. Research on the Riemann Hypothesis

. Wenlong Du

Abstract: This paper studies the relationship between the prime divisor and Stirling's approximation. We get prime number theorem and its corrected value. We get bound for the error of the prime number theorem. Riemann hypothesis is established. Researchon the Riemann Hypothesis

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Key words:the Riemann hypothesis, prime number, error of the prime number theorem, prime divisor

1. INTRODUCTION

In mathematics, the Riemann hypothesis, proposed by Bernhard Riemann (1859), is a conjecture that the non-trivial zeros of the Riemann zeta function all have real part 1/2.

The Riemann zeta function is defined for complex *s* with real part greater than 1 by the absolutely convergent infinite series^[1]

$$
\zeta(s) = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \dots = \sum_{n=1}^{\infty} \frac{1}{n^s}
$$

The Riemann hypothesis is one of the most important conjectures in mathematics^[2]. Hilbert listed the Riemann Hypothesis as one of his 23 problems for mathematicians of the twentieth century to work on^[3].

Von Koch (1901) showed that the Riemann hypothesis is equivalent to $[4]$:

$$
\pi(x) = Lix + O\left(\sqrt{x}\log x\right)
$$

2. RESEARCH ON THE PRIME NUMBER THEOREM

If P is a prime number, the number of prime divisor P which is contained by the natural number is less than N is: conjecture that
the absolutely
¹. Hilbert listed
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atural number
 $\left[\frac{N}{P}\right]$.
 $\left[\frac{N}{P^2}\right]$. conjecture that
the absolutely
¹. Hilbert listed
ieth century to
atural number
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th century to
 $\frac{V}{P^2}$ $\left[\frac{N}{P^3}\right]$.
 $\left[\frac{N}{P^3}\right]$. ^{2]}. Hilbert listed
tieth century to
natural number
 $\left[\frac{N}{P}\right]$.
 $\left[\frac{N}{P^2}\right]$.
is $\left[\frac{N}{P^3}\right]$.

.

The number of natural number which contains more than one prime divisor $\left| \frac{N}{P} \right|$. *P*

The number of natural number which contains more than two prime divisors P is $\left[\frac{N}{P^2}\right]$ P^2 The Riemann zeta function is defined for complex *s* with real part greater than 1 by the absolutely
uvergent infinite series⁽¹⁾
 $(s) = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^3} + \dots = \sum_{n=1}^{\infty} \frac{1}{n^3}$

The Riemann hypothesis is on P^3 $\int \frac{1}{V} = \frac{1}{V} + \frac{1}{2V} + \frac{1}{3V} + \dots = \sum_{n=1}^{\infty} \frac{1}{n^k}$

Eximinan hypothesis is so so of the the most important conjectures in mathematics¹²¹. Hilbert listed

Eximenn Hypothesis is sone of his 23 problems for mat TheNotional Hypologicals is one of this 23 problems for multimatricians of the two-first contents.
 N **Rivernan Hypologicals is one of this 23 problems for multimatricians of the twentieth century to the one¹¹.
** *N***O** 901) showed that the Riemann hypothesis is equivalent to^[4]
 $O(\sqrt{x} \log x)$
 $O(\sqrt{x} \log x)$
 ION THE PRIME NUMBER THEOREM

number, the number of prime divisor P which is contained by the natural number

is:

of natural nu

The number of natural number which contains more than m prime divisors $\left| \frac{N}{P^m} \right|$. *P*^{*m*} $\left| \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \right|$

$$
N \ge P^m, \quad N \le P^{m+1}
$$

So the number of prime divisors P which is contained by the natural number less than N is:

