

Seven conjectures on the squares of primes involving the number 4320 respectively deconcatenation

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Abstract. In this paper I make three conjectures regarding a certain relation between the number 4320 and the squares of primes respectively four conjectures on squares of primes involving deconcatenation.

Conjecture 1:

There exist an infinity of primes of the form $p^2 + 4320$, where p is prime.

Such primes are:

: $4339 = 4320 + 19^2$;
: $4441 = 4320 + 11^2$;
: $5281 = 4320 + 31^2$;
: $5689 = 4320 + 37^2$;
: $6529 = 4320 + 47^2$;
: $7129 = 4320 + 53^2$;
: $9649 = 4320 + 73^2$;
: $12241 = 4320 + 89^2$;
: $13729 = 4320 + 97^2$;
: $14929 = 4320 + 103^2$;
: $21481 = 4320 + 131^2$.

Conjecture 2:

There exist an infinity of semiprimes of the form $q_1 \cdot q_2 = p^2 + 4320$, where p is prime, such that $q_2 - q_1 + 1$ is prime.

Such semiprimes are:

: $4369 = 4320 + 7^2 = 17 \cdot 257$ ($257 - 17 + 1 = 241$, prime);
: $4609 = 4320 + 17^2 = 11 \cdot 419$ ($419 - 11 + 1 = 409$, prime);
: $6001 = 4320 + 41^2 = 17 \cdot 353$ ($353 - 17 + 1 = 337$, prime);
: $7801 = 4320 + 59^2 = 29 \cdot 269$ ($269 - 29 + 1 = 241$, prime);

: $11209 = 4320 + 83^2 = 11 \cdot 1019$ ($1019 - 11 + 1 = 1009$,
 prime);
 : $15769 = 4320 + 107^2 = 13 \cdot 1213$ ($1213 - 13 + 1 =$
 1201 , prime);
 : $16201 = 4320 + 109^2 = 17 \cdot 953$ ($953 - 17 + 1 = 937$,
 prime);
 : $23089 = 4320 + 137^2 = 11 \cdot 2099$ ($2099 - 11 + 1 =$
 2089 , prime);
 : $23641 = 4320 + 139^2 = 47 \cdot 503$ ($503 - 47 + 1 = 457$,
 prime);
 : $28969 = 4320 + 157^2 = 59 \cdot 491$ ($491 - 59 + 1 = 433$,
 prime);
 : $32209 = 4320 + 167^2 = 31 \cdot 1039$ ($1039 - 31 + 1 =$
 1009 , prime);
 : $34249 = 4320 + 173^2 = 29 \cdot 1181$ ($1181 - 29 + 1 =$
 1153 , prime).

Conjecture 3:

There exist an infinity of semiprimes of the form $q_1 \cdot q_2 = p^2 + 4320$, where p is prime, such that $q_2 - q_1 + 1$ is a power of prime.

Such semiprimes are:

: $4681 = 4320 + 19^2 = 31 \cdot 151$ ($151 - 31 + 1 = 121 =$
 11^2);
 : $4849 = 4320 + 23^2 = 13 \cdot 373$ ($373 - 13 + 1 = 361 =$
 19^2);
 : $6169 = 4320 + 43^2 = 31 \cdot 199$ ($199 - 31 + 1 = 169 =$
 13^2);
 : $8809 = 4320 + 67^2 = 23 \cdot 383$ ($383 - 23 + 1 = 361 =$
 19^2);
 : $10561 = 4320 + 79^2 = 59 \cdot 179$ ($179 - 59 + 1 = 121 =$
 11^2);
 : $26521 = 4320 + 149^2 = 11 \cdot 2411$ ($2411 - 11 + 1 = 2401$
 $= 7^4$).

Note:

For the squares of the 27 from the first 35 primes p greater than or equal to 7 the number $p^2 + 4320$ is either prime either semiprime $q_1 \cdot q_2$ such that $q_2 - q_1 + 1$ is prime or square of prime. For other two primes p the number $p^2 + 4320 = q_1 \cdot q_2 \cdot q_3$ such that $q_1 + q_2 + q_3$ is prime ($8041 = 61^2 + 4320 = 11 \cdot 17 \cdot 43$ and $11 + 17 + 43 = 71$; $9361 = 71^2 + 4320 = 11 \cdot 23 \cdot 37$ and $11 + 23 + 37 = 71$) and for other two primes p the number $p^2 + 4320$ is a square ($13^2 + 4320 = 4489 = 67^2$ and $127^2 + 4320 = 20449 = 11^2 \cdot 13^2$).

