Zero-point energy in the Johnson noise of resistors: Is it there?

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There is a longstanding debate about the zero-point term in the Johnson noise voltage of a resistor: Is it indeed there or is it only an experimental artifact due to the uncertainty principle for phase-sensitive amplifiers? We show that, when the zero-point term is measured by the mean energy and force in a shunting capacitor and, if these measurements confirm its existence, two types of perpetual motion machines could be constructed. Therefore an exact quantum theory of the Johnson noise must include also the measurement system used to evaluate the observed quantities. The results have implications also for phenomena in advanced nanotechnology.

1. Introduction

The thermal noise (Johnson noise) in resistors was discovered [1] by Johnson and explained [2] by Nyquist in 1927, a year after the foundations of quantum physics were completed. The Johnson-Nyquist formula states that

$$S_u(f) = 4R(f)hfN(f,T) \tag{1}$$

where $S_u(f)$ is the *one-sided* power density spectrum of the voltage noise on the open-ended complex impedance $Z(f)$ with real part $\mathrm{Re}\left[Z(f)\right] = R(f)$; and h is the Planck constant. The Planck number $N(f,T)$ is the mean number of hf energy quanta in a linear harmonic oscillator with resonance frequency f, at temperature T:

$$N(f,T) = \left[\exp(hf / kT) - 1\right]^{-1} \quad , \tag{2}$$

which is $N(f,T) = kT / (hf)$ for the classical physical range $kT >> hf$. Eq. 2 results in an exponential cut-off of the Johnson noise in the quantum range $f > f_P = kT / h$, in accordance with Planck's thermal radiation formula. In the deeply classical (low-frequency) limit, $f << f_P = kT / h$, Eqs. 1-2 yield the familiar form used at low frequencies:

$$S_{u,J}(f) = 4kTR(f) \tag{3}$$

where the Planck cut-off frequency f_P is about 6000 GHz at room temperature, well-beyond the reach of today's electronics.

The quantum theoretical treatment of the *one-sided* power density spectrum of the Johnson noise was given only 24 years later by Callen and Welton [3] (often called Fluctuation-Dissipation Theorem (FDT). The quantum version [3] of the Johnson-Nyquist formula has an additive 0.5 to the Planck number, corresponding to the zero-point energy of linear harmonic oscillators:

$$S_{u,q}(f) = 4R(f)hf\left[N(f,T) + 0.5\right] \quad . \tag{4}$$

Thus the quantum correction of Eq. 1 is a temperature-independent additive term of Callen-Welton's *one-sided* power density spectrum (Eq. 2) is

$$S_{u,ZP}(f) = 2hfR(f) \quad , \tag{5}$$

which linearly depends on the frequency and it exists for any $f > 0$ frequency, even in the deeply classical, $f << f_P = kT / h$, frequency regime, and even at zero temperature. The zero-point term described by Eq. 5 has acquired a wide theoretical support during the years, e.g. [4-8].

A note of clarification: the Callen-Walton's derivation works solely with the *one-sided* spectrum while subsequent quantum theoretical approaches often utilize asymmetrical power density spectra of fluctuations, e.g. [7,8] and they are in full agreement with the Callen-Welton result.

2. The debate

2.1 The ground state

However, there have also been contra-arguments and debates. MacDonald [9] and Harris [10] argued that extracting energy/power from the zero-point energy is impossible thus Eq. 5 should not exist.

2.2 Planck's black-body radiation

Grau and Kleen [11] and Kleen [12] (similarly to the original treatment of Nyquist [2]), argued that the Johnson noise of a resistor connected to an antenna, see Fig. 1, must satisfy Planck's thermal radiation formula thus the noise must be zero at zero temperature, which would imply that Eqs. 4,5 are invalid. It is obvious even by naked-eye observations: at 6000 K temperature, at 600 nm (orange color), the Planck number $N = 0.0164$. Thus the zero-point term (0.5) in Eq.

4 is 30 times greater than the classical term, yet it is invisible for the eye and a photocell.

