The theory of electrodynamic space-time relativity（Revision 3）

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Abstract: The theory of electrodynamic space-time relativity (TESTR) is the study of the transformation of time and space between two electrodynamic inertial frames of reference, which have both inertial velocity difference and electric potential difference. It is a fundamental space-time theory of theoretical physics based on the Einstein's special theory of relativity (STR), the electric potential limit postulate and the high-precision experimental facts of the inversion proportional square law of Coulomb's force. It also proposed new basic physical concepts, such as electric potential limit, quaternion velocity, quaternion electric potential and etc., revealing the inherent relationships between the quaternion velocity, the quaternion electric potential and time-space. This paper discusses in detail the process of establishing the theory of complex electrodynamic space-time relativity and theory of quaternion electrodynamic space-time relativity as well as their various conversions and transformations. At the same time, proved that the special theory of relativity is a special case of TESTR, and found another important special case: the theory of electric potential relativity (TEPR). The basic effects of TESTR were also discussed, proposing the equations for superposition of quaternion velocity, and quaternion potential of TESTR respectively. It predicts some important new space-time change effects, which provides theoretical basis for the experimental validation of TESTR.

Keyword: Postulate of electric potential limit, theory of quaternion electrodynamic space-time relativity, theory of complex electrodynamic space-time relativity, special theory of relativity, theory of electric potential relativity

Part 1. Postulate of electric potential limit

Experiments have shown that electron is a point particle that does not occupy any space [1], and Coulomb's inverse square law is established by high precision [2]. However, based on these experimental facts and current electromagnetism, electron's self-energy is calculated to be infinite. This is a recognized insuperable problem in modern electromagnetism. Even in the current quantum field theory, there are the more serious divergence difficulties – both electron self-energy and the charge are infinite. Even though through renormalization this problem can be avoided temporarily, but this divergence issue has not been solved fundamentally, and is a general occurrence in fundamental particles. From this, one can see that the basics theories of modern physics are not complete, and they need further development.

Based on current electromagnetism, the superposition of electric potential is linear, that is, there is not an upper limit on electric potential or it can be infinite. This means that the energy of unit electric charge can be infinite. This conclusion is close related to the infinite electron self-energy. From the calculations based on Coulomb's law, the electric potential at the center of electron is also infinite. After comprehensive analyses, an important conjecture can be obtained: There is an upper limit on electric potential. At present there is no experiment that can deny this conjecture. Therefore, it is logical to elevate this conjecture to become one of the basic postulates of a new physical theory.

If there is a limit on electric potential, the Maxwell's equations in electromagnetics will be modified. At the same time, because the Maxwell's equations and the special theory of relativity are covariant, hence there must be a corresponding new expansion of the modern theory of space-time.

Part 2. The theory of complex electrodynamic space-time relativity (TCESTR**)**

Einstein's special theory of relativity (STR) is based on two basic postulates about inertial motion, that is:

1. The principle of relativity: physical laws should be the same in every inertial frame of reference
2. The principle of invariant speed of light: in any inertial system, the speed of light in vacuum is a constant.

According to these two postulates, the famous Lorentz transform equations can be derived.

$$X' = \gamma(X - V_X t) \tag{1}$$

$$Y' = Y \tag{2}$$

$$Z' = Z \tag{3}$$

$$t' = \gamma\left(t - \frac{V_X}{{C_0}^2}X\right) \tag{4}$$

$$\gamma = \frac{1}{\sqrt{1-\frac{{V_X}^2}{{C_0}^2}}} \tag{5}$$

Here, the Lorentz transformation equations are scalar equations, where $V_X$is the speed along x-axis of inertial frame $\Sigma_R{}'(X', Y', Z', t')$ relative to the inertial frame $\Sigma_R(X, Y, Z, t)$. C_0 is the speed of light. X, Y, Z, t and X', Y', Z', t' are the length and time of the observing system and the observed system, respectively.

Upon analyzing the basic postulates of the special theory of relativity, a question can be asked: is our understanding of motion complete? Is there another physical reference frame besides the inertial reference frame, where the same the physical laws still hold true in each frame of reference when it is under different states? In this physical reference frame, the state of such reference frame cannot be determined through any experiment. Does the related physical parameters have limit? And would such limit lead to the relativistic effect of time and space? After further study, it was proposed that equipotential bodies are of such physical reference systems. Based on Coulomb's inverse proportional square law and its high-precision experiments[2], we know that within a closed conductor of any shape, the electric potential at any point is the same regardless of how much electric charge its surface carries. In addition, the interior electric field strength E is zero. This is a corollary of Gauss' law of electric field.

Thus, a thought experiment can be carried out: there are two identical closed metal carriages A and B, and they are motionless relative to each other and are insulated from each other with only carriage B being grounded. Suppose there is an electrode of an ultra-high voltage static electric generator connected to B, and the other electrode is connected to the carriage A. Let the electric potential of the ground be zero. Once the ultra-high voltage electrostatic generator starts running and continues to charge the carriage A with electrical charges (either positive or negative), the electric potential on the surface and in the interior of the carriage increases with it. When the surface electric charges reach Q, the interior and surface electric potential ϕ is the same everywhere. At the same time, the electric field intensity is zero. Therefore, when an observer is isolated inside carriage A, it is impossible for the observer to know the values of electric potential or whether it is positive or negative relative to carriage B through any experiment. Same can be said for the observer in the carriage B where the electric potential is zero, even when there is a very high electric potential difference between them. Please note that same experimental results can be obtained in the inertial frames of reference. Therefore, we can put forth two postulates of equipotential reference system:

3. Relative principle of electric potential: physical law has the same form in any electric equipotential frame of reference;

4. Postulate of electric potential limit: in any stationary and equipotential frame of reference, electric potential limit is a constant Φ_0 at any point in vacuum.

The value for such electric potential limit Φ_0 can only be determined by experiment. In the Planck's units, Planck voltage is a very large value of 1.04295×10^{27} volts. It could be used as one of the reference values of the electric potential limitΦ_0.

By comparing the relative principle of electric potential and the postulate of electric potential limit with the two postulates of the special theory of relativity, it can be seen that their forms are very similar. When the two postulates are established, then, the current Maxwell's equation must be modified. The superposition of the electric potential will be non-linear, and many physical parameters must have function relation with the electric potential limit. In physics, there must be a theory of electric potential relativity (TEPR). It will be specifically used to describe the existence of the relationships between physical quantities such as space-time and the electric potential in the frames of reference with electric potential differences. STR is the study of physical parameters relationships such as space-time with speed in the inertia frames of references. Hence, there must be a higher theory of relativity that studies the relationship of the frames of reference with both speed difference and also electric potential difference. This theory can be called the theory of electrodynamic space-time relativity, where STR and TEPR are both its special cases. In order to establish such "higher level theory of relativity", both the electric-charge state (equipotential) and the inertial motion state (inertial speed) of the same frame of reference must be unified. In modern physics, however, the concepts of electric potential and speed have no direct correlation. Through in-depth study, it was discovered that in order to solve this contradiction, new physical concepts such as imaginary motion, complex motion and etc., must be introduced.