So the number of prime divisors *P* which is contained by the natural number less than *N* is:
\n
$$
\left[\frac{N}{P}\right] + \left[\frac{N}{P^2}\right] + \left[\frac{N}{P^3}\right] + \dots + \left[\frac{N}{P^m}\right]
$$
\n
$$
\approx N \frac{\frac{1}{P}\left[1-\left(\frac{1}{P}\right)^n\right]}{1-\frac{1}{P}}
$$
\n
$$
= N \frac{1}{P-1}
$$
\n
$$
\approx \frac{N-1}{P-1}
$$
\nWe assume that the primes less than N is:
\n $P_1, P_2, \dots, P_m, P_1 < P_2 < \dots, < P_m$
\nWe assume that the number of prime factor *P* which is contained^[5] by the natural number
\nless than *N* is α .
\nso $N! = P_1^{\alpha_1} P_2^{\alpha_2} \dots P_m^{\alpha_m}$
\nWe can get $N! \approx P_1^{\frac{N}{2} + \frac{N}{2} + \frac{N}{2} + \frac{N}{2} + \dots + \frac{$

We assume that the primes less than N is:

 $P = 1$

e assume that the primes less than N is:
 P_2, \ldots, P_m , $P_1 < P_2 < \ldots, < P_m$.

We assume that the number of prime factor P which is contained^[5] by the natural number

than N is α .

so $N! = P_1^{\alpha_1} P_2^{\alpha_2}$ 1 2 3 1 primes less than N is:
 $P_2 < \dots < P_m$

the number of prime factor P which is contained^[5] by the natural number
 $\dots P_m^{\alpha_m}$
 $\dots P_n^{\alpha_m}$
 $\approx P_1^{\frac{N-1}{R-1}} P_2^{\frac{N-1}{2-2}} P_3^{\frac{N-1}{2-3}} \dots P_m^{\frac{N-1}{m}}$, because of $\alpha \$ r of prime factor P which is contained^[5] by the natural number
 $\frac{1}{2} P_3^{\frac{N-1}{2}}$
 $\frac{1}{2} P_3^{\frac{N-1}{2}}$ $P_m^{\frac{N-1}{2}}$, because of $\alpha \approx \frac{N-1}{P-1}$.

(*n*)
 $\frac{1}{2} P_2^{\frac{N-1}{2}} P_3^{\frac{N-1}{2}}$, P

less than *N* is α .

$$
{}_{\rm SO}N!=P_1^{\alpha_1}P_2^{\alpha_2} \dots P_m^{\alpha_m}
$$

We can get
$$
N! \approx P_1^{\frac{N-1}{P_1-1}} P_2^{\frac{N-1}{P_2-2}} P_3^{\frac{N-1}{P_3-3}} \dots P_m^{\frac{N-1}{P_m-1}}
$$
, because of $\alpha \approx \frac{N-1}{P-1}$.

$$
= N \frac{N-1}{P-1}
$$

\n
$$
\approx \frac{N-1}{P-1}
$$

\nWe assume that the primes less than N is:
\n $P_2, \dots, P_m, P_1 < P_2 < \dots < P_m$
\nWe assume that the number of prime factor P which is contained^[5] by the natural number
\n P_1 when N is α .
\nSo $N! = P_1^{a_1} P_2^{a_2} \dots P_m^{a_m}$
\nWe can get
$$
N! = P_1^{a_1} P_2^{a_2} P_2^{a_3} \dots P_m^{a_m}
$$

\nHence $N! = P_1^{a_1} P_2^{a_2} P_2^{a_3} \dots P_m^{a_m}$, because of $\alpha \approx \frac{N-1}{P-1}$.
\nPrime number density is $\rho(n)$
\n
$$
P_1^{a_1} P_2^{a_2} P_2^{a_3} \dots P_m^{a_m}
$$

\n
$$
\approx 2^{n(2)\frac{N-1}{2-1}} 3^{n(3)\frac{N-1}{3}} 4^{n(4)\frac{N-1}{4-1}} \dots N^{n(N)\frac{N-1}{N-1}}
$$

