2 + r^2 \sin^2\theta \mathrm{d}\phi^2 \tag{8}$$

where r, θ and ϕ are the standard polar coordinates. The Dirac operator in this metric is written as

$$\mathcal{D} = i\gamma^0 \left(\partial_t + \frac{3\dot{R}(t)}{2R(t)}\right) + \frac{1}{R(t)} \begin{pmatrix} 0 & \mathcal{D}_{S^3} \\ -\mathcal{D}_{S^3} & 0 \end{pmatrix}, \tag{9}$$

where γ^0 is the standard Dirac matrix, and D_{S^3} is the Dirac operator on the unit 3-sphere. The operator D_{S^3} has discrete eigenvalues $\lambda = \pm\frac{3}{2}, \pm\frac{5}{2}, \ldots$, corresponding to quantization of momenta of the particles. The Dirac equation is separate with the ansatz

$$\Psi_\lambda = R(t)^{-\frac{3}{2}} \left[\frac{8\pi\kappa}{3}\left(\lambda^2 - \frac{1}{4}\right)\right]^{-\frac{1}{2}} \begin{pmatrix} \alpha(t)\,\psi_\lambda(r,\vartheta,\varphi) \\ \beta(t)\,\psi_\lambda(r,\vartheta,\varphi) \end{pmatrix}, \tag{10}$$

where α and β are complex functions. For a homogenous system the components of the energy-momentum tensor simplify and the time component is

$$8\pi\kappa T^t_t = \left[m\Big(|\alpha|^2 - |\beta|^2\Big) - \frac{2\lambda}{R}\,\mathrm{Re}(\alpha\overline{\beta})\right]. \tag{11}$$

Substituting ψ and T^j_i into the Einstein-Dirac equation one gets

$$i\frac{d}{dt}\begin{pmatrix}\alpha \\ \beta\end{pmatrix} = \begin{pmatrix} m & -\lambda/R \\ -\lambda/R & -m \end{pmatrix}\begin{pmatrix}\alpha \\ \beta\end{pmatrix} \tag{12}$$

$$\dot{R}^2 + 1 = \frac{m}{R}\left(|\alpha|^2 - |\beta|^2\right) - \frac{\lambda}{R^2}\left(\overline{\beta}\alpha + \overline{\alpha}\beta\right). \tag{13}$$

With the ansatz all single particle wave functions have the same time dependence thus they form a coherent macroscopic quantum state. The fermionic many-particle state is a spin condensate.

The ED equations further reduce to ordinary differential equations involving the scale function $R(t)$ and the complex functions $\alpha(t)$ and $\beta(t)$. In the limits $\lambda = 0$ and m $= 0$ the equations reduce to the Friedmann equations for dust and radiation universes, respectively. For large R the universe behaves classically as in the dust case. But near the singularities big bang and big crunch quantum effects change the situation. Under certain conditions $\dot{R}$ can become zero and change sign even for small values of R. Now the formation of a big bang or big crunch is prevented. This effect is called the bouncing scale function.

4 Asymptotically Free Quantum Gravity

A modern view is that general relativity forms a quantum effective field theory at low energies. This view is applied in this and the next section, also to high energies.

Building on higher derivative terms in the Einstein-Hilbert action, super-renormalizable and asymptotically free (AF) theories of gravity have been discussed in the literature [13] (see also [14]). Asymptotic freedom removes the singularity. Secondly, asymptotic freedom due to higher derivative form factor causes an effective negative pressure. Repulsive gravity at high density produces a bounce of a black hole. Black holes in fact never form. A distant observer sees a long lifetime for the trapped surface and interprets it as a black hole. The bounce is not given by Heisenberg uncertainty but follows from the dynamics of the system.

In [13] the following non-polynomial extension of the quadratic gravitational action of [15] has been considered

$$\mathcal{S} = \int d^4x \frac{2\sqrt{|g|}}{\kappa^2}\left[R - G_{\mu\nu}\frac{V(-\Box/\Lambda^2)^{-1} - 1}{\Box}R^{\mu\nu}\right], \tag{14}$$

where $\kappa^2 = 32\pi G_N$ and Λ is the Lorentz invariant energy scale. Its value is of the order of Planck mass. The form factor, an entire function V contains the non-polynomial property of the theory. V cannot have poles in the complex plane to ensure unitarity and it must have at least logarithmic behavior in the UV to give super-renormalizability at the quantum level. The theory reduces to general relativity in the low energy limit since all the corrections to the Einstein-Hilbert action are suppressed by the factor Λ^{-1}.

