General Quantum Matrix Theory

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ABSTRACT

The objective is to formulate a foundation from which SM, GR and the dark sector are emergent. It is proposed that matrix generators with single entries ±1, ±i over a complex field are the building blocks of all physical states. The axiom that vacuum states have non-zero minima requires that all matrix elements are non-zero. In the paper [1], an extended curvature tensor was calculated and N=6d spaces were found. The 4d Weyl spinors are local gauge invariant under SU(4) and a 4d space metric. The transition of a space-like gamma matrix to time like gamma matrix results in the emergence of Space-Time. The dark sector consists of SU(4) gluons modifying the gravitational potential and the cosmological constant is the ground state of the asymmetric curvature scalar. SM fermion sector with 3 generations of quarks & leptons emerges from symmetry breaking of SU(4).

1 Introduction

The objective is to formulate a foundation from which the Standard Model (SM), General Relativity (GR) and the dark sector are emergent. Physical states such as spinors and metrics are emergent from fundamental entities – the G matrices:

Axiom 1

- The infinite set of matrices G_n called generators with single-entry ±1, ±i over the field $\mathbb{C}$

The generators G_n satisfy the anti-commutation relation:

$$\tfrac{1}{2}\{G_a^\dagger, G_b\} = \delta_{ab} M_{ab} \quad \textbf{1.1}$$

M_{ab} are single-entry matrices with entry +1. Thus the generators G_n form CAR algebra. The generators are also orthonormal. A state S is a linear combination of the generators G_n

$$\mathrm{S} = a_k \omega_{kn} G_n \quad \textbf{1.2}$$

where $a_k, \omega_{kn} \in \mathbb{C}$

Re-write **1.2** terms of a 1d complex density function π_{nk} so Ψ is a 1d complex density – a general quantum matrix (GQM)

$$\Psi = a_k \pi_{kn} G_n \quad \textbf{1.3}$$

Normalise the basis vectors $\pi_n G_n$

$$\langle \pi_n G_n | \pi_m G_m \rangle = \delta_{nm} \quad \textbf{1.4}$$

The transition of a matrix element from an initial state Ψ_{inm} to the final state Ψ_{fnm} has probability

$$p_{nm} = \langle \Psi_{fnm} | \Psi_{inm} \rangle \quad \textbf{1.5}$$

Normalise Ψ_{nm}

$$\langle \Psi_{nm} | \Psi_{nm} \rangle = 1 \quad \textbf{1.6}$$

A special case of **1.5** is when the transition probabilities $p_{nm} = p$ then a state Ψ_i has a probability *p* of transition to the state Ψ_f

$$p = \langle \Psi_f | \Psi_i \rangle \quad \textbf{1.7}$$

Expectation values of an operator $\hat{O}$ where $\hat{O}$ is a linear combination of the generators G_n is

$$\langle \hat{O} \rangle = \langle \Psi | \hat{O} | \Psi \rangle \qquad \textbf{1.8}$$

The expectation values of the gamma matrices and generators of SU(n) are the matrices

$$X^m = \langle \Psi | \gamma^m | \Psi \rangle \qquad \Omega^a = \langle \Psi | T^a | \Psi \rangle \qquad \textbf{1.9}$$

where $\gamma^m \in Cl_{p,q}$ and $T^a \in SU(n)$, are special quantum matrices (SQM)

The generalised co-ordinates of $Cl_{p,q}$ and $SU(n)$ are special cases of **1.9** with the reduction to. eigenequations

$$\langle \Psi | \gamma^m | \Psi \rangle = x^m \gamma^m \qquad \langle \Psi | T^a | \Psi \rangle = \omega^a T^a \qquad \textbf{1.10}$$

So Ψ has generalised co-ordinates x^m also ω^a has generalised co-ordinates x^m

The expectation value of the matrix operator $\hat{g}_{mn}$ a symmetric matrix with elements 1 is the metric:

$$\langle \hat{g}_{mn} \rangle = \langle \Psi | \hat{g}_{mn} | \Psi \rangle \qquad \textbf{1.11}$$

Axiom 2

For the matrix elements of a state Ψ to have a minimum the π_{mk} satisfy a quadratic potential

$$V(\pi_{mk}^{\dagger}\pi_{mk}) = -\mu^2(\pi_{mk}^{\dagger}\pi_{mk}) + \lambda(\pi_{mk}^{\dagger}\pi_{mk})^2 \qquad \textbf{1.12}$$

This implies that the ground states of the matrix elements of Ψ are dependent on the minima of **1.12.** Consequently the zero matrix elements of the gamma matrices γ^m and SU(n) generators T^a are replaced with minima $\epsilon_{mn} > 0$.

