A modified circle-cutting strategy for conceptualizing n! and its application to derive yet another well-known mathematical result: the approximate sum of a convergent series involving factorials equals unity

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Abstract: n! is defined as the product 1.2.3………n and it popularly represents the number of ways of seating n people on n chairs. In a previous paper we conceptualized a new way of describing n!, using sequential cuts to an imaginary circle and derived a well known result. In this paper we use the same intuitive approach but reverse the cutting strategy by starting with n-cuts to the circle. We observe that this method leads us to estimate the approximate sum of an infinite convergent series involving factorials as unity.

Results:

n! is defined as the product of n consecutive positive integers from 1 to n, each considered exactly once. It has been popularly described as the number of ways in which n people can be seated in n chairs (1).

We use yet another way to visualize the factorial function (modification of the method in reference 2).

n!=(n)(n-1)(n-2)…………..4.3.2.1=(n)(n-1)(n-2)…………..4.3.2

We can consider cutting a circle into n! pieces as a sequential step wise process.

Let us consider an intact circle.

In the first step let us cut it at n positions. This will give rise to n pieces.
(In this step we have used n cuts and end up with n arcs or pieces of the circle. Note that until this step we have used a total of n cuts).

In second step let us cut each of the n pieces/arcs from the first step at {(n-1)-1}=(n-2) positions. In the second step we use n(n-2)cuts and we end up with n(n-1) pieces. Until now we have used n+n(n-2) cuts to end up with n(n-1) pieces or arcs.

In the third step let us cut each n(n-1) pieces from the second step at {(n-2)-1}=(n-3) positions. In the third step we use n(n-1)(n-3) cuts and end up with n(n-1)(n-2) pieces. Until now we have used n+n(n-2)+ n(n-1)(n-3) cuts to end up with n(n-1)(n-2) pieces or arcs.

In the fourth step let us cut each of the n(n-1)(n-2) pieces from the third step at {(n-3)-1}=(n-4) positions. In the fourth step we use n(n-1)(n-2)(n-4) cuts and we end up with n(n-1)(n-2)(n-3)pieces. Until now we have used n+n(n-2)+ n(n-1)(n-3) + n(n-1)(n-2)(n-4) cuts and end up with n(n-1)(n-2)(n-3) pieces or arcs.

Proceeding in this manner……..

In the $(n-1)^{th}$ step we will cut each of the n(n-1)(n-2)….5.4.3 pieces from previous step at {{n-(n-2)}-1} =(2-1)=1 position. Thus in the $(n-1)^{th}$ step we use n(n-1)(n-2)….5.4.3 .1 cuts and we end up with n(n-1)(n-2)….5.4.3.2 pieces. Until now we have used a total of

n+n(n-2)+ n(n-1)(n-3) + n(n-1)(n-2)(n-4) +…….. +n(n-1)(n-2)….5.4.2+ n(n-1)(n-2)….5.4.3 .1 cuts to end up with n(n-1)(n-2)….5.4.3.2 =n! pieces or arcs.

Since a total of x-cuts to circle would result in x-pieces or arcs therefore

n+n(n-2)+ n(n-1)(n-3) + n(n-1)(n-2)(n-4) +……………..... +{n(n-1)(n-2)…….5.4.2}+ {n(n-1)(n-2)….5.4.3 .1}

= n!

or

n+n![{(n-2)/(n-1)!}+ {(n-3)/(n-2)!}+ {(n-4)/(n-3)!}+...............{4/5!}+{3/4!} +{2/(3!)}+{1/(2!)}]

= n!

dividing both sides by n! we obtain

{1/(n-1)!}+ [{(n-2)/(n-1)!}+ {(n-3)/(n-2)!}+ {(n-4)/(n-3)!}+...............{4/5!}+{3/4!} +{2/(3!)}+{1/(2!)}]

=1

As n becomes large and approaches ∞, {1/(n-1)!} approaches 0.

Then [{(n-2)/(n-1)!}+ {(n-3)/(n-2)!}+ {(n-4)/(n-3)!}+...............{4/5!}+{3/4!} +{2/(3!)}+{1/(2!)}]

≈1

Rewriting and replacing (n-2) by N we derive that the sum of the infinite series (1/2!)+(2/3!)+(3/4!)+(4/5!)+…………+{N/(N+1)!} ≈ 1

This is a well documented convergent series whose approximate sum we have derived using a novel strategy and the sum equals unity. Its sum may represent the sum of probabilities of selecting any one of (N-1) permissible arrangements of possible N! arrangements for an infinite large number of objects that have been grouped into infinite sets with increasing number of objects from 2 to N and likely to have applications in fields such as chemistry, astronomy, etc.

Reference:

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