The form factor is related to the propagator and to the effective potential of the theory. An example of a form factor is

$$V(z)^{-1} = \exp(\mathrm{z}^{\mathrm{n}}) \tag{15}$$

where $z = -\Box/\Lambda^2$ and n is a positive integer. String theory suggests n = 1. These theories have only the graviton pole. There are no ghosts or tachyons. The UV is dominated by the bare action, counterterms are negligible. Further details of these theories are discussed in [13].

It is known that if one adds all quadratic curvature invariants to the Einstein-Hilbert action the resulting theory is renormalizable at the price of ghost modes

[15]. In string theory the Einstein-Hilbert action is the first term of an infinite series containing powers of the curvature tensor and its derivatives.

According to Narain and Anishetty [16] the behavior of running coupling constant in the coupled system of higher derivative gravity and gauge fields is renormalizable to all order loops. The leading contribution to the gauge coupling beta function comes entirely from quantum gravity effects and it vanishes to all order loops.

In [16] the authors study fourth order higher derivative gravity which is claimed to be renormalizable to all loops [15] and is unitary [17]. The motivation for their study came from the realization that at one loop four kinds of divergences appear $\sqrt{-g}, \sqrt{-g}R, \sqrt{-g}R_{\mu\nu}R^{\mu\nu} and \sqrt{-g}R^2$. They consider the following higher derivative gravity action in dimensions $2 \leq d \leq 4$

$$S = \int \frac{d^4x\sqrt{-g}}{16\pi G}\left[-R - \frac{1}{M^2}\left(R_{\mu\nu}R^{\mu\nu} - \frac{d}{4(d-1)}R^2\right) + \frac{(d-2)\omega}{4(d-1)M^2}R^2\right] \tag{16}$$

where M has dimension of mass and ω is dimensionless. There are negative norm states, the propagator of the spin 2 massive mode appears with wrong sign violating unitarity at tree level. It was found though that in a certain domain of coupling parameter space, large enough to include known physics, the one loop running of gravitational parameters makes the mass of spin 2 massive mode behave in such a way that it is always above the energy scale being studied.

For our scheme asymptotically free quantum gravity is extremely interesting. There may not be at the moment general consensus of it.

5 Asymptotic Safety

5.1 Functional Renormalization Group Method

Asymptotic safety was proposed by Weinberg [18] in 1976 as a condition of renormalizability. It is based on a nontrivial, or non-Gaussian, fixed point (NGFP) of the underlying renormalization group (RG) flow for gravity. It is nonperturbative in character and it guaranties finite results for measureable quantities. The method for investigation of this scenario is functional renormalization group equation (FRGE) for gravity. The FRGE defines a Wilsonian RG flow on a theory space which consists of all diffeomorphism invariant functionals of the metric $g_{\mu\nu}$ of the type occuring in the action of general relatvity. From this construction emerges a theory called Quantum Einstein Gravity (QEG). QEG is not a quantization of classical general relativity, but it is consistent and predictive theory within the framework of quantum field theory. A modern view is that general relativity forms a quantum effective field theory at low energies.

The method of Reuter [19] uses the effective average action Γ_k, which is background independent. The RG scale dependence is governed by the FRGE of Wetterich

[20]

$$k\partial_k\Gamma_k[\Phi,\bar{\Phi}] = \frac{1}{2}\mathrm{Str}\left[\left(\frac{\delta^2\Gamma_k}{\delta\Phi^A\delta\Phi^B} + R_k\right)^{-1} k\partial_k R_k\right] . \tag{17}$$

where Φ^A is the collection of all dynamical fields and $\bar{\Phi}^A$ denotes their background counterparts. R_k is an infrared cutoff which vanishes for $p^2 \gg k^2$ and provides a k-dependent mass term for fluctuations with momenta $p^2 \ll k^2$. Solutions of the FRGE give families of effective field theories $\Gamma_k[g_{\mu\nu}], 0 \leq k < \infty$, labeled by the coarse graining scale k. The solution Γ_k interpolates between the microscopic action at $k \to \infty$ and the effective action $\Gamma_{k\to 0}$.