To investigate this problem, suppose at random point P in the equipotential space:

$$\beta = \frac{\phi}{\Phi_0}$$

Where Φ_0 is the electric potential limit, ϕ is electric potential of point P, and β is the ratio between the electric potential and the electric potential limit. Even though equipotential frame of reference is very similar to the inertial frame of reference, but the equipotential frame of reference is stationary in the three-dimensional space. It cannot be defined as a regular inertial reference frame. Equipotential reference frame will be defined as the inertial frame of reference of the imaginary speed. In addition, equipotential space-time and inertial space-time are homogeneous and isotropic, therefore corresponding relationship exists between electric potential and imaginary speed. The electric potential limitΦ_0 is equivalent to the imaginary speed of light $C_0 i$. The potential ϕ is equivalent to the imaginary speed $V_\phi i$. From the perspective of complex number, both C_0 and V_ϕ are scalar. That is:

$$\beta = \frac{V_\phi i}{C_0 i}$$

Where, imaginary number $i = \sqrt{-1}$,

Hence the following formula is established:

$$\frac{\phi}{\Phi_0} = \frac{V_\phi i}{C_0 i} \tag{6}$$

Let, $V_\phi i = K\phi$ (7)

Here, $K = \frac{C_0}{\Phi_0} i$, is an imaginary constant, and can be named as the electrodynamic conversion factor.

To further investigate more universal space-time relationship, we must extend our concept of motion and space-time from the real number domain to the complex number domain. The more general motion can be abstractly understood as the motion state in the complex plane. If the motion in the coordinate reference system possesses both real and imaginary motion, then we call its motion state the complex motion state. The frame of reference situated in the complex motion state is called complex inertial electrodynamic frame of reference; it has both equipotential and inertial motion. The theory to describe this space-time relation of such motion is called the theory of complex electrodynamic space-time relativity.

As shown in Figure 1, let there be two complex coordinate systems of reference $\Sigma_C(X, F, t)$and $\Sigma_C'(X', F', t')$in the same complex plane. Their imaginary axis F and F′, real axis X and X′ are all parallel to each other. Since in complex number there is no vector, therefore all the relevant physical parameters of motion must be scalars quantities such as speed and distance. Let $\Sigma_C(X, F, t)$ be the stationary observing reference frame that is, the imaginary speed is zero (the electric potential is zero) and real speed is also zero. While $\Sigma_C'(X', F', t')$ is the observed reference frame and in complex motion state relative to $\Sigma_C(X, F, t)$, its complex speed is V_w, its imaginary speed is $V_\phi i$, and real speed is V_X, and θ is the complex angle. When $O'$and$O$origin of coordinates overlap, let $t = t' = 0$.

$$V_w = V_X + V_\phi i = |V_w|\, e^{\theta i} \tag{8}$$

The modulus of complex speed is: $|V_w| = \sqrt{V_X{}^2 + V_\phi{}^2}$ (9)

Where, $V_\phi = \frac{\phi C_0}{\Phi_0}$ (10)

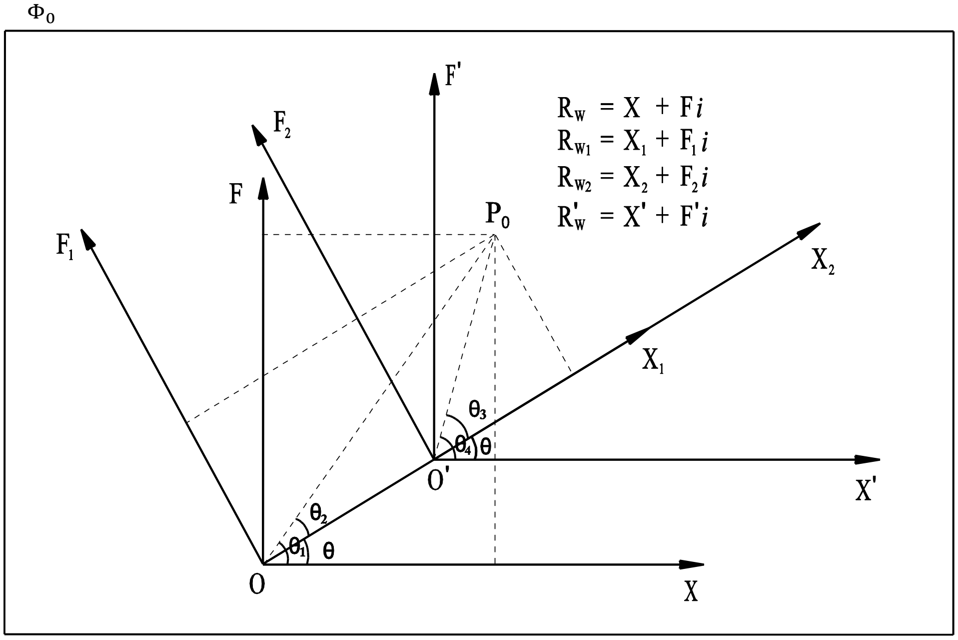

Figure 1

Since real electric potential can be converted into imaginary speed, then by symmetry principle, real speed can be converted into imaginary electric potential $\phi_X i$. Therefore, $\phi_X i = V_X \frac{1}{K} = -\frac{V_X \Phi_0}{C_0} i$,where real electric potential ϕ and imaginary electric potential $\phi_X i$ together forms complex electric potential ϕ_w:

$$\phi_w = \phi + \phi_X i \tag{11}$$

The modulus of complex electric potential is:

$$|\phi_w| = \sqrt{\phi^2 + \phi_X{}^2} \tag{12}$$

Where, $\phi_X = -\frac{V_X \Phi_0}{C_0}$ (13)

Multiplying both sides of the equation (11) with the imaginary constant K, and comparing with equation (8), we get:

$V_w = K\phi_w$ (14)

This shows that complex speed and the complex electric potential are inter-convertible. Therefore the complex reference frame of electric potential is a type of complex electrodynamic inertial reference frame. To take it one step further, the aforementioned four basic postulates will be combined into two basic postulates of TCESTR. Since complex numbers cannot be compared in magnitude, but their moduli can, we have:

5. Relative principle of complex electrodynamic space-time: physical law has the same form in any complex electrodynamic inertial frame of reference;

6. The postulate of complex electrodynamic time-space limit: in any complex electrodynamic inertial reference system, the limit of complex speed's modulus of any point in vacuum is a constant, C_0; or the limit of complex electric potential's modulus of any point in vacuum is a constant , Φ_0.