\n
$$
= \prod_{n=2}^{N} n^{p(n)\frac{N-1}{n-1}} = N!
$$

\n(1)
$$
\prod_{n=2}^{N} n^{p(n)\frac{N-1}{n-1}} = N!
$$

(2)
$$
\prod_{n=2}^{N+1} n^{\rho(n)} \frac{1}{n-1} = [(N+1)!]^{\frac{1}{N}}
$$

 $(2) \div (1)$

$$
(N+1)^{o(N+1)\frac{1}{N}} = \frac{[(N+1)!]^{\frac{1}{N}}}{[N!]^{\frac{1}{N-1}}} = [N!]^{\frac{1}{N} \cdot \frac{1}{N-1}} (N+1)^{\frac{1}{N}}
$$

\n
$$
\frac{\rho(N+1)}{N} \ln(N+1) = (\frac{1}{N} - \frac{1}{N-1}) \ln(N!) + \frac{1}{N} \ln(N+1)
$$

\n
$$
\rho(N+1) = \frac{N}{\ln(N+1)} \left[(\frac{1}{N} - \frac{1}{N-1}) \ln(N!) + \frac{1}{N} \ln(N+1) \right]
$$

\n
$$
= \frac{N}{\ln(N+1)} \left[\frac{-1}{N(N-1)} (\ln \sqrt{2\pi N} + N \ln N - N + \frac{\theta}{12N}) + \frac{1}{N} \ln(N+1) \right]
$$

\n
$$
= \frac{N}{\ln(N+1)} \left\{ \left[-\frac{\ln N}{N-1} + \frac{1}{N} \ln(N+1) \right] - \frac{\ln \sqrt{2\pi N} + \frac{\theta}{12N}}{N(N-1)} + \frac{1}{N-1} \right\} (3)
$$

\n
$$
- \frac{\ln N}{N-1} + \frac{1}{N} \ln(N+1)
$$

\n
$$
= \frac{\ln(N+1)^{N-1} - \ln N^N}{N(N-1)}
$$

\n
$$
= \frac{\ln \frac{(N+1)^{N-1}}{N(N-1)}}{N(N-1)}
$$

\n
$$
= \frac{\ln \left[\left(\frac{N+1}{N} \right)^N \frac{1}{N+1} \right]}{N(N-1)}
$$

\n
$$
= \frac{1 - \ln(N+1)}{N(N-1)}
$$

\n
$$
< \frac{1}{N-1}
$$

$$
\frac{\ln\sqrt{2\pi N} + \frac{\theta}{12N}}{N(N-1)} << \frac{1}{N-1}
$$

(3) $\approx \frac{N}{\ln(N+1)} \frac{1}{N-1} \approx \frac{1}{\ln(N+1)}$

$$
\rho(N+1) \approx \frac{1}{\ln(N+1)} \rho(N) \approx \frac{1}{\ln N}
$$

3. DICUSSION ON THE NUMBER OF PRIME NUMBERS' EXPECTED VALUE

 $(N+1) \approx \frac{1}{\ln(N+1)}$ $\rho(N) \approx \frac{1}{\ln N}$
USSION ON THE NUMBER OF PRIME NUMBERS' EXPECTED VALUE
ume that the Prime number density is $\frac{1}{\ln N}$ The product of all prime factors is $\frac{1}{\ln(N+1)}$, $\rho(N) \approx \frac{1}{\ln N}$.