Suppose there is a set of basic functionals $P_\alpha[\cdot]$. Any functional can be written as a linear combination of the P_α's. The the solutions Γ_k of the FRGE have expansions of the form

$$A[\Phi,\bar{\Phi}] = \sum_{\alpha=1}^{\infty} \bar{u}_\alpha\, P_\alpha[\Phi,\bar{\Phi}]\,. \tag{18}$$

The basis $P_\alpha[\cdot]$ may include local field monomials and non-local invariants. We use the generalized couplings $\bar{u}_\alpha$ as local coordinates. Or better, we use a subset of couplings, so called essential couplings which cannot be absorbed by a field reparametrization. Though the method is non-perturbative truncations have to be made to the expansions of solutions.

Expandin Γ_k as above and inserting into FRGE we obtain a system of infinitely many coupled differential equations for the $\bar{u}_\alpha$'s

$$k\partial_k\, \bar{u}_\alpha(k) = \overline{\beta}_\alpha(\bar{u}_1, \bar{u}_2, \cdots ; k)\ , \quad \alpha = 1, 2, \cdots\ . \tag{19}$$

which can be solved using analytical or numerical methods.

A simple ansatz for action is the Einstein-Hilbert action where Newton's constant G_k and the cosmological constant Λ_k depend on the RG scale k. Let $g_\mu\nu$ and $\bar{g}_{\mu\nu}$ denote the dynamical and background metric, respectively. The effective action then satisfies in arbitrary spacetime dimension d

$$\Gamma_k[g,\bar{g},\xi,\bar{\xi}] = \frac{1}{16\pi G_k}\int d^dx\sqrt{g}\Big(-R(g)+2\Lambda_k\Big) + \Gamma_k^{gf}[g,\bar{g}] + \Gamma_k^{gh}[g,\bar{g},\xi,\bar{\xi}] \tag{20}$$

where $R(g)$ is the scalar curvature from metric $g_{\mu\nu}$, Γ_k^{gf} denotes the gauge fixing action and Γ_k^{gh} the ghost action with the ghost fields ξ and $\bar{\xi}$.

The corresponding β-functions describing the evolution of the dimensionless Newton constant $g_k = k^{d-2}G_k$ and dimensionless cosmological constant $\lambda_k = k^{-2}\Lambda_k$, were derived the first time by Reuter in [19] for any value of the spacetime dimensionality. The most important result is the existence of a non-Gaussian fixed point suitable for asymptotic safety (AS). It is UV-attractive both in g- and λ-directions (roughly $\lambda \approx .35$ and $g \approx .4$).

In the study of [21] it was shown that for $r \to 0$ the RG improved black hole metric approaches that of de Sitter space. This means that the quantum corrected spacetime is completely regular, free from any curvature singularity unlike the Schwarzschild black hole. The improved regularity comes because the 1/r-behavior of $f_{class} = 1 - 2G_0 M/r$ is tamed by very rapidly vanishing of the Newton constant at small distances.

A very heavy black hole obeys the classical relation $T_{BH} \sim 1/M$. The mass of the hole is reduced by the radiation the temperature increases. This tendency is opposed by quantum effects. Once the mass is as small as $M_{cr} \sim M_{\text{Planck}}$ the temperature reaches its maximum value $T_{BH}(M_{cr})$ [21]. For even smaller masses it drops very rapidly and vanishes at or below the M_{Planck}. The microscopic black hole could have a remnant which does not Hawking radiate any more.

Asymptotic safety is an important theoretical tool for quantum gravity. The methods used to derive the result are relevant to our scheme, but the analysis does not support asymptotic freedom. On the other hand, the FRGE analysis necessitates approximations, like series truncations, and contains a number of field theory subtleties.

5.2 Starobinsky Model

A very interesting model for gravitation was suggested by Starobinsky [22]. The model was originally studied to evaluate the one-loop corrections to the Einstein-Hilbert action resulting from vacuum quantum fluctuations in the matter sector at sufficiently high energies [22, 23], which can be taken into account effectively by adding an R^2-term to the gravitational action. It was also the first model for inflation in big bang, at least without assuming any new source for inflation but the gravitational sector. The model is even now consistent with the latest Planck satellite data [24]. The action of the model is obtained by adding a square of the Ricci scalar R term to the standard action

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{16\pi G} R + \frac{1}{b} R^2 \right) \tag{21}$$

with the dimensionless coupling b usually expressed as $b = 6M^2/M^2_{\text{Planck}}$, with M a constant of mass dimension one, where $M_{\text{Planck}} = G^{-1/2}$ is the Planck mass, G is Newton's constant which will become scale dependent and g is the determinant of the metric.