Where, C_0 is the speed of light in the vacuum with zero electric potential, and is equal to 299,792,458 m/sec. The limit of complex electric potential's modulus Φ_0 can only be determined by experiment.

One of the inference can be derived from postulate 6 is that in the frame of reference where electric potential is not zero, the speed of light C_0' in the real three-dimensional space is less than C_0. However, the complex speed's modulus of the light in the complex space is C_0. (This is proven in subsequent paper regarding expansion of the Maxwell's equations).

When $V_X = 0,\ V_w = V_\phi i$, the two above postulates become the postulates of the theory of electric potential relativity. When $V_\phi = 0,\ V_w = V_X$, the two above postulates become the fundamental postulates of the special theory of relativity. Hence, special theory of relativity and the theory of electric potential relativity are two special cases of TCESTR.

The special theory of relativity is commonly referred to the motion of the observed system relative to the observing system along an axis and is called one-dimensional special theory of relativity. In fact, such motion can have two-dimensional or three-dimensional forms, and corresponding the special theory of relativity becomes more intricate but also more universal. There are already detailed discussions in literatures on this matter, showing that in the real space two-dimensional [3] and three-dimensional special theory of relativity [4] in arbitrary direction can be derived through rotation and translation of the coordinate system of one-dimensional special theory of relativity; or three-dimensional special theory of relativity [5], [6] can be derived through vector transformation of one-dimensional special theory of relativity. Although they are difference in their derivation methods, there is a common theme, that is, through the use of one-dimension special theory of relativity and appropriate mathematical approach, the higher dimension real form of special theory of relativity can be derived. Therefore, based on the above mentioned postulates and one-dimension special theory of relativity, TCESTR may be derived by a way of transforming the complex coordinate system.

As shown in Figure 1, let there be a random point P_0 be in the complex plane. In different complex coordinate systems, it can be represented by different coordinate parameters. In the complex coordinate system $\Sigma_C(X, F, t)$, the coordinate of P_0 is represented by complex distance $R_w = X + Fi$ with complex angle is θ_1. In the coordinate system $\Sigma_C'(X', F', t')$, the coordinate of point P_0 is represented as complex distance $R_w' = X' + F'i$ with complex angle θ_4. Refer to method of two dimensional coordinate transformation of real numbers [4], and expanding it into the complex planar space, reference system $\Sigma_C'(X', F', t')$ can be obtained through three times coordinate transformations from reference system $\Sigma_C(X, F, t)$.

(1) In the complex coordinate system $\Sigma_C(X, F, t)$, its time is t, rotating the coordinate system by θ degree counterclockwise, we get complex coordinate system $\Sigma_{C1}(X_1, F_1, t_1)$, , whose time is t_1;

(2) Complex coordinate system $\Sigma_{C1}(X_1, F_1, t_1)$ is translated along axis X_1 of real number in the quantity equal to the modulus of the complex speed $|V_w|$, we get complex coordinate system $\Sigma_{C2}(X_2, F_2, t_2)$., whose time is t_2;

(3) Complex coordinate system $\Sigma_{C2}(X_2, F_2, t_2)$. is rotated by θ degree clockwise, we get complex coordinate system $\Sigma_C{}'(X', F', t')$, whose time is t'.

The detailed derivation steps, numbered corresponding to the above, are as follow:

(1) Rotate the coordinate system $\Sigma_C(X, F, t)$ by θ degree counter clockwise, making the real axis X_1 of the coordinate system $\Sigma_{C1}(X_1, F_1, t_1)$ pass through the origin O' of the reference system $\Sigma_C{}'(X', F', t')$. Point P_0 in $\Sigma_C(X, F, t)$ complex coordinate system has complex distance R_w, whose complex angle is θ_1. Point P_0 in the coordinate system $\Sigma_{C1}(X_1, F_1, t_1)$ has complex distance R_{w_1}, whose complex angle is θ_2. That is,

$$R_w = X + Fi = |R_w|e^{\theta_1 i} \tag{15}$$

$$R_{w_1} = X_1 + F_1 i = |R_{w_1}|e^{\theta_2 i} \tag{16}$$

Also, $|R_w| = |R_{w_1}|$,therefore,

$$R_{w_1} = R_w\, e^{(\theta_2 - \theta_1)i}$$

$$R_{w_1} = R_w\, e^{-\theta i} \tag{17}$$

$$R_{w_1} = (X + Fi)(\cos\theta - i\sin\theta) \tag{18}$$

From equations (18) and (16),

$$X_1 = X\cos\theta + F\sin\theta \tag{19}$$

$$F_1 = F\cos\theta - X\sin\theta \tag{20}$$

(2) Let $\Sigma_{C1}(X_1, F_1, t_1)$ be the stationary frame of reference whose time is t_1. $\Sigma_{C2}(X_2, F_2, t_2)$ is the moving reference system whose time is t_2, and real axes $X_1$and X_2 are overlapping. The reference system $\Sigma_{C2}(X_2, F_2, t_2)$ moves along the positive X_1 direction relative to $\Sigma_{C1}(X_1, F_1, t_1)$ with the magnitude of $|V_w|$, hence $F_2 = F_1$. Since X_1 is the real number axis of $\Sigma_{C1}(X_1, F_1, t_1)$, it can be thought as the real motion. This is the same state that the special theory of relativity studies. Hence the Lorentz transform equations (1), (2), (4) and (5) can be directly used, but the parameters in the equations need to be replaced by the parameters of $\Sigma_{C1}(X_1, F_1, t_1)$ and $\Sigma_{C2}(X_2, F_2, t_2)$.