However, defenders of Eq. 4 may say that the same zero-point term exists also in the thermal radiation field and that makes the net energy flow between the resistor and the radiation field zero for the zero-point term just like it does for the classical term by satisfying the Second Law of Thermodynamics. Nevertheless, this argument is unable to against a more advanced objection based on fluctuations, even if we neglect the obvious problem that photon absorption is irreversible. The zero-point terms in Eqs. 4,5 represent noises and that means statistical fluctuations [12, 13] of their finite-time mean square values. The implication is that for independent zero-point noises in the resistor and the radiation field (it is easy to shield the resistor to make that sure) a "zero-point energy flow" with fluctuating direction and value of the short-time average were observable between the antenna's input and its radiation field. This is not the case and it is a hard experimental fact that neither zero-point term nor its fluctuations are observable in the thermal radiation.

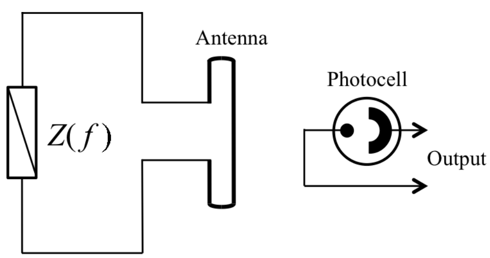

Figure 1. Measurement scheme based on an antenna and a photon counter, which does not show the presence of the zero-point term (Eq. 5) or its fluctuation in the Johnson noise at its output [11,12].

2.3 Divergent noise energy

Later, Kish [14] pointed out that the existence of the zero-point term, which has and "*f*"-noise implies a 1/f noise and related logarithmic divergence of the energy of a shunt capacitor in the high-frequency limit. While this does not disprove the existence of Eq. 5, it may indicate that the problem is a renormalization problem, a mathematical artifact, which is not actually present at measurements.

2.4 The crucial experimental proof

Yet, on the contrary of all the criticisms above, the experimental test by Koch, van Harlingen and Clarke [15] *fully* confirmed Eqs. 4,5 by measurements on resistively shunted Josephson-junctions, which is a heterodyne measurement method (required by the high frequency), see Fig. 2. The scheme is understood to be equivalent to the standard linear amplifier/filter method that determines the one-sided power density spectrum of the noise but allows accessing very high frequencies.

2.5 The uncertainty principle argument

However, Haus [16] and Kleen [17], by using Heffner's theory [18] of the uncertainty principle in linear amplifiers, state that the zero-point term (Eq. 5) in Eq. 4 is the direct consequence of the uncertainty principle at phase-sensitive amplitude measurement (Fig. 2). The same argumentation implies that the antenna arrangement [11,12] (Fig. 1) will not show uncertainty (and zero-point term) in the photon number. Nevertheless, the uncertainly principle argument *cannot disprove* Eqs. 4,5. The claimed zero-point term in the noise voltage may still exist and also satisfy the uncertainty principle instead of being solely an experimental artifact.

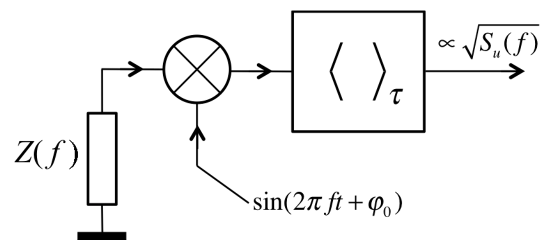

Figure 2. Heterodyne measurement scheme [15] based on a Josephson junction that mixes down the noise in the frequency range of interest. The mixing is represented by an analog multiplier driven by the noise and by the sinusoidal voltage oscillation at the Josephson frequency $f = 2qU_{\mathrm{dc}}/h$, where q is charge quantum and U_{dc} is the dc voltage on the Josephson junction. The dc component of the down-converted noise is proportional to $S_u^{0.5}(f)$ and it is extracted by time average unit of time constant τ. Other filters and devices are not shown.

2.6 Criticism of the Callen-Welton theory

Recently, Reggiani, et al. [19] generalized the derivations [3-6] of Eq. 4 by including the case of a discrete eigenvalue spectrum of the physical system under interest. They proposed (see their [19] Eq. 8) how Callen Welton relation, and the associated zero point contribution to the noise spectrum, should be modified in this new context.