N THE NUMBER OF PRIME NUMBERS' EXPECTED VALUE

Prime number density is $\frac{1}{\ln N}$. The product of all prime factors is $\rho(N+1) \approx \frac{1}{\ln(N+1)}$ $\rho(N) \approx \frac{1}{\ln N}$
CUSSION ON THE NUMBER OF PRIME NUMBERS' EXPECTED VALUE
ssume that the Prime number density is $\frac{1}{\ln N}$ The product of all prime factors is $(N) \approx \frac{1}{\ln N}$
 EER OF PRIME NUMBERS' EXPECTED VALUE

density is $\frac{1}{\ln N}$ The product of all prime factors is We assume that the Prime number density is $\frac{1}{1}$ The product of all prime factors is

$$
s_0 \t N + 1) \approx \frac{1}{\ln(N+1)} \t P(N) \approx \frac{1}{\ln N}
$$

\n3. DICUSION ON THE NUMBER OF PRUNE NUMBERS' EXPECTED VALUE
\nWe assume that the Prime number density is $\frac{1}{\ln N}$. The product of all prime factors is
\n
$$
P_1^{\frac{N-1}{2}} P_2^{\frac{N-1}{2}} P_2^{\frac{N-1}{2}} ... P_m^{\frac{N-1}{2}}
$$
\n
$$
\approx \frac{\lambda^{N-1}}{2^{m(1-1)}} \t \frac{\lambda^{N-1}}{n^{m(1-1)}} \t \frac{\lambda^{N-1}}{n^{m(1-1)}} \t \frac{\lambda^{N-1}}{n^{N-1}}
$$
\n
$$
= \prod_{n=2}^{N} \frac{\lambda^{N-1}}{n^{m(n-1)}}
$$
\n
$$
= \prod_{n=2}^{N} \frac{\lambda^{N-1}}{n^{m(n-1)}}
$$
\n
$$
= \left[\frac{\lambda^{N-1}}{n} \right]^{\lambda^{N-1}}
$$
\nBecause of $\left(\frac{N}{N-1} \right)^{N-1} \approx e$, we get $\left(\frac{N-1}{M-1} \right)^{N-1} = \frac{N^{N-1}}{e(m-1)^{N-1}}$.
\nBecause of $\left(\frac{N}{N-1} \right)^{N-1} \approx e$, we get $\left(\frac{N-1}{M-1} \right)^{N-1} = \frac{e^{\frac{N^{N-1}}{2}}}{(e^{\frac{N^{N-1}}{2}} - \sqrt{2\pi N^2}} \t \text{ is small. We can ignore the error.}$
\nThe product of all prime divisor P is $P^{\frac{N-1}{N-1}}$. What is a local value is less than $P^{\frac{N-1}{N-1}}$. When $P > \sqrt{N}$, the product is about $P^{\frac{N-1}{N-1}}$. When $\sqrt{N} > P > \frac{1}{\sqrt{N}}$, the product is about $P^{\frac{N-1}{N-1}}$.
\nThe decrease of prime divisor product is about $e^N e^{\frac{\sqrt{N}}{N}} e^{\frac{1}{N}} \sqrt{e^{\frac{1}{N}} \sqrt{w}}$, the product is about number is
\n<math display="</math>

 $\frac{12N}{2N-1} (m-1)^{N-1} \sqrt{2\pi N^2}$ \overline{N} (*m* - 1)^{*N*-1} - ³ N^2 θ πN^2 $\frac{(-1)^{N-1}}{-1} \sqrt{2\pi N^{\frac{3}{2}}}$

 e^{N-1} $\qquad \qquad$ is small. We is small.We can ignore the error.

The product of all prime divisor P is P^{P-1} , but The actual value is less than P^{P-1} . 1 $1 - 1$ 1.1 The actual value is *N* P^{P-1} , but The actual value is less than P^{P-1} . When -1 $^{-1}$, but The actual value is less than P^{P-1} . When 1 1 When *N* P^{P-1} . When -1 $^{-1}$. When $\left(\frac{\sqrt{1} - 1}{1} \right)^{N-1} = \frac{N^{N-1}}{e(m-1)^{N-1}}$