According to the postulate of complex electrodynamic time-space limit, the moduli of complex speed of light in all directions are a constant C_0. When a coordinate system rotates around itself origin to an angle, the two origins of coordinate systems before and after the rotation have not been displaced. Their complex distances' moduli are equal between the origins and point P_0. When the light beamed from origins to P_0 point, the times spent are the same. Therefore time is not related to the angle of coordinate system rotation, that is $t = t_1$, $t_2 = t'$. We get,

$$X_2 = \gamma(X_1 - |V_w|t) \tag{21}$$

$$F_2 = F_1 \tag{22}$$

$$t' = \gamma\left(t - \frac{|V_w|}{C_0{}^2} X_1\right) \tag{23}$$

$$\text{Where, } \gamma = \frac{1}{\sqrt{1 - \frac{|V_w|^2}{C_0{}^2}}} \tag{24}$$

(3) In complex coordinate reference system$\Sigma_{C2}(X_2, F_2, t_2)$., P_0 is represented by complex distance R_{w_2} with complex angle θ_3, rotating clockwise θ degree, we get coordinate system $\Sigma_C{}'(X', F', t')$. In $\Sigma_C{}'(X', F', t')$, P_0 is represented by complex distance $R_w{}'$ with complex angle θ_4:

$$R_{w_2} = X_2 + F_2 i = |R_{w_2}|e^{\theta_3 i}$$

$$R_w{}' = X' + F'i = |R_w{}'|e^{\theta_4 i}$$

And also, $|R_w{}'| = |R_{w_2}|$, hence,

$$\mathrm{R_w}' = \mathrm{R_{w_2}}\, e^{(\theta_4-\theta_3)i} = \mathrm{R_{w_2}}\, e^{\theta i} \tag{25}$$

Because $\mathrm{R_{w_2}} - \mathrm{R_{w_1}} = \mathrm{X_2} - \mathrm{X_1}$, therefore,

$$\mathrm{R_w}' = (\mathrm{R_{w_1}} - (\mathrm{X_1} - \mathrm{X_2}))\, e^{\theta i} \tag{26}$$

Substituting equation (17) into equation(26),

$$\mathrm{R_w}' = \mathrm{R_w} - (\mathrm{X_1} - \mathrm{X_2})\, e^{\theta i}$$

Because $\mathrm{V_w} = |\mathrm{V_w}| e^{\theta i}$, therefore,

$$\mathrm{R_w}' = \mathrm{R_w} - \mathrm{V_w}\frac{(\mathrm{X_1}-\mathrm{X_2})}{|\mathrm{V_w}|} \tag{27}$$

Let $\mathrm{t_w} = \frac{(\mathrm{X_1}-\mathrm{X_2})}{|\mathrm{V_w}|}$ (28)

$\mathrm{t_w}$ is a real number, and can be called the system time of complex electrodynamic inertial reference frames. It is not a special time of any particular reference frame, but related to the time of both the observing system and the observed system. It is the “time” needed to solve number of fundamental physic problems.

To sum up the above discussion, we obtained the basic equations of TCESTR as the following:

$$\mathrm{R_w}' = \mathrm{R_w} - \mathrm{V_w t_w} \tag{29}$$

$$\mathrm{t}' = \gamma\left(\mathrm{t} - \frac{|\mathrm{V_w}|}{C_0{}^2}\mathrm{X_1}\right) \tag{30}$$

Where, $\gamma = \frac{1}{\sqrt{1-\frac{|\mathrm{V_w}|^2}{C_0{}^2}}}$ (31)

$$\mathrm{t_w} = \frac{(\mathrm{X_1}-\mathrm{X_2})}{|\mathrm{V_w}|} \tag{32}$$

$$\mathrm{X_2} = \gamma(\mathrm{X_1} - |\mathrm{V_w}|\mathrm{t}) \tag{33}$$

$$\mathrm{X_1} = \mathrm{X}\cos\theta + \mathrm{F}\sin\theta \tag{34}$$

Depending on different applications, the above relationship can be further expanded into different forms.

According to equation (8), $\cos\theta = \frac{\mathrm{V_X}}{|\mathrm{V_w}|}$, $\sin\theta = \frac{\mathrm{V_\phi}}{|\mathrm{V_w}|}$. Substituting them into equation (34), we get:

$$\mathrm{X_1} = \frac{1}{|\mathrm{V_w}|}(\mathrm{XV_X} + \mathrm{FV_\phi}) \tag{35}$$

By substituting equation (32), (33) and (35) into equation (29), we can obtain expanded equations describing TCESTR with complex speed,

$$\mathrm{R_w}' = \mathrm{R_w} - \frac{\mathrm{V_w}}{|\mathrm{V_w}|}\left((1-\gamma)\mathrm{X_1} + |\mathrm{V_w}|\gamma\mathrm{t}\right) \tag{36}$$

$$\mathrm{t}' = \gamma\left(\mathrm{t} - \frac{|\mathrm{V_w}|}{C_0{}^2}\mathrm{X_1}\right) \tag{37}$$

Where, $\gamma = \frac{1}{\sqrt{1-\frac{|\mathrm{V_w}|^2}{C_0{}^2}}}$

Speed and electric potential has a corresponding relationship. Therefore, we can also obtain another extended equations describing TCESTR with complex electric potential. This will be discussed later in the quaternion electrodynamic space-time relativity.

By decomposing (36) into equations of real number and imaginary number, we obtain:

$$\mathrm{X}' = \mathrm{X} - \frac{\mathrm{V_X}}{|\mathrm{V_w}|}\left((1-\gamma)\mathrm{X_1} + |\mathrm{V_w}|\gamma\mathrm{t}\right) \tag{38}$$

$$\mathrm{F}'i = \mathrm{F}i - i\frac{\mathrm{V_\phi}}{|\mathrm{V_w}|}\left((1-\gamma)\mathrm{X_1} + |\mathrm{V_w}|\gamma\mathrm{t}\right) \tag{39}$$

Therefore when $V_\phi = 0$, then $|V_w| = V_X$, according to (35) can get $X_1 = X$, and, Substituting the equations into (38), (39), (30) and (31), the special theory of relativity can be obtained:

$$X' = \gamma(X - V_X t)$$
$$t' = \gamma\left(t - \frac{V_X}{{C_0}^2}X\right)$$
$$F' = F$$
$$\gamma = \frac{1}{\sqrt{1-\frac{{V_X}^2}{{C_0}^2}}}$$

Let $V_X = 0$, then $|V_w| = V_\phi$, $|V_w| = \frac{C_0}{\Phi_0}\phi$ can be obtained based on equation (6) and according to (35) can obtain $X_1 = F$. Substituting the equations into (38), (39), (30) and (31), a new special theory of relativity can be obtained:

$$F'i = \gamma\left(Fi - \frac{C_0}{\Phi_0}\phi i t\right) \tag{40}$$
$$X' = X \tag{41}$$
$$t' = \gamma\left(t - \frac{\phi}{\Phi_0 C_0}F\right) \tag{42}$$
$$\gamma = \frac{1}{\sqrt{1-\frac{\phi^2}{{\Phi_0}^2}}} \tag{43}$$

This is the aforementioned the theory of electric potential relativity (TEPR), whose form can be seen here as symmetric to that of the special theory of relativity, but they have different physical meaning.