It follows that the contravariant metric states have a minimum and hence there inverses the covariant metric states have a maximum. Consequently singularities are avoided.

2 Space-Time and SM fermion sector

Calculation of the curvature tensor with the connection extended by inclusion of an asymmetric connection leads to 6d spaces for massless asymmetric metric fields, Appendix A & B. 6d Riemann spaces have 2 4d irreducible Weyl spinors - *f*-spinors with chirality ± 1 (f_1, f_{-1}). The 6d Lagrangian **A12** is local gauge invariant under a phase transformation with phase θ

$$\theta = \omega_a T_a + g \qquad \textbf{2.1}$$

where g is the Nd symmetric metric. Thus if the dimension of the complex space of SU(n) is n and the dimension of the metric space is N, then from **2.1** if follows that N=n. The gauge fields are SU(4) and massless spin 2 4d gravitation. The transition from a 4d Riemann space to a pseudo-Riemann space is a consequence of the following transition

$$p = \langle \gamma^0 | \gamma^i \rangle \qquad \textbf{2.2}$$

The Riemann space (p,0) makes a transition to the pseudo-Riemann space (p-1,1). Space-Time (1,3) emerges from (4,0) via the transition **2.2.**

In Appendix C, the set of 6d spaces leads to 3 generations of and particle-antiparticles.

A $\mathbb{C}^4$ scalar field Φ forms with probability

$$p = \langle \Phi | \Psi \rangle \qquad \textbf{2.3}$$

The symmetry breaking of SU(4) to SU(3) via the Higgs et al mechanism with $\mathbb{C}^4$ scalar Φ results in 6 quarks and 6 leptons. *f*-spinors form a chiral doublet of chirality -1 only by coupling to chiral -1 spinons ((1,3) Weyl spinors). These chiral doublets are gauged under the SU(2)xU(1) electroweak interaction.

3 Dark Sector

The generator of U(1) Electromagnetism is proportional to the diagonal generator T_{15} of SU(4) after symmetry breaking. SU(4) potential is of the form:

$$V(r) = -\frac{A}{r} + Br \quad \textbf{3.1}$$

Solving the Poisson equation for the Gravitational potential $\Phi(r)$ gives

$$\Phi(r) = -\frac{Gm}{r} + \frac{G}{c^2} Bln(r) - \frac{GA}{2c^2r^2} + C \quad \textbf{3.2}$$

Dark matter dominates the gravitational potential when

$$Bln(r) > \frac{mc^2}{r} \quad \textbf{3.3}$$

Write $B = \frac{\hbar c}{\lambda_0^2}$ the condition **3.3** is

$$rln(r) > \frac{mc^2\lambda_0^2}{\hbar c} = \frac{\lambda_0^2}{\lambda} = \hbar c\frac{E}{E_0^2} \quad \textbf{3.4}$$

The gravitational interaction between photons $\lambda = \lambda_0$ results in

$$rln(r) > \lambda_0 \quad \textbf{3.5}$$

Using the upper bound of $10^{-27}\mathrm{eV/c^2}$ for the rest-mass of a photon, the dark matter dominates when $r > 10^{18}\mathrm{m}{\sim}10^2\mathrm{Ly}$. The rest-mass of photon $10^{-29}\mathrm{eV/c^2}$ gives $r > 10^{20}\mathrm{m}{\sim}10^4\mathrm{Ly}$. For protons the condition is $r > 10^{58}\mathrm{m}$. It follows that the gravitational potential is modified on the galactic scale by photons with non zero rest mass.

From the appendix A the curvature scalar in (1,3) is $G = R + S$. Variation with respect to symmetric metric $g_{\mu\nu}$ gives

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \frac{1}{2}g_{\mu\nu}S \quad \textbf{3.6}$$

Identify S as dark energy field and hence the cosmological constant as

$$\Lambda = \frac{1}{2}S(\langle 0|0\rangle) \quad \textbf{3.7}$$

$S(\langle 0|0\rangle)$ is the ground state of the dark energy field. Inflation drives the dark energy field from near Planck density to its ground state.