th is caused by $\frac{e^{\frac{\theta}{12N}} (m-1)^{N-1}}{e^{N-1}} \sqrt{2\pi N^{\frac{3}{2}}}$ is small. We
 $\frac{N-1}{e^{N-1}}$, but The actual value is less than $P^{\frac{N-1}{p-1}}$. When $\sqrt{N} > P > \frac{\frac{1}{3}N}{\$ $\left(\frac{N-1}{M-1}\right)^{N-1} = \frac{N^{N-1}}{e(m-1)^{N-1}}$
 $\frac{e^{\frac{\theta}{12N}}(m-1)^{N-1}}{e^{N-1}} \sqrt{2\pi N^{\frac{3}{2}}}$ is small. We
 $\frac{N-1}{P^{P-1}}$, but The actual value is less than $P^{\frac{N-1}{P-1}}$. When $\sqrt{N} > P > \frac{1}{\sqrt[3]{N}}$, the product is a $N-1$ 1 1 P^{P-1} ² When $\sqrt{N} > P > \sqrt[3]{N}$ the product is a $\frac{1}{\sqrt{1}}$ When $\sqrt{N} > P > \frac{1}{\sqrt[3]{N}}$ the product is ab ^{$\frac{\delta}{2N}$ $\frac{1}{\left(\frac{2N}{N}\right)^{N-1}}$, $\frac{1}{\sqrt{2\pi N^2}}$ is small. We
actual value is less than $P^{\frac{N-1}{P-1}}$. When
 $\frac{1}{N} > P > \frac{1}{\sqrt[3]{N}}$, the product is about
 $\frac{1}{\sqrt[3]{N}} \frac{1}{e^{\sqrt[3]{N}}}$, The decrease of Prime} $\frac{1}{1}$ -1 $N-1$, P^{P-1} …… $= \frac{N^{N-1}}{e(m-1)^{N-1}}$

And by $\frac{e^{\frac{\beta}{12N}}(m-1)^{N-1}}{e^{N-1}}\sqrt{2\pi N^{\frac{3}{2}}}$ is small. We

but The actual value is less than $P^{\frac{N-1}{P-1}}$. When

then $\sqrt{N} > P > \frac{1}{\sqrt[3]{N}}$, the product is about
 $e^N e^{\sqrt{N}} e^{\frac{1}{$ $P^{\frac{N-1}{P-1}}$, but The actual value is less than $P^{\frac{N-1}{P-1}}$. When
 $\frac{1}{2}$. When $\sqrt{N} > P > \frac{1}{\sqrt[3]{N}}$, the product is about

about $e^N e^{\sqrt{N}} e^{\frac{1}{\sqrt[3]{N}}} e^{\frac{1}{\sqrt[3]{N}}}$ The decrease of Prime
 $\frac{1}{\sqrt[$ $P^{\frac{N-1}{P-1}}$, but The actual value is less than $P^{\frac{N-1}{P-1}}$. When
 $\sqrt{N} > P > \frac{1}{\sqrt[3]{N}}$, the product is about

about $e^N e^{\sqrt{N}} e^{\frac{1}{\sqrt[3]{N}}} e^{\frac{1}{\sqrt[3]{N}}}$ The decrease of Prime
 $\frac{1}{\sqrt[3]{n}}$ and $\ln \frac{$ $=\left(\frac{N-1}{M-1}\right)^{N-1}$ $\approx e$ $\approx e$ $\approx e \text{ where } \sqrt{M-1}\bigg)^{N-1} = \frac{N^{N-1}}{e(m-1)^{N-1}}$ $\frac{e^{\frac{\theta}{12N}}(m-1)^{N-1}}{e^{N-1}} \sqrt{2\pi N^{\frac{3}{2}}}$ is small. We

divisor P is $\frac{\sum_{i=1}^{N-1} p_i}{P^{N-1}}$, but The actual value is less than $x^{k+1} \approx e$
 $x^k e^{i\omega t} \propto \text{erf}(\frac{N-1}{M-1})^{N-1} = \frac{N^{N-1}}{e(m-1)^{N-1}}$