Part 3. The theory of quaternion electrodynamic space-time relativity

TCESTR describes relationship between the space-time and the state of complex motion of the reference frames in the complex plane, but the real axis in the complex plane is one-dimensional. In reality, our real number space is three-dimensional. Therefore two-dimensional space of TCESTR must be expanded into four dimensions. In mathematics, the higher form of complex number is quaternion. Let A be a quaternion,

$$A = z_1 + z_2\mathbf{i} + z_3\mathbf{j} + z_4\mathbf{k} \tag{44}$$

Where, $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are the unit vectors, and they satisfy the multiplication rules of quaternion. If $z_n (n = 1, 2, 3, 4,)$ are all real number, then A is a real quaternion. If there is at least one complex number (or one imaginary number) in z_n, then A is called biquaternion. The real quaternion is a special case of biquaternion. The modulus $|A|$ of the quaternion A is

$$|A|^2 = {z_1}^2 + {z_2}^2 + {z_3}^2 + {z_4}^2 \tag{45}$$

If A is biquaternion, then under normal condition $|A|$ is a complex number. However, because complex numbers cannot be compared in magnitude, The paper need to define another modulus $||A||$ of the quaternion A, where

$$||A||^2 = |z_1|^2 + |z_2|^2 + |z_3|^2 + |z_4|^2 \tag{46}$$

Here $|z_n|$ $(n = 1,2,3,4)$ is the absolute value of z_n, $||A||$ is a norm. Therefore, $||A||$ can be called the norm-modulus of quaternion A, and it is a real number that is greater or equal to zero. When A is real quaternion, we get,

$$||A|| = |A| \tag{47}$$

Hence, in order to further expand TCESTR into TQESTR, the physical parameters within the TCESTR must be expanded to become quaternion physical parameters. For example, quaternion velocity, quaternion electric potential and etc, whose corresponding reference system would be collectively referred to as the quaternion electrodynamic inertial reference system. At same time, the two basic postulates 5 and 6 of TCESTR can be further expanded into the basic postulates of TQESTR:

7. Relative principle of quaternion electrodynamic space-time: in any quaternion electrodynamic inertial reference system, physical laws have the same form;

8. Postulate of quaternion electrodynamic space-time limit: in any quaternion electrodynamic inertial reference system, the limit of quaternion velocity's norm-modulus of any point in vacuum is a constant, C_0; or the limit of quaternion electric potential's norm-modulus of any point is a constant , Φ_0.

Where C_0 is the speed of light in the vacuum with zero electric potential, and is equal to $299{,}792{,}458\ \mathrm{m/sec}$. The limit of quaternion electric potential's modulus Φ_0 only can be determined by experiment.

According to the basic equation (29) of TCESTR, it can be expanded and separated into real number equation and imaginary number equation:

$$\mathrm{F}'i = \mathrm{F}i - \mathrm{V}_\phi i \mathrm{t}_\mathrm{w} \tag{48}$$

$$\mathrm{X}' = \mathrm{X} - \mathrm{V}_\mathrm{X}\mathrm{t}_\mathrm{w} \tag{49}$$

Notice that equation (49) is a real scalar expression. However, the fact is that the observed reference frame $\Sigma_\mathrm{C}'(\mathrm{X}', \mathrm{F}', \mathrm{t}')$ moves along the real axis X of the observing reference frame $\Sigma_\mathrm{C}(\mathrm{X}, \mathrm{F}, \mathrm{t})$ with vector velocity $\mathbf{V_x}$. Let its unit vector be $\mathbf{i}$, because axes X' and X are the same direction as $\mathbf{i}$. Multiply both sides of equation (49) by $\mathbf{i}$, equation (49) becomes a vector equation:

$$\mathbf{X}' = \mathbf{X} - \mathbf{V_x}\mathrm{t}_\mathrm{w} \tag{50}$$

Equations (48) and (50) describe the physical nature more objectively than the complex equations (29). So the physical quantities about motion will also use vector, such as velocity, displacement and etc. Equation (50) can be expanded into three-dimensional. If the vector $\mathbf{X}'$ and $\mathbf{X}$ in equation (50) are defined as vector $\mathbf{r}'$ and $\mathbf{r}$ in the three-dimensional space $\mathrm{X}', \mathrm{Y}', \mathrm{Z}'$ and $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$, corresponding velocity $\mathbf{V_x}$ is defined as $\mathbf{V_r}$**,** and have the same direction as $\mathbf{r}'$**,** $\mathbf{r}$ and $\mathbf{V_r}$. Let $\mathbf{i}, \mathbf{j}, \mathbf{k}$ be the unit vectors in the coordinate of three-dimensional space along the axis $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$, respectively, we have:

$$\mathbf{r} = \mathrm{X}\mathbf{i} + \mathrm{Y}\mathbf{j} + \mathrm{Z}\mathbf{k} \tag{51}$$

$$\mathbf{r}' = \mathrm{X}'\mathbf{i} + \mathrm{Y}'\mathbf{j} + \mathrm{Z}'\mathbf{k} \tag{52}$$

$$\mathbf{V_r} = \mathrm{V_X}\mathbf{i} + \mathrm{V_Y}\mathbf{j} + \mathrm{V_Z}\mathbf{k} \tag{53}$$

$\mathbf{X}', \mathbf{X}, \mathbf{V_x}$ becomes $\mathbf{r}', \mathbf{r}, \mathbf{V_r}$**.** At the same time, let $\mathrm{t_w}$ become $\mathrm{t_q}$ accordingly. Therefore, equation (48) and (50) can be transformed into:

$$\mathrm{F}'i = \mathrm{F}i - \mathrm{V}_\phi i \mathrm{t}_\mathrm{q} \tag{54}$$

$$\mathbf{r}' = \mathbf{r} - \mathbf{V_r}\mathrm{t}_\mathrm{q} \tag{55}$$

Equation (54) and (55) form a quaternion space system composed of three real number vectors and one imaginary number, and is called type A_i quaternion space. This expands the state of motion of the reference system in the complex plane into motion of the type A_i quaternion space, that is, the reference system $\Sigma_\mathrm{Q}'(\mathrm{F}', \mathrm{X}', \mathrm{Y}', \mathrm{Z}', \mathrm{t}')$ moves with quaternion velocity $\mathrm{V_q}$ relative to the reference frame $\Sigma_\mathrm{Q}(\mathrm{F}, \mathrm{X}, \mathrm{Y}, \mathrm{Z}, \mathrm{t})$. Suppose quaternion distance R'_q and $\mathrm{R_q}$ are quaternion displacement coordinates at any point P_0 in the inertial reference system $\Sigma_\mathrm{Q}'(\mathrm{F}', \mathrm{X}', \mathrm{Y}', \mathrm{Z}', \mathrm{t}')$ and $\Sigma_\mathrm{Q}(\mathrm{F}, \mathrm{X}, \mathrm{Y}, \mathrm{Z}, \mathrm{t})$ in the A_i type quaternion space, respectively, then:

$$\mathrm{R_q} = \mathrm{F}i + \mathbf{r} = \mathrm{F}i + \mathrm{X}\mathbf{i} + \mathrm{Y}\mathbf{j} + \mathrm{Z}\mathbf{k} \tag{56}$$