Conclusion

Using the generators G_n defined in section 1, all physical states are linear combinations of these generators. The Lagrangians for curvature scalar and spinors are derived from the curvature of N dimension space. A solution to the major problem of time is the transition of a space-like gamma matrix to a time-like matrix. Space-Time emerges as a gauge field acting on 4d spinors on a set of 6d spaces. Fermion sector of SM with 3 generations follows from symmetry breaking of SU(4). Lagrangians of GR and the fermion sector of the SM follow from curvature of (1,3) Space-Time. Dark matter consists of SU(4) potential modifying gravity in the ultra-IR and the asymmetric metric states determine the dark energy of inflation and the cosmological constant. The scalar sector of the SM and inflation are to be considered in another paper.

Appendix A

With reference to [1] here are re-stated key results of extending the connection to include an asymmetric metric and a completely asymmetric connection.

Connection:

$$\mathrm{A}_{\sigma\mu\nu} = \Gamma_{\sigma\mu\nu} + i\hat{\Gamma}_{\sigma\mu\nu} \quad \textbf{A1}$$

$$\hat{\Gamma}_{\sigma\mu\nu} = \frac{1}{2}\left(\partial_\sigma \hat{g}_{\mu\nu} + \partial_\nu \hat{g}_{\sigma\mu} + \partial_\mu \hat{g}_{\nu\sigma}\right) \quad \textbf{A2}$$

$\hat{g}_{\mu\nu}$ is the antisymmetric metric.

Curvature tensor:

$$\left[D_\sigma, D_\rho\right]B_\nu = G^{\alpha}_{\nu\rho\sigma}B_\alpha + iH^{\alpha}_{\nu\rho\sigma}B_\alpha + i2\hat{\Gamma}^{\alpha}_{\rho\sigma}\partial_\alpha B_\nu \quad \textbf{A3}$$

$$G^{\alpha}_{\nu\rho\sigma} = R^{\alpha}_{\nu\rho\sigma} + S^{\alpha}_{\nu\rho\sigma} \quad \textbf{A4}$$

$$R^{\alpha}_{\nu\rho\sigma} = \partial_\sigma \Gamma^{\alpha}_{\nu\rho} - \partial_\rho \Gamma^{\alpha}_{\nu\sigma} + \Gamma^{\beta}_{\nu\rho}\Gamma^{\alpha}_{\beta\sigma} - \Gamma^{\beta}_{\nu\sigma}\Gamma^{\alpha}_{\beta\rho} \quad \textbf{A5}$$

$$S^{\alpha}_{\nu\rho\sigma} = -\hat{\Gamma}^{\beta}_{\nu\rho}\hat{\Gamma}^{\alpha}_{\beta\sigma} + \hat{\Gamma}^{\beta}_{\nu\sigma}\hat{\Gamma}^{\alpha}_{\beta\rho} + 2\hat{\Gamma}^{\beta}_{\rho\sigma}\hat{\Gamma}^{\alpha}_{\nu\beta} \quad \textbf{A6}$$

$$H^{\alpha}_{\nu\rho\sigma} = \partial_\sigma \hat{\Gamma}^{\alpha}_{\nu\rho} - \partial_\rho \hat{\Gamma}^{\alpha}_{\nu\sigma} + \Gamma^{\beta}_{\nu\rho}\hat{\Gamma}^{\alpha}_{\beta\sigma} + \Gamma^{\alpha}_{\beta\sigma}\hat{\Gamma}^{\beta}_{\nu\rho} - \Gamma^{\beta}_{\nu\sigma}\hat{\Gamma}^{\alpha}_{\beta\rho} - \Gamma^{\alpha}_{\beta\rho}\hat{\Gamma}^{\beta}_{\nu\sigma} + 2\Gamma^{\alpha}_{\nu\beta}\hat{\Gamma}^{\beta}_{\sigma\rho} \quad \textbf{A7}$$

Write the vector $B_\nu = \gamma_\nu \psi$ equate the imaginary terms to zero and multiply by $g_{\alpha\beta}$

$$\hat{\Gamma}_{\beta\rho\sigma}\left(\partial_\alpha \gamma_\nu \psi - \Gamma^{\mu}_{\mu\nu}\gamma_\alpha \psi\right) = 0 \quad \textbf{A8}$$

$\hat{\Gamma}_{\beta\rho\sigma} \neq 0$ and multiplying by $g^{\alpha\nu}$, **A8** simplifies to

$$\gamma^{\mu}\left(\partial_\mu - \Gamma^{\sigma}_{\sigma\mu}\right)\psi = 0 \quad \textbf{A9}$$