Inflation occurs in the model at sufficiently high curvature regime in the early universe the R^2 term dominating in the action causing an unstable inflationary period with exponential expansion. As the curvature decreases with time, the first term of the action eventually dominates, and inflation ends with a graceful exit.

In [25] the authors study the quantum fluctuations in gravity using the renormalization group approach. They take the Newton constant G and the R^2-term coupling

b as running parameters. They start with the generating functional

$$Z[J] = \int Dg_{\mu\nu}\, e^{i\left(S[g_{\mu\nu}] + \int J^{\mu\nu} g_{\mu\nu} + S_{GF} + S_{gh} + \Delta S_k\right)} \tag{22}$$

where $S[g_{\mu\nu}]$ corresponds to the bare gravitational action, $J^{\mu\nu}$ is an appropriate external source with S_{GF}, S_{gh} denoting appropriate gauge-fixing and ghost terms, respectively. The bare action is further modified by the presence of the scale-dependent infrared regulator ΔS_k. The latter is chosen such as to suppres momenta lower than the infrared cut-off k, and the integrating out of degrees of freedom proceeds in a Wilsonian fashion, i.e. shell by shell in momenta [27].

The coarse-grained effective action $\Gamma_k[g_{\mu\nu}]$ corresponding the above generating functional can be derived using a Legendre transformation

$$k\partial_k \Gamma_k[g_{\mu\nu}] = \frac{1}{2}\mathrm{Tr}\left[\left(\Gamma_k^{(2)} + R_k\right)^{-1} \partial_t R_k\right], \tag{23}$$

with $\Gamma_k^{(2)}$ denoting the inverse, full propagator, and $\partial_k = \partial/\partial k$, where the infrared (IR) cut-off scale k sets the coarse graining or RG scale. This looks like equation (17), apart from some notational differences, and is here called exact renormalization group equation (ERGE). In particular, momentum modes below k are suppressed, while those above k are not, and therefore integrated out. The regulator R_k ensures IR regularisation as well as finiteness of the trace under very generic requirements [27].

An effective action associated with a Wick-rotated Starobinsky action (21), the two beta functions are

$$k\frac{d}{dk}\widetilde{G}(k) = \beta_{\widetilde{G}}(\widetilde{G}, b), \quad k\frac{d}{dk}b(k) = \beta_b(\widetilde{G}, b), \tag{24}$$

where we have introduced the dimensionless Newton's coupling $\widetilde{G}(k) \equiv k^2 G(k)$. The beta functions are calculated partly numerically together with analytic approximations in [25].

If one would consider the action only the R^2 term the corresponding beta function in leading order in b is

$$k\frac{d}{dk}b = -\frac{1117}{8640\pi^2}b^2, \tag{25}$$

and is qualitatively similar to the quantum chromodynamics (QCD) case. The beta function (25) exhibits one fixed point, $b = 0$, with an associated eigenvalue equal to zero. The vanishing of the coupling b in the UV persists after the inclusion of the linear curvature term.

The fixed points of the system of beta functions (24) can be found by setting the corresponding right hand sides to zero. Using the cut-off scheme of ref. [26] we find that they exhibit three real-valued fixed points, a Gaussian Fixed Point (GFP) labeled (a), and two UV (or non-Gaussian) Fixed Points (UVFP) labeled (b) and (c), respectively

$$(a)\ \ (\widetilde{G}_{\rm fp}, b_{\rm fp}) = (0,0), \tag{26}$$

$$\begin{aligned} (b)\ \ (\widetilde{G}_{\rm fp}, b_{\rm fp})_1 &= (2.451, 914.57), & (27)\\ (c)\ \ (\widetilde{G}_{\rm fp}, b_{\rm fp})_2 &= (24\pi/17 \approx 4.44, 0). & (28)\end{aligned}$$

The fixed point (28) is rather special, and one of the most important results of that work: it describes an asymptotically safe (AS) Newton's coupling and a vanishing R^2 coupling in the UV. Apart from its interest from a pure RG perspective, this fixed point gives rise to RG trajectories along which Starobinsky inflation can be viably realized.

We summarize some the results of [25]. The second fixed point (28) is rather special, and one of the most important results of this work:

It describes an asymptotically safe Newton's coupling and a vanishing R^2 coupling in the UV. Apart from its interest from a pure RG perspective, this fixed point gives rise to RG trajectories along which Starobinsky inflation can be viably realized.

The existence of the fixed point (28) is crucial for the resulting inflationary behavior as it provides us with a mechanism for naturally producing small inflationary fluctuations at the perturbative level. This type of behavior is able to overcome some previously found problems in the context of AS inflation, i.e. combining a sufficient number of e-folds with the requirement of obtaining the correct amplitude for the metric fluctuations.