The norm-modulus of $\mathrm{R_q}$ is $||\mathrm{R_q}||$, that is $||\mathrm{R_q}|| = \sqrt{\mathrm{F}^2 + \mathrm{X}^2 + \mathrm{Y}^2 + \mathrm{Z}^2}$ (57)

$$R'_q = F'i + \mathbf{r}' = F'i + X'\mathbf{i} + Y'\mathbf{j} + Z'\mathbf{k} \quad (58)$$

The norm-modulus of R'_q is $||R'_q||$, i.e., $||R'_q|| = \sqrt{F'^2 + X'^2 + Y'^2 + Z'^2}$ (59)

Type A_i quaternion velocity V_q and its norm-modulus $||V_q||$ are:

$$V_q = V_\phi i + \mathbf{V_r} = V_\phi i + V_X\mathbf{i} + V_Y\mathbf{j} + V_Z\mathbf{k} \quad (60)$$

$$||V_q|| = \sqrt{{V_r}^2 + {V_\phi}^2} = \sqrt{{V_X}^2 + {V_Y}^2 + {V_Z}^2 + {V_\phi}^2} \quad (61)$$

Where $i = \sqrt{-1}$

Adding equation (54) with (55), and substituting (56), (58) and (60) into it to obtain equation (62). Also, replacing X', X_1, X_2, X and $|V_w|$ of equations (30), (31), (32), (33) and (34)with r', r_1, r_2, r and $||V_q||$, the quaternion basic equations can be obtained for TQESTR:

$$R_q{}' = R_q - V_q t_q \quad (62)$$

$$t' = \gamma\left(t - \frac{||V_q||}{{C_0}^2} r_1\right) \quad (63)$$

$$\text{where,} \gamma = \frac{1}{\sqrt{1 - \frac{||V_q||^2}{{C_0}^2}}} \quad (64)$$

$$r_1 = r\cos\theta + F\sin\theta \quad (65)$$

$$r_2 = \gamma\big(r_1 - ||V_q||t\big) \quad (66)$$

$$t_q = \frac{(r_1 - r_2)}{||V_q||} \quad (67)$$

Depending on the need, the above basic equation can be transformed into different variations.

Because, $\cos\theta = \frac{V_r}{||V_q||}$, $\sin\theta = \frac{V_\phi}{||V_q||}$, $r = X\frac{V_X}{V_r} + Y\frac{V_Y}{V_r} + Z\frac{V_Z}{V_r}$, then,

$$r_1 = \frac{1}{||V_q||}\left(FV_\phi + XV_X + YV_Y + ZV_Z\right) \quad (68)$$

The r_1 can also be expressed with quaternions or four-dimensional vector's dot product.

Hence the quaternion velocity equations can be obtained for TQESTR,

$$R_q{}' = R_q - \frac{V_q}{||V_q||}\Big((1-\gamma)r_1 + ||V_q||\gamma t\Big) \quad (69)$$

$$t' = \gamma\left(t - \frac{||V_q||}{{C_0}^2} r_1\right) \quad (70)$$

$$\text{Where, } \gamma = \frac{1}{\sqrt{1 - \frac{||V_q||^2}{{C_0}^2}}} \quad (71)$$

When $V_\phi = 0$, Based on equations(47), (56), (58) and (60) $||V_q|| = |\mathbf{V_r}|$, $R_q = \mathbf{r}$, $R_q{}' = \mathbf{r}'$, $V_q = \mathbf{V_r}$ can be obtained. According to equation (68), then ,

$$r_1 = \frac{\mathbf{r} \cdot \mathbf{V_r}}{|\mathbf{V_r}|} \quad (72)$$

Therefor the special theory of relativity in any direction $\mathbf{r}(\mathbf{x}, \mathbf{y}, \mathbf{z})$ in three-dimensional space can be obtained [5] [6]:

$$\mathbf{r}' = \mathbf{r} - \frac{\mathbf{V_r}}{|\mathbf{V_r}|}\left((1-\gamma)\frac{\mathbf{r} \cdot \mathbf{V_r}}{|\mathbf{V_r}|} + |\mathbf{V_r}|\gamma t\right)$$

$$t' = \gamma\left(t - \frac{\mathbf{r} \cdot \mathbf{V_r}}{{C_0}^2}\right)$$

$$\text{Where, } \gamma = \frac{1}{\sqrt{1 - \frac{|\mathbf{V_r}|^2}{{C_0}^2}}}$$

Because imaginary speed V_ϕ and real electric potential ϕ are inter-convertible, and real velocity can be converted into imaginary electric potential vector, hence the quaternion velocity equations can be converted into another type of quaternion electric potential equations. The quaternion is composed of one real and three imaginary vectors. It is called B_i type quaternion. Through further analysis, both A_i type quaternion and B_i type quaternion are a type of biquaternion [7].

Let B_i type quaternion electric potential be ϕ_h, then:

$$\phi_h = \frac{V_q}{K} = (V_\phi i + V_X\mathbf{i} + V_Y\mathbf{j} + V_Z\mathbf{k})\frac{\Phi_0}{C_0}(-i) \tag{73}$$

Let $\phi = \frac{V_\phi \Phi_0}{C_0}, \phi_X = \frac{V_X\Phi_0}{C_0}, \phi_Y = \frac{V_Y\Phi_0}{C_0}, \phi_Z = \frac{V_Z\Phi_0}{C_0}$, then:

$$\phi_h = \phi + (\phi_X\mathbf{i} + \phi_Y\mathbf{j} + \phi_Z\mathbf{k})(-i) \tag{74}$$

This shows that B_i type quaternion potential ϕ_h is composed of one scalar electric potential and three imaginary components of the vector electric potential.

