The notion of s-primes and a generic formula of 2-Poulet numbers

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Abstract. In Addenda to my previous paper "On the special relation between the numbers of the form 505+1008k and the squares of primes" I defined the notions of c/m-integers and g/s-integers and showed some of their applications. In a previous paper I conjectured that, beside few definable exceptions, the Fermat pseudoprimes to base 2 with two prime factors are c/m-primes, but I haven't defined the "definable exceptions". However, in this paper I confirm one of my constant beliefs, namely that the relations between the two prime factors of a 2-Poulet number are definable without exceptions and I make a conjecture about a generic formula of these numbers, namely that the most of them are s-primes and the exceptions must satisfy a given Diophantine equation.

Definition of a s-prime:

We name s-primes the semiprimes of the form p*q, p < q, with the property that q can be written as k*p - k + 1, where k is positive integer.

Preliminary conjecture:

All 2-Poulet numbers but a set of few definable exceptions are s-primes.

Note:

For a list of 2-Poulet numbers see the sequence A214305 submitted by me on OEIS.

Verifying the conjecture (for the first thirty 2-Poulet numbers):

For 341 = 11*31 we have:
:   11*3 – 2 = 31. The number 341 is a s-prime.

For 1387 = 19*73 we have:
:   19*4 – 3 = 73. The number 1387 is a s-prime.

For 2701 = 37*73 we have:
:   37*2 – 1 = 73. The number 2701 is a s-prime.

For 3277 = 29*113 we have:
:   29*4 - 3 = 113. The number 3277 is a s-prime.

For 4033 = 37*109 we have:
:   37*3 - 2 = 109. The number 4033 is a s-prime.

For 4369 = 17*257 we have:
:   17*16 - 15 = 257. The number 4369 is a s-prime.

For 4681 = 31*151 we have:
:   31*3 - 2 = 151. The number 4681 is a s-prime.

For 5461 = 43*127 we have:
:   43*3 - 2 = 127. The number 5461 is a s-prime.

The number 7957 = 73*109 is an exception (we will try to define it when more exceptions will occur)

For 8321 = 53*157 we have:
:   53*3 - 2 = 157. The number 4681 is a s-prime.

For 10261 = 31*331 we have:
:   31*11 - 10 = 331. The number 10261 is a s-prime.

For 13747 = 59*233 we have:
:   59*4 - 3 = 233. The number 13747 is a s-prime.

For 14491 = 43*337 we have:
:   43*8 - 7. The number 14491 is a s-prime.

For 15709 = 23*683 we have:
:   23*31 - 30 = 683. The number 15709 is a s-prime.

For 18721 = 97*193 we have:
:   97*2 - 1 = 193. The number 18721 is a s-prime.

For 19951 = 71*281 we have:
:   71*3 - 2 = 281. The number 19951 is a s-prime.

The number 23377 = 97*241 is an exception (we will try to define it when more exceptions will occur)

For 31417 = 89*353 we have:
:   89*4 - 3 = 353. The number 31417 is a s-prime.

For 31609 = 73*433 we have:
:   73*6 - 5 = 433. The number 31609 is a s-prime.

For 31621 = 103*307 we have:
:   103*3 - 2 = 307. The number 31621 is a s-prime.

The number 35333 = 89*397 is an exception (we will try to define it when more exceptions will occur)

The number 42799 = 127*337 is an exception (we will try to define it when more exceptions will occur)

For 49141 = 157*313 we have:
: 157*2 – 1 = 313. The number 49141 is a s-prime.

The number 49981 = 151*331 is an exception (we will try to define it when more exceptions will occur)

For 60701 = 101*601 we have:
: 101*6 – 5 = 601. The number 60701 is a s-prime.

The number 60787 = 89*683 is an exception (we will try to define it when more exceptions will occur)

For 65281 = 97*673 we have:
: 97*7 – 6 = 673. The number 65281 is a s-prime.

For 80581 = 61*1321 we have:
: 61*22 – 21 = 1321. The number 80581 is a s-prime.

For 83333 = 167*499 we have:
: 167*3 – 2 = 499. The number 83333 is a s-prime.

Conclusion:

I studied the exceptions and I found one thing common to them: they satisfy the equation a*q = b*p + c, where p and q are the two prime factors, p < q, a and b positive integers and c integer that satisfy the condition a = b + c:

: 7957 = 73*109: satisfies for (a,b,c) = (3, 2, 1)
Indeed, 3*73 = 2*109 + 1 and 3 = 2 + 1;

: 23377 = 97*241: satisfies for (a,b,c) = (5, 2, 3)
Indeed, 5*97 = 2*241 + 3 and 5 = 2 + 3;

: 35333 = 89*397: satisfies for (a,b,c) = (8, 36, -28)
Indeed, 8*397 = 36*89 - 28 and 8 = 36 - 28;

: 42799 = 127*337: satisfies for (a,b,c) = (8, 3, 5)
Indeed, 8*127 = 3*337 + 5 and 8 = 3 + 5;

: 49981 = 151*331: satisfies for (a,b,c) = (5, 11, -6)
Indeed, 5*331 = 11*151 - 6 and 5 = 11 – 6.

Conjecture on a generic formula of 2-Poulet numbers:

All 2-Poulet numbers p*q, p < q (or equal in the two cases known, the squares of the Wieferich primes) satisfy at least one of the following two conditions:

(i) q can be written as k*p - k + 1, where k is positive integer;

(ii) they satisfy the equation a*q = b*p + c, where a and b are positive integers and c integer that satisfy the condition a = b + c.