3. Energy and force in a capacitor

Regarding our present considerations, the main conclusion of the debates described above is that the actual measurement scheme has a crucial role in the outcome of the observation. Thus the natural question emerges: can we use other types of measurements and check if the implications of Eq. 4,5 are visible in those experiment, or not?

Here we design two new measurement schemes utilizing the energy and force in a capacitor shunting a resistor.

3.1 Energy in a shunting capacitor

Consider first the mean energy in a capacitor shunting the resistor. Fig. 3 shows this system, which is a first-order low-pass filter with a single pole at frequency $f_L = (2\pi RC)^{-1}$.

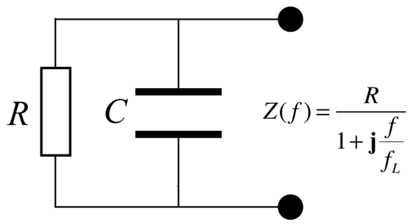

Figure 3. Resistor shunted by a capacitor.

The real part of the impedance is given as $\mathrm{Re}\left[Z(f)\right] = R\left(1+\mathbf{j}ff_L^{-1}\right)^{-1}$ thus, in accordance with Callen-Welton [3] and Equation 4, the *one-sided* power density spectrum $S_{u,C}(f)$ of the voltage on the impedance (and that on the capacitor) is:

$$S_{u,C}(f) = \frac{4RhfN(f,T)}{1+f^2 f_L^{-2}} + \frac{2Rhf}{1+f^2 f_L^{-2}} \quad , \qquad (6)$$

where the first term is classical physical while the second one is its quantum (zero-point) correction.

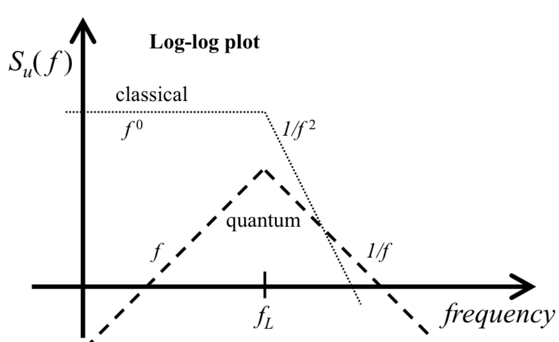

Figure 4. The Boode plot (with the low and high frequency asymptotes) of the classical and quantum (zero-point) component of the power density spectrum of the voltage on the capacitor at a finite temperature. The classical Lorentzian spectrum has white and $1/f^2$ spectral regimes. At zero temperature, only the quantum term exists, which is an *f*-noise at low frequencies and converges to $1/f$ at $f > f_L$.

The mean energy in the capacitor is given as:

$$\langle E_C \rangle = 0.5C\left\langle U_C^2(t)\right\rangle = 0.5C\int_0^{f_c} S_{u,C}(f)\,df \quad , \qquad (7)$$

where $f_c >> f_L$ is the cut-off frequency of the transport in the resistor. At near-to-zero temperature, the classical component $\left\langle U_{C,c}^2(t)\right\rangle$ of $\left\langle U_C^2(t)\right\rangle$ vanishes:

$$\lim_{T\to 0}\left\langle U_{C,c}^2(t)\right\rangle = \lim_{T\to 0}\left\{4Rh\int_0^{f_c}\frac{f\left[\exp(hf/kT)-1\right]^{-1}}{1+f^2 f_L^{-2}}df\right\} = 0 \quad (8)$$

but the quantum (zero-point) term remains:

$$\left\langle U_{C,q}^2(t)\right\rangle = \int_0^{f_c}\frac{2hfR}{1+f^2 f_L^{-2}}df = hRf_L^2\ln\left(1+\frac{f_c^2}{f_L^2}\right). \qquad (9)$$

Thus the energy in the capacitor, in the zero-temperature approximation, is:

$$\langle E_C \rangle = \frac{h}{8\pi^2 RC}\ln\left(1+4\pi^2 R^2 C^2 f_c^2\right) \ . \qquad (10)$$

Eq. 10 implies that by choosing different resistance values, the capacitor is charged up to different mean energy levels. This energy can be measured by, for example, switching the capacitor between two resistor of different resistance values and evaluating the dissipated heat, see Section 4.1.