Conjecture 4:

There exist an infinity of primes formed by deconcatenating a square of a prime and inserting the digit 0 between the first of its digits and the others.

Such primes are:

: 409 formed from $49 = 7^2$;
 : 1021 formed from $121 = 11^2$;
 : 1069 formed from $169 = 13^2$;
 : 2089 formed from $289 = 17^2$;
 : 3061 formed from $361 = 19^2$;
 : 10369 formed from $1369 = 37^2$;
 : 20809 formed from $2809 = 53^2$;
 : 50329 formed from $5329 = 73^2$;
 : 60889 formed from $6889 = 83^2$;
 : 70921 formed from $7921 = 89^2$;
 : 100609 formed from $10609 = 103^2$;
 : 101449 formed from $11449 = 107^2$;
 : 102769 formed from $12769 = 113^2$;
 : 106129 formed from $16129 = 127^2$;
 : 108769 formed from $18769 = 137^2$;
 : 109321 formed from $19321 = 139^2$;
 : 202201 formed from $22201 = 149^2$.

Conjecture 5:

There exist an infinity of semiprimes $q_1 \cdot q_2$ such that $q_2 - q_1 + 1$ is prime or square of prime formed by deconcatenating a square of a prime and inserting the digit 0 between the first of its digits and the others.

Such semiprimes are:

: $5029 = 47 \cdot 107$ formed from $529 = 23^2$ ($107 - 47 + 1 = 61$, prime);
 : $10681 = 11 \cdot 971$ formed from $1681 = 41^2$ ($971 - 11 + 1 = 961 = 31^2$);
 : $30721 = 31 \cdot 991$ formed from $3721 = 61^2$ ($991 - 31 + 1 = 961 = 31^2$);
 : $40489 = 19 \cdot 2131$ formed from $4489 = 67^2$ ($2131 - 19 + 1 = 2113$, prime);
 : $60241 = 107 \cdot 563$ formed from $6241 = 79^2$ ($563 - 107 + 1 = 457$, prime);
 : $90409 = 11 \cdot 8219$ formed from $9409 = 97^2$ ($8219 - 11 + 1 = 8209$, prime);
 : $100201 = 97 \cdot 1033$ formed from $9409 = 101^2$ ($1033 - 97 + 1 = 937$, prime);
 : $107161 = 101 \cdot 1061$ formed from $17161 = 131^2$ ($1061 - 101 + 1 = 961 = 31^2$);

- : 202801 = 139*1459 formed from 22801 = 151² (1459 - 139 + 1 = 1321, prime);
- : 204649 = 19*10771 formed from 24649 = 157² (10771 - 19 + 1 = 10753, prime).

Conjecture 6:

There exist an infinity of semiprimes q_1*q_2 such that $q_2 - q_1 + 1 = q_3*q_4$ where $q_4 - q_3 + 1$ is prime or square of prime formed by deconcatenating a square of a prime and inserting the digit 0 between the first of its digits and the others.

Such semiprimes are:

- : 10849 = 19*571 formed from 1849 = 43² (571 - 19 + 1 = 553 = 7*79 and 79 - 7 + 1 = 73, prime);
- : 20209 = 7*2887 formed from 2209 = 47² (2887 - 7 + 1 = 2881 = 43*67 and 67 - 43 + 1 = 25 = 5²);
- : 50041 = 163*307 formed from 5041 = 71² (307 - 163 + 1 = 145 = 5*29 and 29 - 5 + 1 = 25 = 5²).

Conjecture 7:

There exist an infinity of composites $q_1*q_2*q_3$ such that $q_1 + q_2 + q_3$ is prime formed by deconcatenating a square of a prime and inserting the digit 0 between the first of its digits and the others.

Such composites are:

- : 8041 = 11*17*43 formed from 841 = 29² (11 + 17 + 43 = 71, prime);
- : 9061 = 13*17*41 formed from 961 = 31² (13 + 17 + 41 = 71, prime);
- : 30481 = 11*17*163 formed from 3481 = 59² (11 + 17 + 163 = 71, prime);
- : 101881 = 13*17*461 formed from 11881 = 109² (13 + 17 + 461 = 491, prime);
- : 206569 = 11*89*211 formed from 26569 = 163² (11 + 89 + 211 = 311, prime).

Note:

For all 35 from the first 35 primes greater than or equal to 7 the number formed in the way mentioned satisfies one of the conditions defined in the four conjectures above.