To conclude this subsection we would like to mention - to the joy of friends of quantum chromodynamics - that an approximate system of beta functions for $\widetilde{G}(k) = k^2 G(k)$ and $b(k)$ can be solved analytically to find

$$\begin{aligned} \widetilde{G}(k) &\simeq \frac{\widetilde{G}_0}{1+\frac{41\widetilde{G}_0}{72\pi}(\frac{k}{k_0})^2}\left(\frac{k}{k_0}\right)^2, \\ b(k) &\simeq \frac{b_0}{1+\frac{41\widetilde{G}_0}{72\pi}(\frac{k}{k_0})^2}, \end{aligned} \tag{29}$$

with $\widetilde{G}_0$ and b_0 being constants of integration, and k_0 a constant non-vanishing reference scale, which we will choose to be the Planck mass as it is measured today, i.e. $k_0 = M_{\rm Planck}$. This choice allows us to measure everything in units of the Planck mass. The values of all physical observables are then defined with respect to the chosen scale. It is easy to see that in the IR limit, i.e as $k \to 0$, $b \to b_0$, and $\widetilde{G} \to 0$, respectively. This behavior is in very good agreement with the full numerical solution of equations in the vicinity of the GFP.

6 Planck Stars

In [28] the quantum gravitational effects come from quantum cosmology. In loop cosmology the scale factor $a(t)$ of the universe is modified by quantum gravitational effects

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_{\text{Planck}}}\right) \tag{30}$$

where G is the Newton constant, ρ the energy density of matter and

$$\rho_{\text{Planck}} \sim M_{\text{Planck}}/l^3_{\text{Planck}} \sim c^5/(\hbar G^2) \tag{31}$$

where $\hbar$ is the reduced Planck constant. Quantum gravity corrections appear when $\rho \sim \rho_c$. This may happen before $l \sim l_{\text{Planck}}$. A collapsing universe may bounce back into expansion. This repulsion is due to Heisenberg uncertainty relations. In a matter dominated universe the volume of the universe at the bounce is

$$V \sim m/M_{\text{Planck}}\ l^3_{\text{Planck}} \tag{32}$$

where m is the total mass of the universe. This volume of the universe is estimated to be about 75 orders of magnitude larger than the Planck volume.

Quantum analysis of a collapsing star leads to similar results. The gravitational collapse of a star does not lead to singularity but to a new phase of the star where the large gravitational attraction is balanced by large quantum pressure. The authors call stars in this phase Planck stars, and they estimate that a stellar mass black hole could have a radius of the order of 10^{-10} cm. This is very small, of course, compared to the original star but still more than 20 orders of magnitude larger than the Planck length.

The lifetime of a Planck star is very long for a distant observer since it is determined to the Hawking evaporation time of the black hole. But if measured on the surface of the star it is very short, the time light takes to cross the star.

A primordial black hole with mass about 10^{12} kg has a lifetime of the order of the age of the universe $t_{\text{H}} \sim 14 \times 10^9$ years. So they would be at the end of their lifetime now and be detectable at present. The size of this kind of object is

$$r = \sqrt[3]{t_{\text{H}}/(348\ \pi t_{\text{Planck}})}\, l_{\text{Planck}} \sim 10^{-14}\ \text{cm} \tag{33}$$

The size of the black hole is the only scale in the process and it therefore fixes the energy scale of the emitted particles in the last stage. Assuming that all fundamental particles emitted with about the same energy taken at

$$E_{burst} = \frac{hc}{2r} \sim 3.9\ \text{GeV} \tag{34}$$

From detectional viewpoint it is natural to measure emitted gamma-rays. Only directly emitted gamma-rays (estimated to be about 3 per cent) are at the energy

E_{burst}. Most gamma-rays come from decays of hadrons, mainly from neutral pions. The authors [28] have made a Pythia [29] analysis of secondary gamma-rays emitted by a Planck star at the end of its life. The mean energy is of the order 0.03 $\times E_{burst}$, which is in the tens of MeV range. The multiplicity is quite high at about 10 photons per $q\bar{q}$. A major hindrance comes from the maximum distance at which bursts can be detected. For measuring say 10 photons using a 1 m^2 detector surface the estimated distance of burst origin is only about 200 light years.