3.2 Force in the capacitor

In a plane capacitor, where the distance x between the planes is much smaller than the smallest diameter d of the planes, $x << d$, the attractive force between the planes is given as [20]:

$$F = \frac{E_C}{x} \qquad (11)$$

From Eqs. 10 and 11, the mean force in the plane capacitor shunting a resistor (see Fig. 3) is:

$$\langle F(x)\rangle = \frac{\langle E_C\rangle}{x} = \frac{1}{x}\frac{h}{8\pi^2 RC(x)}\ln\left[1+4\pi^2 R^2 C^2(x) f_c^2\right], \quad (12)$$

where the x-dependence of the capacitance is given by $C(x) = \varepsilon\varepsilon_0 A/x$, and A is the surface of the planes.

Eq. 12 indicates that, at a given plane distance x, different resistance values result in different forces.

4. Two "perpetual motion machines"

The above energy and force effects, *if the zero-point term were visible at these kinds of measurements*, could be used to build two different perpetual motion machines.

4.1 Zero-point noise based "perpetual heat generator"

In Fig. 5, the "*heat-generator*" is shown. It is an ensemble of *N Units*, each one containing two different resistors and one capacitor. The capacitors in the Units are *periodically* alternated between the two resistors by centrally controlled switches, in a synchronized fashion, that makes the relative control energy negligible [20]. The duration τ_h of the period is selected so long that the capacitors are "thermalized" by the zero-point noise, that is, $\tau_h >> \max\left\{R_1C, R_2C\right\}$. Suppose: $R_1 < R_2$ and that the parameters satisfy $\max\left\{\left(4\pi R_i C\right)^{-1}\right\} << f_c$. In this case, whenever the switch makes the $1 \Rightarrow 2$ transition, the energy difference will dissipate in the system of R_2 resistors:

$$0 < E_h = N\frac{h}{8\pi^2 C}\left[\frac{\ln\left(1+4\pi^2 R_1^2 C^2 f_c^2\right)}{R_1} - \frac{\ln\left(1+4\pi^2 R_2^2 C^2 f_c^2\right)}{R_2}\right] \tag{13}$$

After the reverse, $2 \Rightarrow 1$ transition, the capacitors will be recharged by the system of R_1 resistors to their higher mean energy level.

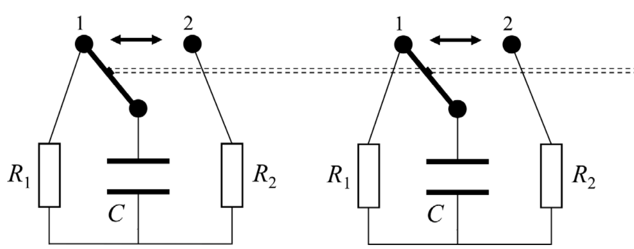

Figure 5. The heat generator based "perpetual motion machine". The switch is periodically switched between the two states. If the zero-point term existed in the Jonson noise voltage then energy were pumped from the system of resistors of the smaller resistance into the system of resistors of the greater resistance.

In conclusion the "*heat-generator*" system is pumping energy from the system of R_1 resistors to the system of R_2 resistors where this energy will dissipate to heat, which can be utilized to drive the switches of this perpetual motion machine. This process violates not only the Second Law of Thermodynamics by its negentropy production but it directly violates the Energy Conservation Law, too.

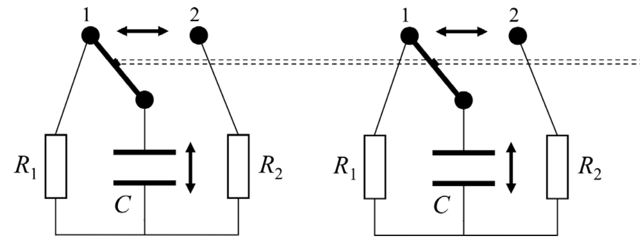

Figure 6. The moving-plate capacitor piston based "perpetual motion machine". The switch is periodically switched between the two states. See also Figure 7.