According to equation (73) from the definition of ϕ_h, we get:

$$\frac{V_q}{||V_q||} = i\frac{\phi_h}{||\phi_h||} \tag{75}$$

Because $||V_q|| = \frac{C_0}{\Phi_0}||\phi_h||$ (76)

Substituting equations(75),(76) into (69),(70),(71) ,the quaternion electric potential equations can be obtained for TQESTR:

$$R_q{}' = R_q - i\frac{\phi_h}{||\phi_h||}\left((1-\gamma)r_1 + \frac{C_0}{\Phi_0}||\phi_h||\gamma t\right) \tag{77}$$

$$t' = \gamma\left(t - \frac{||\phi_h||}{\Phi_0 C_0}r_1\right) \tag{78}$$

Where, $\gamma = \frac{1}{\sqrt{1-\frac{||\phi_h||^2}{\Phi_0{}^2}}}$, (79)

The above equation can be further simplified into theory of complex electric potential relativity, the theory of electric potential relativity (TEPR) and other forms.

Part 4. Discussion on the basic effects of TQESTR

Although TQESTR have various expression forms, but the forms can all be obtained through conversion and simplification of basic the equations (62), (63), (64), (65), (66) and (67). Therefore, they will be the main focus for the discussions of the basic effects of TQESTR. The effects are called electrodynamic space-time effect.

1. Superposition principle of TQESTR

By analyzing the above equations, one can see that the equations (66) and (63) have the same form as that of Lorentz transform equations (1) and (4). Therefore, the equations(66),(63) and (64) can also be written in the form of hyperbolic functions [5],

$$r_2 = r_1\cosh(\varphi) - C_0 t\sinh(\varphi) \tag{80}$$

$$C_0 t' = -r_1\sinh(\varphi) + C_0 t\cosh(\varphi) \tag{81}$$

Where, $\cosh(\varphi) = \frac{1}{\sqrt{1-(\frac{||V_q||}{C_0})^2}}$ (82)

Because, $\cosh^2(\varphi) - \sinh^2(\varphi) = 1$ (83)

From(82) and(83) it can be obtained: $\tanh(\varphi)=\frac{||V_q||}{C_0}$ (84)

Assume: $\varphi=\varphi_1+\varphi_2$, $\frac{||V_{q1}||}{C_0}=\tanh(\varphi_1)$ and $\frac{||V_{q2}||}{C_0}=\tanh(\varphi_2)$

$$\frac{||V_q||}{C_0}=\tanh(\varphi_1+\varphi_2) \tag{85}$$

According to equation(85), the formula for superposition of velocity of TQESTR can be obtained:

$$||V_q||=\frac{||V_{q2}||+||V_{q1}||}{1+\frac{||V_{q1}||\,||V_{q2}||}{C_0{}^2}} \tag{86}$$

Where, $||V_{q1}||=\sqrt{V_{\phi1}{}^2+V_{X1}{}^2+V_{Y1}{}^2+V_{Z1}{}^2}$ (87)

$$||V_{q2}||=\sqrt{V_{\phi2}{}^2+V_{X2}{}^2+V_{Y2}{}^2+V_{Z2}{}^2} \tag{88}$$

When the direction of motion of the reference frame is along X axis, and, V_{X1}, V_{X2} are positive real numbers. $||V_q||=V_X$, $||V_{q1}||=V_{X1}$, $||V_{q2}||=V_{X2}$,All other terms are zero. By substituting the above-mentioned formula of superposition, the formula for velocity superposition of special theory of relativity can be obtained:

$$V_X=\frac{V_{X2}+V_{X1}}{1+\frac{V_{X1}V_{X2}}{C_0{}^2}} \tag{89}$$

According to the equation (76), the quaternion velocity equations can be converted into quaternion electric potential equations. Substituting equation (76) into (86), The formula for superposition of quaternion electric potential of TQESTR can be obtained:

$$||\phi_h||=\frac{||\phi_{h2}||+||\phi_{h1}||}{1+\frac{||\phi_{h1}||\,||\phi_{h2}||}{\Phi_0{}^2}} \tag{90}$$

When the only the scalar electric potential is considered, and, $\phi_1, \phi_1$are positive real numbers. $||\phi_h||=\phi$, $||\phi_{h1}||=\phi_1$, $||\phi_{h2}||=\phi_1$, other terms are all zero. By substituting the above-mentioned formula, the formula for superposition of scalar electric potential can be obtained:

$$\phi=\frac{\phi_2+\phi_1}{1+\frac{\phi_1\phi_2}{\Phi_0{}^2}} \tag{91}$$

Thus a conclusion that differs from the modern physics is deduced: the superposition of electric potential is nonlinear. However, if the electric potential is far lower than the electric potential limit, the equation (91) will change back to linear equation of the current electromagnetics, that is $\phi=\phi_2+\phi_1$.

2. The time relationship of TQESTR

(1) Time effects

In any electrodynamic inertia frame of reference, time is isotropic. The time measured by the clock put in the real space of the inertial frame is the time of the electrodynamic inertial frame. If two events happen at two different moments t_1 and t_2 at the same location r_1 of the electrodynamic stationary inertial reference frame, their time difference is $\Delta t=t_2-t_1$; for the moments $t_1{}'$ and $t_2{}'$ corresponding to the electrodynamic relative inertial reference frame, the time difference is $\Delta t'=t_2{}'-t_1{}'$. From equations(63), (64) and (61), the equation of velocity-time effect of TQESTR is:

$$\Delta t'=\frac{1}{\sqrt{1-\frac{V_\phi{}^2+V_X{}^2+V_Y{}^2+V_Z{}^2}{C_0{}^2}}}\Delta t \tag{92}$$

It shows that the expansion effect of the time is not just about speed, but with potential.

The expression for the electric potential-time effect of TQESTR is:

$$\Delta t' = \frac{1}{\sqrt{1-\frac{\phi^2+\phi_X{}^2+\phi_Y{}^2+\phi_Z{}^2}{\Phi_0{}^2}}}\Delta t \tag{93}$$

In the reference frame of complex electrodynamic inertia, i.e., there are only electric potential ϕ and one dimensional speed V_X, from $V_\phi = \frac{C_0}{\Phi_0}\phi$, it can be obtained:

$$\Delta t' = \frac{1}{\sqrt{1-\frac{V_X{}^2}{C_0{}^2}-\frac{\phi^2}{\Phi_0{}^2}}}\Delta t \tag{94}$$

In(94), when $\phi = 0$, the formula for time expansion of special relativity is obtained:

$$\Delta t' = \frac{1}{\sqrt{1-\frac{V_X{}^2}{C_0{}^2}}}\Delta t \tag{95}$$

In(94),$V_X = 0$, the formula for the time expansion effect of electric potential relativity theory is obtained:

$$\Delta t' = \frac{1}{\sqrt{1-\frac{\phi^2}{\Phi_0{}^2}}}\Delta t \tag{96}$$

This is a new important physical effect, called it the potential time expansion effect.