7 Cosmology

In the schemes considered for quantum gravitation the initial singularity is smoothed into a very high but finite density and temperature objects. After inflationary phase the standard model particles are formed together with occasional black holes associated with the gravon. Their relative presence is a free parameter depending on the properties of Planck scale black holes but a first guess is equal amount of each standard model fermions.

This does not lead to major deviations from the standard cosmological model. The abundance of primordial black holes contributes to the distribution of dark matter of the universe. Quantitative differences to the standard model should be looked for from gamma-ray spectra. The formation of stellar size black holes proceeds as in general relativity theory.

8 Experimental Tests

As mentioned, high accuracy measurements of gamma-ray signals from the sky [30] is at present the most promising key to observe new physics. With the Planck scale at about 10^{19} GeV, all particles coupling to gravity, any particle with energy half the Planck energy is a clear signal. A remnant is expected to have two and many particle decay channels, of which the few particle channels have rather clear signals. Information and global charge conservation are imteresting questions.

There are models which may bring the relevant energy scale down within reach of the LHC. These include models of ref. [31]. Provided the Planck scale is brought down to TeV scale black holes with mass in the TeV region may be produced by gravitational interaction. The basic idea discussed here does not depend on the value of the Planck scale.

Cross sections and decay channels have been extensively calculated in [32]. Detailed analysis indicates best few body decay channels, in particular eμ pair, for black hole production.

9 Discussion and Conclusions

The present note contains some tentative thoughts, and references elsewhere, how to go a short step beyond the standard model towards a model of Planck scale phenomena, assuming the standard model is valid up to that scale. At the Planck scale black holes are the key objects of quantum gravity to study. Unfortunately existing model calculation results concerning Planck mass region black holes are still unreliable. Furthermore, the literature is scattered in time over about half a century to pre-arXivian times, independent researchers are locally over all continents, and the results cover several branches of special fields and journal publications. (A modest hope of this concise note is to collect a few of the interesting results together.) Nevertheless, it turns out that more is known of quantum gravity as an effective quantum field theory at low energies than any single piece of work let us expect. But a key idea is still missing.

The details of the theoretical models discussed in Sections 2-6 vary somewhat but the singularity softening trend seems to be on rather sound basis. Information loss can be avoided in one way or another. Dark matter has reasonable candidate models while dark energy is harder to explain.

Our assumption (i) of section 1 may or may not be true (see last paragraph below) though it is endorsed by the Section 4 models. Assumption (ii) is open with respect to the remnant but is likely true because of the above mentioned non-existence of singularity.

The next task is to find a specific action for the model scheme of this note as a field theory, or other type of theory, first for pure gravity later one and more standard model particles included. Pure gravity should be taken in this model as gravon and graviton terms in a quantum bound state that will correspond the Einstein equation (5). On the other hand, the bound state scheme for the black hole could also be checked against the phase transition picture like the the gravastar model [33], and the one developed by Laughlin et al [34].

The phases of a black hole late life start with thermal Hawking radiation, followed by emission of gravitational quanta as discussed in subsection 2.1. The remaining remnant of black hole, without a horizon, is coupled to all other particles of the standard model, or whatever a more complete model is. Its decay is interesting to try to observe. It should be searched as a long lived heavy decaying particle.

The scheme discussed here can be summarized as having the gravon and the graviton the fundamental elementary particles of quantum gravity, to be included to the standard model. The dressed gravon is a natural candidate for dark matter.

A realistic model of quantum gravity should start from the microscopic entities operating at the quantum scale, like the Planck scale. Then the methods of the new model theory, be it quantum field theory or something else, will be introduced to calculate the properties of the model like the UV behavior of the interaction.

The physical picture would be as follows. It is closer to quantum chromodynamics rather than quantum electrodynamics. But for color and electric charge the relevant objects, hadrons and atoms, respectively, are neutral, of course, as seen from a proper distance while mass cannot be similarly "hidden". Even for a single massless core in vacuum, very close to the core virtual particle pairs are created and destroyed, making a cloud of mass/energy around the core, all objects interacting gravitationally with the core and each other. At shortest scales mainly neighboring gravons and gravitons interact. When the scale is increased gradually larger blocks of gravons and virtual particles interact. With the renormalization group techniques, or whatever the proper method is, it is expected that an equivalent of curved spacetime is created. With high enough energy density a horizon is expected to surround the system. In the classical limit general relativity would be obtained. But all this happens within the gravitational field of everything else in the universe. And it has to be taken into account. It may mean that quantum cosmology needs be simultaneously developed.

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