Dirac operator $\gamma^{\mu}\partial_\mu$ is recovered. The spinor ψ is coupled to curved spaces via the $\Gamma^{\sigma}_{\sigma\mu}$

Taking account of spin when the spinors ψ couple to spinons requires the spin connection and $\partial_\mu \rightarrow \nabla_\mu$

The real part of **A1** is

$$G^{\alpha}_{\nu\rho\sigma}\gamma_\alpha \psi \quad \textbf{A10}$$

multiply **A10** by $g_{\alpha\beta}$ and contracting to form the curvature scalar G, **A10** reduces to

$$G\gamma_\alpha \psi \quad \textbf{A11}$$

The Lagrangians then follow:

$$\mathcal{L} = \bar{\psi}\gamma^{\mu}\left(D_\mu - \Gamma^{\sigma}_{\sigma\mu}\right)\psi = 0 \quad \textbf{A12}$$

$$\mathcal{L} = \bar{\psi}G\psi \quad \textbf{A13}$$

D_μ is the SU(n) gauge covariant derivative

Appendix B

The asymmetric metric is massless then the traceless tensor $T_{\mu\nu}$ can be constructed [1]:

$$T_{\mu\nu} = S_{\mu\nu} - \frac{1}{N}S \quad \textbf{B1}$$

The divergence of the traceless tensor is:

$$\hat{\Gamma}^{\nu}_{f\beta}\left[\left(1 - \frac{3}{N}\right)D_\nu \hat{\Gamma}^{f}_{\sigma\beta} + \frac{3}{N}D_\beta \hat{\Gamma}^{f}_{\sigma\nu}\right] \quad \textbf{B2}$$

f=3 to N. The expression **B2** vanishes for $\left(1 - \frac{3}{N}\right) = \frac{3}{N}$. It follows that the dimension N=6. The metrics are physical states constructed from the generators G_n and the complex fields π_{nk} so the complete set of 6d spaces $\{(p,q): p + q = 6\}$ is necessary. The $(0,6)$ and $(6,0)$ are isomorphic to SU(4).

Appendix C

Apply Wick rotations which are elements of U(1), to (p,q) the result is (q,p) which suggests the *f*-spinors on (p,q) and (q,p) form charge ant-charge pairs of 2 generations. When applied to $(3,3)$ the result is $(3,3)$ and hence the *f*-spinors are charge chiral thus forming 1st generation fermions.

For each *f*-spinor, the electric charge quantum numbers Q_i are assumed proportional to the diagonal matrix T_{15} of $SU(4)$ after symmetry breaking,

$$Q_i = c_i diag(-1,-1,-1,3) \quad \textbf{C1}$$

Where i=1,2 denotes the 2 *f*-spinors and c_i are constants. The sum of the electric charges equal to zero is equivalent to the condition:

$$Tr(Q_1) + Tr(Q_2) = 0 \quad \textbf{C2}$$

The electric charges on the sub-group SU(3) are

$$Q_i = c_i diag(-1,-1,2) \quad \textbf{C3}$$

Since $Tr(Q_i) = 0$ set $c_i = 1$

Electric charge quantum number is in the range $Q \in [-1,1]$

Dividing these 2 matrices by 3 results in the 2 electric charges -1/3 and 2/3

$$\begin{aligned} Q_1 &= diag(-1/3,-1/3,-1/3,q_1) \\ Q_2 &= diag(2/3,2/3,2/3,q_2) \end{aligned} \quad \textbf{C4}$$

Using the conditions:

$$\begin{aligned} Tr(Q_1) + Tr(Q_2) &= 0 \\ Q_1 - Q_2 &= -1 \end{aligned} \quad \textbf{C5}$$

It follows that:

$$\begin{aligned} q_1 - q_2 &= -1 \\ q_1 + q_2 &= -1 \end{aligned} \quad \textbf{C6}$$

Hence $q_1 = -1$ and $q_2 = 0$

Thus the electric charge quantum numbers are $q \in [-1,-1/3,0,2/3]$

The 2 *f*-spinors have electric quantum numbers:

$$Q_1 = \left(-\frac{1}{3},-\frac{1}{3},-\frac{1}{3},-1\right), \qquad Q_2 = \left(\frac{2}{3},\frac{2}{3},\frac{2}{3},0\right) \quad \textbf{C7}$$

Thus Quarks are the triplet states of electric charge -1/3 and 2/3. Leptons are the -1 and 0 electric charge states.

References

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