Three conjectures on a sequence based on concatenation and the odd powers of the number 2

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Abstract. In this paper I make three conjectures regarding the infinity of prime terms respectively the infinity of a certain kind of semiprime terms of the sequence obtained concatenating the odd powers of the number 2 to the left respectively to the right with the digit 1.

The sequence of the numbers obtained concatenating the odd powers of the number 2 to the left respectively to the right with the digit 1 (see A004171 in OEIS for the odd powers of the number 2):

121, 181, 1321, 11281, 15121, 120481, 181921, 1327681, 11310721, 15242881, 120971521, 183886081, 1335544321, 11342177281, 15368709121, 121474836481, 185899345921, 1343597383681, 11374389534721, 15497558138881, 121990232555521, 187960930222081, 1351843720888321, 11407374883553281, 15629499534213121 (...)

Conjecture 1:

There exist an infinity of primes of the form 1n1 (where 1n1 is a number formed by concatenation, not 1*n*1), where n is an odd power of 2.

Such primes are:

181, 1321, 15121, 1335544321, 121474836481, 1351843720888321, 194447329657392904273921,
1405648192073033408478945025720321,
125961484292674138142652481646100481,
1425352958651173079329218259289710264321,
16805647338418769269267492148635364229121 (...)

Conjecture 2:

There exist an infinity of semiprimes q1*q2 of the form 1n1, where n is an odd power of 2, such that q2 – q1 + 1 is prime or square of prime.

: 11281 = 29*389 (389 – 29 + 1 = 361 = 19^2);

: 120481 = 211*571 (571 - 211 + 1 = 361 = 19^2);
: 1327681 = 467*2843 (2843 - 467 + 1 = 2377, prime);
: 11310721 = 2777*4073 (4073 - 2777 + 1 = 1297, prime);
: 185899345921 = 61*3047530261 (3047530261 - 61 + 1 = 3047530201, prime);
: 1272225893536750770770699685945414569164 81 = 535583191189*237540295227039642622315748029 (237540295227039642622315748029 - 535583191189 + 1 = 237540295227039642086732556841, prime).

Conjecture 3:

There exist an infinity of semiprimes q1*q2 of the form 1n1, where n is an odd power of 2, such that q2 - q1 + 1 = q3*q4, where q4 - q3 + 1 is prime, square of prime or semiprime with the property that, reiterating the operation described, it's finnaly reached a prime or a square of prime.

: 181921 = 109*1669 (1669 - 109 + 1 = 1561 = 7*223 and 223 - 7 + 1 = 217 = 7*31 and 31 - 7 + 1 = 25 = 5^2);
: 15242881 = 331*46051 (46051 - 331 + 1 = 45721 = 13*3517 and 3517 - 13 + 1 = 3505 = 5*701 and 701 - 5 + 1 = 697 = 17*41 and 41 - 17 + 1 = 25 = 5^2);
: 120971521 = 11*10997411 (10997411 - 11 + 1 = 10997401 = 137*80273 and 80273 - 137 + 1 = 80137 = 127*631 and 631 - 127 + 1 = 505 = 5*101 and 101 - 5 + 1 = 97, prime);
: 11407374883553281 = 61*187006145632021 (187006145632021 - 61 + 1 = 187006145631961 = 19813*9438557797 and 9438557797 - 19813 + 1 = 9438537985 = 5*1887707597 and 1887707597 - 5 + 1 = 1887707597, prime).