There theorem which is caused by $\frac{e^{i2N}(m-1)^{N-1}}{e^{N-1}} \sqrt{2\pi} N^{\frac{3}{2}}$ is small. We

time divisor P is P^{k-1} , but The actual value is les $\left(\frac{N-1}{M-1}\right)^{N-1} = \frac{N^{N-1}}{e(m-1)^{N-1}}$
 de get $\left(\frac{N-1}{M-1}\right)^{N-1} = \frac{e^{\frac{\theta}{(2N)}}(m-1)^{N-1}}{e^{N-1}} \sqrt{2\pi N^2}$ is small. We

or *P* is $P^{\frac{N-1}{P-1}}$, but The actual value is less than $P^{\frac{N-1}{P-1}}$. When
 $\frac{$ = $\left(\frac{N-1}{M-1}\right)^{N-1}$
 $\approx e_{\text{av}} \left(\frac{N-1}{M-1}\right)^{N-1}$
 $\text{er theorem which is caused by } \frac{e^{\frac{NN-1}{k}}(m-1)^{N-1}}{e^{N-1}} \sqrt{2\pi N^{\frac{3}{2}}}$
 $\text{if a similar. We
\ndivisor } P \text{ is } P^{\frac{N-1}{k-1}}$, but The actual value is less than $P^{\frac{N-1}{k-1}}$. When
 $\text{is about } P$ $\left(\frac{1}{n-1}\right)^{N-1} = \frac{N^{N-1}}{e(m-1)^{N-1}}$
 n is caused by $\frac{e^{\frac{\beta}{12N}}(m-1)^{N-1}}{e^{N-1}}\sqrt{2\pi N}^{\frac{3}{2}}$ is small. We
 $P^{\frac{N-1}{P-1}}$, but The actual value is less than $P^{\frac{N-1}{P-1}}$. When
 $\frac{1}{e^{N}}$, When $\sqrt{N$

The decrease of prime divisor product is about $e^N e^{\sqrt{N}} e^{\frac{1}{\sqrt[3]{N}}} e^{\frac{1}{\sqrt[4]{N}}}$ The decrease of Prime number is

$$
\int \frac{1}{2\sqrt{n} \ln n} dn + \int \frac{1}{2\sqrt[3]{n} \ln n} dn + \int \frac{1}{2\sqrt[3]{n} \ln n} dn + \dots
$$

Prime number which is corrected is closer to the actual value.

4. DISCUSS ON THE ERROR OF THE PRIME NUMBER THEOREM

The product of all prime factors P is P^{P-1} . The maximum error is N We assume that 1 and 1 1 The maximum error is 1 *N* P^{P-1} . The maximum error is N We assume that the -1 ⁻¹ The maximum error is N . We assume that the The product of all prime factors P is P^{N-1} . The maximum error is N . We assume that the
primes is $P_1, P_2, P_3,.....,P_l$. The remainder that N is divided by $P_1, P_2, P_3,.....,P_l$ is
 $\delta_1, \delta_2, \delta_3,....., \delta_l$. The value ran tors *P* is $P^{\frac{N-1}{P-1}}$. The maximum error is *N* We assume that the

. The remainder that *N* is divided by $P_1, P_2, P_3,.....,P_l$ is

value range of δ_1 is $(0, P_1 - 1)$. The value range of
 $N = P_1 P_2 P_3.....,P_l$, $\delta_1, \$ is The product of all prime factors P is P^{N-1} . The maximum error is N We assume that the
primes is $P_1, P_2, P_3, \dots, P_l$. The remainder that N is divided by $P_1, P_2, P_3, \dots, P_l$ is
 $\delta_1, \delta_2, \delta_3, \dots, \delta_l$. The value r The product of all prime factors P is $P^{\frac{N-1}{P-1}}$. The maximum error is N we assume that the
primes is $P_1, P_2, P_3, \dots, P_t$. The remainder that N is divided by $P_1, P_2, P_3, \dots, P_t$ is
 $\delta_1, \delta_2, \delta_3, \dots, \delta_t$. The

