

# On Kurt Gödel and Hyperbolic math.

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## Abstract

Reputed Austrian American mathematician Kurt Gödel formulated two extraordinary propositions in mathematical logic. Accepted by all mathematicians they have revolutionized mathematics, showing that mathematical truth is more than logic and computation. These two ground breaking theorems changed mathematics, logic, and even the way we look at our Universe. The cognitive scientist Douglas Hofstadter described Gödel's first incompleteness theorem as that in a formal axiomatic mathematical system there are propositions that can neither be proven nor disproven. The logician and mathematician Jean van Heijenoort summarizes that there are formulas that are neither provable nor disprovable. According to Peter Suber, in a formal mathematical system, there are undecidable statements. S. M. Srivatsava formulates that formulations of number theory include undecidable propositions. And Miles Mathis describes Gödel's first incompleteness theorem as that in a formal axiomatic mathematical system we can construct a statement which is neither true nor false. [Mathematical variance of liar's paradox] In this short work, the author attempts to show these equivalent propositions to Gödel's incompleteness theorems by applying elementary arithmetic operations, algebra and hyperbolic geometry. [1 – 6 ]

**Key words:** Non – Euclidean geometries, Gödel's Incompleteness theorems

## Results

It is well known that the sum of the interior angles of a hyperbolic triangle is less than 180 degrees. Let us assume that  $y$ ,  $z$  and  $m$  denote the interior angle sums of three equal hyperbolic triangles.

$$\text{Let the sum of the interior angles of } y \text{ and } z \text{ be, } y + z = 360^0 - a \quad (1)$$

$$\text{And Let the sum of the interior angles of } m \text{ and } z \text{ be, } m + z = 360^0 - b \quad (2)$$

$$(1) - (2) \text{ gives, } y - m = b - a \quad (3)$$

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Multiplying (1) and (2),  $y(m+z) + z(m+z) = 360^0 a^2 - 360^0 b - 360^0 a + ab$  (4)

Multiplying (3),  $b^2 + a^2 - 2ab = y^2 + m^2 - 2ym$  (5)

Adding (4) and (5),  $y[m+z - y + 2m] + z[z+m] + b[b - 2a - a + 360^0] + a[a + 360^0] = 360^0 a^2 + m^2$

Applying (1), (2) and (3) in the above relation,

$$y[360^0 + m + a - 2b] + z[360^0 - b] + b[b - 3a + 360^0] + a[a + 360^0] = 360^0 a^2 + m^2$$

Rearranging ,

$$m(y-m) + 360^0(y+z+b+a-360^0) + a(y+a-3b) + b(b-2y-z) = 0$$

Once again putting (1) and (3) in the above equation,

$$m(b-a) + 360^0(360^0 - b + b + a - 360^0) + a(y+a-3b) + b(b-z-2y) = 0$$

$$\text{i.e } a[y+a+360^0-3b-m] + b[b-z-2y+m] = 0$$

Assuming (3) in the first factor, and (1) and (3) in the second factor ,

$$a[360^0 - 2b] + b[b + a - b + a - 360^0] = 0$$

$$\text{i.e } 360^0[a-b] - 2ab + 2ab = 0$$

$$\text{i.e } 360^0[a-b] = 0$$

$$\text{i.e } a = b \tag{6}$$

$$\text{Comparing (1), (2) and (6) we get that } y = m \tag{7}$$

$$\text{If } y \text{ and } m \text{ are equal, this establishes the fifth Euclidean postulate. [ 7] \tag{8}}$$

## Discussion

Addition, subtraction, multiplication and division are the four fundamental operations of arithmetic. Multiplication is the shortest form for addition and division is the easiest way of subtraction. In this short work, by applying these basic operations, we have derived equations 1 to 8. Needless to say, there is no error in the algebra. So, our equations are consistent. So, beyond any doubt eqn. (8) establishes Euclid's parallel postulate. But the mere existence of no Euclidean geometries and their successful physical and cosmological applications demonstrate that the parallel axiom of Euclidean geometry is a special case. Also, Beltrami, Cayley, Klein, Poincare and others showed that it is impossible to deduce Euclid V from Euclid I to V. Comparing this theorem with our eqn. (8) yields that Gödel's first incompleteness theorem holds. *I request reads to go through references [8] to [16]*

## References

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