(2)The system time

The system time t_q is a new concept in TQESTR. It unified the observe reference time t and the observed reference time t' together. And let basic equations of TQESTR become very concise form.

From equations (67), (66) and(63), it can be obtained:

$$t_q = \left((1-\gamma)\frac{C_0{}^2}{||V_q||^2}+\gamma\right)t - \left(\frac{1}{\gamma}-1\right)\frac{C_0{}^2}{||V_q||^2}t' \tag{97}$$

3. Space change effect of TQESTR

In the stationary electrodynamic reference frame Σ_Q,event A $(F_1, X_1, Y_1, Z_1, t_1)$ and event B $(F_2, X_2, Y_2, Z_2, t_2)$ occur, the corresponding events in the moving electrodynamic frames of reference $\Sigma_Q{}'$ are event A′ $(F_1{}', X_1{}', Y_1{}', Z_1{}', t_1{}')$ and event B′ $(F_2{}', X_2{}', Y_2{}', Z_2{}', t_2{}')$, V_q is the relative quaternion velocity of two reference frames. Therefore from equation(62), the expressions of the space coordinates of the two events are:

$R_{q1}{}' = R_{q1} - V_q t_{q1}$

$R_{q2}{}' = R_{q2} - V_q t_{q2}$

So there is,$\Delta R_q{}' = \Delta R_q - V_q \Delta t_q$ (98)

Where, $\Delta R_q = R_{q2} - R_{q1}$, $\Delta R_q{}' = R_{q2}{}' - R_{q1}{}'$

From equation (97),$t_{q1} = f(t_1, t_1{}')$ and $t_{q2} = f(t_2, t_2{}')$ can be obtained,

$$\Delta t_q = \left((1-\gamma)\frac{C_0{}^2}{||V_q||^2}+\gamma\right)\Delta t - \left(\frac{1}{\gamma}-1\right)\frac{C_0{}^2}{||V_q||^2}\Delta t' \tag{99}$$

Where, $\Delta t_q = t_{q2} - t_{q1}, \Delta t = t_2 - t_1, \Delta t' = t_2{}' - t_1{}'$

From equation (63), $\Delta t' = \gamma\left(\Delta t - \frac{||V_q||}{C_0{}^2}\Delta r_1\right)$ (100)

In the stationary electrodynamic frame of reference, the time can be different when reading the space parameters of the event A and event B. However, in the moving electrodynamic frames of reference, the reading of event A' and event B' must be completed at the same time. Therefore, $\Delta t' = 0$. Substituting $\Delta t' = 0$ into equations (99) and (100) obtains:

$$\Delta t_q = \frac{\left(1-\frac{1}{\gamma}\right)\Delta r_1}{||V_q||} \tag{101}$$

By substituting(101) into(98), the equation of space-velocity effect of TQESTR can be obtained：

$$\Delta R_q{}' = \Delta R_q - \frac{V_q}{||V_q||}\left(1-\frac{1}{\gamma}\right)\Delta r_1 \tag{102}$$

Where $\Delta r_1 = \Delta r\cos\theta + \Delta F\sin\theta$ (103)

Equation (102) is expanded into the equations of imaginary and vector equations of real number：

$$\Delta F'i = \Delta Fi - i\frac{V_\phi}{||V_q||}\left(1-\frac{1}{\gamma}\right)\Delta r_1 \tag{104}$$

$$\Delta \mathbf{r}' = \Delta \mathbf{r} - \frac{\mathbf{V_r}}{||V_q||}\left(1-\frac{1}{\gamma}\right)\Delta r_1 \tag{105}$$

When $V_\phi \neq 0, V_r = 0, \theta = \frac{\pi}{2}, \Delta r_1 = \Delta F$, there is：

$$\Delta \mathbf{r}' = \Delta \mathbf{r} \tag{106}$$

That is, in any space of relative stationary static electric potential, the real space length is not related to the scalar electric potential. It can be called the effect of electric potential space unchanged.

When $V_\phi = 0, \mathbf{V_r} \neq 0, ||V_q|| = V_r, \theta = 0, \Delta r_1 = \Delta r$

$$\Delta r' = \frac{1}{\gamma}\Delta r \tag{107}$$

This is the formula for length contraction of the special theory of relativity.

Substituting equations (75) and (76) into (102), the equation of electric potential-space effect can be obtained. At same time, it can be also simplified under certain conditions.

4. Predictions and verifications of the theory

The above derivations indicate that in the theory of electrodynamic space-time relativity, time and space not only are related to the three-dimensional velocity but also related to electric potential. In principle, different experiments can be designed to verify the theory. Amongst them, the effect of electric potential time expansion can be more easily tested.

Equation (96) shows that under completely stationary conditions, when there is sufficiently high electric potential difference between two reference frames, their time difference will also be found. If the potential is higher, this effect will be more obvious. Therefore, an experiment may be designed: after calibration, two clocks of high precision T_c and T_d are put separately in two identical metallic closed rooms C and D, which are isolated and motionless relative to each other. Room C is grounded and let its electric potential be zero. An electric potential generator of super high voltage is used to charge room D and maintain the super high electric potential ϕ comparing to room C. Equation (96) indicates that given long enough time, after electricity of room D is totally discharged, the electric potential in room D becomes zero again. Then two clocks T_c and T_d are put together to compare their respective reading of the elapsed time, Δt and $\Delta t'$. It can be found that there is a time difference between time Δt and $\Delta t'$ recorded by T_c and T_d that is induced by electric potential ϕ, and $\Delta t < \Delta t'$. Comparing the measured values and the values calculated by equation (96) will verify the theory. If the experiment proves that the theoretic calculation is correct, then from the experiment data, the magnitude of the electric potential limit Φ_0 can be obtained by the equation:

$$\Phi_0 = \frac{\phi}{\sqrt{1-\frac{\Delta t^2}{\Delta t'^2}}} \tag{108}$$

It would be extremely difficult to achieve very high electric potential ϕ in laboratory. However, the advantage of this experimental setup is that through adding the experimental time and improving the accuracy of the time measurement, it lowers the required the electric potential and increases the possibility of success in the experiment. Base on the theory of electrodyanmic space-time relativity, many new physical effects can be derived, which in turn, can prove the validity of the theory through experiments based on the said effects.

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