is product of all prime factors P is $P^{\frac{N-1}{P-1}}$. The maximum error is N . We assume that the
mes is $P_1, P_2, P_3, \dots, P_l$. The remainder that N is divided by $P_1, P_2, P_3, \dots, P_l$ is
 S_2, S_3, \dots, S_l . The value range The maximum error is N We assume that the
 N is divided by $P_1, P_2, P_3,.....P_l$ is
 $s (0, P_1 - 1)$. The value range of
 $\delta_1, \delta_2, \delta_3,.....\delta_l$ contain all different values, and
 n value, $\delta_1, \delta_2, \delta_3,.....\delta_l$ is a fixed va The product of all prime factors P is P^{x-1} . The maximum error is N We assume that the
primes is $P_1, P_2, P_3,.....P_t$. The remainder that N is divided by $P_1, P_2, P_3,.....P_t$ is
 $\delta_1, \delta_2, \delta_3,.....\delta_t$. The value range o The product of all prime factors P is P^{R-1} . The maximum error is N , We assume that the primes is $P_1, P_2, P_3, \dots, P_l$. The remainder that N is divided by $P_1, P_2, P_3, \dots, P_l$ is $\delta_1, \delta_2, \delta_3, \dots, \delta_l$. The value

than *N*, δ is random. When *P* is not much less than *N*, δ is not random. With the increase of N, $\delta^*(P^* \ll m)$ increases and decreases periodically. So location of P is not random. The duct of all prime factors P is $P^{\frac{N-1}{N-1}}$. The maximum error is N . We assume that the
 $P_1, P_2, P_3, \ldots, P_t$. The remainder that N is divided by $P_1, P_2, P_3, \ldots, P_t$ is
 $\delta_3, \ldots, \delta_t$. The value range of δ_1 error of the prime number theorem will be reduced.

The changes of Prime's location change the number of prime numbers. The product of all prime factors *P* is $P^{\frac{N-1}{P-1}}$. When *P* is increased, the product is reduced, so w ⁻¹. When P is increased, the product is reduced, so we need more prime. When

P is reduced, the product is increased, so we need to reduce the number of prime. δ increases and decreases periodically. Prime does not change the position more than two times or less than 1/2. Therefore, the number of prime numbers will not change more than two times or less than $1/2$. So it can't bring magnitude changes.

We analyze the random error. The maximum error is N . We can assume that the probability that error is N is 0.5 and the probability that error is 0 is 0.5. Mean square error of the number of prime

numbers is about $\frac{N^{\frac{1}{2}}}{2(\ln N)^{\frac{1}{2}}}$. The maximum number of primes' error is about $\frac{N^{\frac{1}{2}}}{\sqrt{2}}$. The probability 2 ralue of $\delta_1, \delta_2, \delta_3,.....\delta_l$ change, and tend to disorder. When P is much less
ndom.When P is not much less than N, δ is not random. With the increase
n) increases and decreases periodically. So location of P is not $N^{\overline{2}}$ N^2 . The probability that error is greater than $3N^{\frac{1}{2}}$ tends to 0. The level of the actual value is not more than $\frac{N^{\frac{1}{2}}}{\sqrt{2}}$. 22 N^2 . *x* \sim *x* \sim *x* necesses and decreases periodically. So location of *F* is not random. The
or of the prime umber theorem will be reduced.
The changes of Prime's location change the number of prime numbers. The produ

Von Koch (1901) showed that the Riemann hypothesis is equivalent to:

1

$$
\pi(x) = Lix + O\left(\sqrt{x}\log x\right)
$$

So Riemann hypothesis is established.

5. SUMMARY

This paper analyses error of the prime number theorem. Riemann hypothesis is established. Through the further analysis, we may obtained the error function. This method is effective on the twin prime conjecture, Goldbach's conjecture and Mersenne Primes conjecture.

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