4.2 Zero-point noise based "perpetual motion engine"

The second perpetual motion machine is a two-stroke engine, see Fig. 6. This is the zero-point energy version of the two-stroke Johnson noise engine described earlier [20]. The engine has *N* parallel cylinders with identical elements and parameters as in the system in Fig. 5. The only difference is that the capacitors have a moving plate, which acts as a piston. The moving plates are coupled to a flywheel, which moves them in a periodic, synchronized fashion. When the plate distance reaches its nearest and farthest distance limits, $x_{\min}$ and $x_{\max}$, where the corresponding capacitance values are $C_{\max}$ and $C_{\min}$, respectively, the switch alternates the driving resistor, see Fig. 7. During contraction and expansion, the driver is R_1 and R_2, $(R_1 < R_2)$, respectively. At a given distance *x*, the difference between the attractive force between the capacitors is [16]

$$\left\langle \Delta F(x)\right\rangle = \frac{1}{x}\frac{h}{8\pi^2 C(x)} \left\{\frac{1}{R_1}\ln\left[1+4\pi^2 R_1^2 C^2(x) f_c^2\right] - \frac{1}{R_2}\ln\left[1+4\pi^2 R_2^2 C^2(x) f_c^2\right]\right\} \tag{14}$$

where *x* is the distance between the plates and $C(x)$ is the capacitance versus the distance. Thus the total force difference in *N* cylinders is:

$$\Delta F_N(x) = N\left\langle \Delta F(x)\right\rangle \ . \tag{15}$$

With $R_1 < R_2$, at any given plate distance *x* (and corresponding capacitance value), the force $N\left\langle F(x)\right\rangle$ is stronger during contraction than during expansion, c.f. Fig. 7. During a full cycle, a positive net work is executed by the engine:

$$W = \oint_{x_{\min}, x_{\max}} N\left\langle F(x)\right\rangle dx = \int_{x_{\max}}^{x_{\min}} \Delta F_N(x) dx > 0 \ . \tag{16}$$

Wile this two-stroke engine produces a positive work during its whole cycle, at the switching at C_{max}, the heat-generation effect also kicks in, that is, heat is generated in R_2, similarly to the first perpetual motion machine.

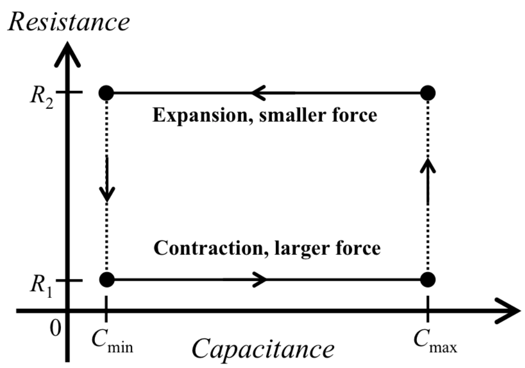

Figure 7. The capacitance-resistance diagram of the two-stroke perpetual motion engine.

5. Conclusions

Both perpetual motion machines directly violate not only the Second Law of Thermodynamics but also the Energy Conservation Law. Thus the key assumption leading to their creation, that is, the presence of the zero-point term (Eqs. 4,5) in the voltage at these experiments, must be incorrect.

As this assumption directly leads to the feasibility of the two perpetual motion machines described in Section 4, we must conclude that, *during these types of measurements, the zero-point term of the Johnson noise voltage spectrum cannot be correct in the form given by Eqs. 4 and 5.*

Note, the Casimir-effect also implies an attractive force between the plates. However, the Casimir-pressure decays [21] with x^{-4}, which implies that the Casimir force *at fixed capacitance* decays with x^{-3}. At the same time, the force due to the zero-point noise decays as x^{-1}. Thus, in the perpetual motion machines introduced above the Casimir effect can always be made negligible by the proper choice of the range of distance x between the plates during operation.

Our main conclusions is as follows:

An exact quantum theory of the Johnson noise must include also the measurement system used to evaluate the observed quantities.

Finally, it is important to mention that the above considerations are not only fundamental scientific but they can also be relevant for technical applications. The issue of the force in a capacitor has potential importance in advanced nanotechnology where the van der Waals/Casimir forces are present [22]. In systems where there is electrical connection between nanostructural conductors forming capacitors, such as coated cantilevers, the zero-point noise would imply forces that could dominate over the van der Waals/Casimir forces.

Acknowledgement

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