A general Partition generating algorithm for a positive integer $k= k_1.k_2.\ldots k_n$

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Abstract: In this paper we present a potentially novel partition generating algorithm for a positive integer $k= k_1k_2\ldots k_{n-1}k_n$. In previous papers we used a similar strategy to derive two important known mathematical results regarding factorials and a novel strategy to partition odd composites(Refs 1-3). Here we will generalize this approach to make it widely applicable to all positive integers. We believe this strategy may be an important tool to mathematicians to attack unsolved conjectures as well to derive alternate possibly simpler proofs of established theorems.

Results:

Let k be a positive integer and $k_1, k_2, \ldots, k_{n-1}, k_n$ be a set of its "n" positive integral factors not necessarily distinct such that $k= k_1k_2\ldots k_{n-1}k_n$.

Consider a circle. We will cut this circle in a sequential step wise manner.

In the first step we will cut it at k_1 distinct positions along its circumference to obtain exactly k_1 arcs or pieces. At the end of this step we have used a total of k_1 cuts to the circle to obtain k_1 pieces or arcs.

In the second step we cut each of the k_1 pieces obtained from the first step at k_2-1 positions to obtain a total of k_1k_2 pieces or arcs. In the second step we have used $k_1(k_2-1)$ cuts. Until this step we have used a total of $k_1+ k_1(k_2-1)$ cuts to obtain k_1k_2 arcs or pieces.

In the third step we cut each of the k_1k_2 pieces obtained from the second step at k_3-1 positions to obtain a total of $k_1k_2k_3$ pieces. In the third step we have used $k_1k_2 (k_3-1)$ cuts. Until this step we have used a total of $k_1+ k_1(k_2-1)+ k_1k_2 (k_3-1)$ cuts to obtain $k_1k_2k_3$ pieces or arcs.

Proceeding in this manner in the (n-1)th step we cut each of the $k_1k_2\ldots\ldots k_{n-3}k_{n-2}$ pieces obtained from the (n-2)th step at $k_{n-1}-1$ positions to get total $k_1k_2\ldots\ldots k_{n-3}k_{n-2}k_{n-1}$ pieces. In the (n-1)th step we have used $k_1k_2\ldots\ldots k_{n-3}k_{n-2}(k_{n-1}-1)$ cuts. Until this step we used a total of $k_1+ k_1(k_2-1)+ k_1k_2 (k_3-1)+\ldots+ k_1k_2\ldots k_{n-3}k_{n-2}(k_{n-1}-1)$ cuts to obtain a total of $k_1k_2\ldots\ldots k_{n-3}k_{n-2}k_{n-1}$ pieces or arcs.

In the nth and last step, we cut each of the $k_1k_2\ldots\ldots k_{n-3}k_{n-2}k_{n-1}$ pieces obtained from the (n-1)th step at k_n-1 positions to obtain a total of $k_1k_2\ldots\ldots k_{n-3}k_{n-2}k_{n-1}k_n$ pieces. In the nth step we have used $k_1k_2\ldots\ldots k_{n-3}k_{n-2}k_{n-1}(k_n-1)$ cuts. Until this step we used a total of $k_1+ k_1(k_2-1)+ k_1k_2 (k_3-1)+\ldots+ k_1k_2\ldots k_{n-3}k_{n-2}(k_{n-1}-1)+ k_1k_2\ldots\ldots k_{n-3}k_{n-2}k_{n-1}(k_n-1)$ cuts to obtain a total of $k_1k_2\ldots\ldots k_{n-3}k_{n-2}k_{n-1}k_n$ pieces or arcs.

Since the total number of positions that a circle is cut will yield as many parts or arcs

Therefore

$k = k_1 k_2 \ldots k_{n-3} k_{n-2} k_{n-1} k_n =$

$k_1 + k_1(k_2-1) + k_1 k_2 (k_3-1) + \ldots + k_1 k_2 \ldots k_{n-3} k_{n-2}(k_{n-1}-1) + k_1 k_2 \ldots\ldots k_{n-3} k_{n-2} k_{n-1}(k_n-1)$

(a total of n-terms in the partition of k)

We would like to point out that algebraic expansion of the terms leads to cancellation of alternate terms leaving only the second last term of the expansion which is $k_1 k_2 \ldots\ldots k_{n-3} k_{n-2} k_{n-1} k_n$ and therefore we believe this algorithm will be applicable to positive non-integral numbers as well.

Although one of us (Prashanth) has imperfect memory that circle cutting analogy or similar may be already extant in literature, our several attempts to search online resources or requests to professional mathematicians to trace anything similar have been futile. Therefore we believe that this particular algorithm may be novel and original. We are however open to be pointed out otherwise.

References:

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