On the special relation between the numbers of the form 505+1008k and the squares of primes

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Abstract. The study of the power of primes was for me a constant probably since I first encounter “Fermat’s last theorem”. The desire to find numbers with special properties, as is, say, Hardy-Ramanujan number, was another constant. In this paper I present a class of numbers, i.e. the numbers of the form n = 505 + 1008*k, where k positive integer, which, despite the fact that they don’t seem to be, prima facie, “special”, seem to have a strong connection with the powers of primes: for a lot of values of k (I show in this paper that for nine from the first twelve and I conjecture that for an infinity of the values of k), there exist p and q primes such that p^2 – q^2 + 1 = n. The special nature of the numbers of the form 505 + 1008*k is also highlight by the fact that they are (all the first twelve of them, as much I checked) primes or g/s-integers or c/m-integers (I define in Addenda to this paper the two new notions mentioned).

The sequence of the squares of primes (A001248 in OEIS):

4, 9, 25, 49, 121, 169, 289, 361, 529, 841, 961, 1369, 1681, 1849, 2209, 2809, 3481, 3721, 4489, 5041, 5329, 6241, 6889, 7921, 9409, 10201, 10609, 11449, 11881, 12769, 16129, 17161, 18769, 19321, 22201, 22801, 24649, 26569, 27889, 29929, 32041, 32761, 36481 (...)

The sequence of the numbers of the form 505 + 1008*k:

505, 1513, 2521, 3529, 4537, 5545, 6553, 7561, 8569, 9577, 10585, 11593 (...)

Conjecture 1:

There exist an infinity of values of k, positive integer, such that the number n = 505 + 1008*k can be written as n = p^2 – q^2 + 1, where p and q are primes.

Note:

The numbers from the sequence above more probably to can be written the way mentioned are the ones that have the last digit 1, 3 or 9, because p^2 and q^2 have, without an exception (I refer only to primes greater than or equal to 5), the number 25, only the values 1 and 9 for the last digit; that means that the numbers from the sequence above ended in digits 5 or 7 can only satisfy the equation if q^2 is 25 (but the numbers 505 and 10585 do satisfy the equation!).

Examples:
(The ways in which n from the examples below can be written as mentioned is revealed just up to p = 191 that means p^2 = 36481)

: The number 505 (obtained for k = 0) can be written as
  : 505 = 23^2 – 5^2 + 1.
: The number 1513 (obtained for k = 1) can be written as
  : 1513 = 41^2 – 13^2 + 1;
  : 1513 = 61^2 – 47^2 + 1.
: The number 2521 (obtained for k = 2) can be written as
  : 2521 = 53^2 – 17^2 + 1;
  : 2521 = 59^2 – 31^2 + 1;
  : 2521 = 73^2 – 53^2 + 1.
: The number 3529 (obtained for k = 3) can be written as
  : 3529 = 67^2 – 31^2 + 1;
  : 3529 = 107^2 – 89^2 + 1.
: The number 6553 (obtained for k = 6) can be written as
  : 6553 = 89^2 – 37^2 + 1;
  : 6553 = 109^2 – 73^2 + 1;
  : 6553 = 131^2 – 103^2 + 1;
  : 6553 = 139^2 – 113^2 + 1.
: The number 7561 (obtained for k = 7) can be written as
  : 7561 = 89^2 – 31^2 + 1.
: The number 8569 (obtained for k = 8) can be written as
  : 8569 = 137^2 – 101^2 + 1;
  : 8569 = 167^2 – 139^2 + 1.
: The number 10585 (obtained for k = 10) can be written as
  : 10585 = 103^2 – 5^2 + 1.
: The number 11593 (obtained for k = 11) can be written as
  : 11593 = 109^2 – 17^2 + 1;
  : 11593 = 149^2 – 103^2 + 1.

ON THE SPECIAL NATURE OF THE NUMBERS OF THE FORM 505 + 1008*K

As I mentioned in Abstract, all the first 12 such numbers are primes or c/m-integers or g/s-integers (I defined in Addenda 1 respectively in Addenda 2, see below, these two new notions).

The numbers 2521, 3521, 6553, 7561, 11593 are primes; the rest of the numbers from the sequence checked (up to the term 11593) are both c/m-integers and s/m-integers.

Conjecture 2:

All the numbers of the form n = 505 + 1008*k, where k positive integer, are either primes either c/m-integers and/or g/s-integers.

Verifying the conjecture:
(for the seven numbers which are not primes from the first twelve from sequence)

: the number 505 = 5*101 is g-prime because 101 = 5*17 + 16; is also c-prime because 101 – 5 + 1 = 97, prime;
: the number 1513 = 17*89 is g-prime because 89 = 17*5 + 4; is also c-prime because 89 – 17 + 1 = 73, prime;
: the number 4537 = 13*349 is a g-prime because 349 = 13*25 + 24; is also c-prime because 349 – 13 + 1 = 337, prime; is also m-prime because 349 + 12 – 1 = 361 = 19^2 and 19 + 19 – 1 = 37, prime;
: the number 5545 = 5*1109 is a g-prime because 1109 = 5*185 + 184;
: the number 9577 = 61*157 is a c-prime because 157 – 61 + 1 = 97, a prime;

: the number 8569 = 11*19*41 is a gs-composite, g-composite because 19*41 = 11*65 + 64 and s-composite because 11*41 = 19*25 – 24; it is also cm-composite, c-composite because 11*41 – 19 + 1 = 433, prime (also 19*41 – 11 + 1 = 769, prime, and 11*91 – 41 + 1 = 169, square of prime) and m-composite because 11*41 + 19 – 1 = 7*67 (m-prime because 7 + 67 – 1 = 73, prime);
: the number 10585 = 5*29*73 is a gs-composite, g-composite because 5*353 + 352 = 29*73 and s-composite because 73*2 – 1 = 5*29 (and also 29*13 – 12 = 5*73); it is also cm-composite, c-composite because 5*29 – 73 + 1 = 73, prime (and also 5*73 – 29 + 1 = 337, prime and 29*73 – 5 + 1 = 2113, prime) and m-composite because 5*29 + 73 – 1 = 217 = 7*31 and 7 + 31 – 1 = 37, prime;

Comment:

Note that the number 10585 (obtained for k = 10) is also a Carmichael number. In a previous paper, namely "Conjecture that states that any Carmichael number is a cm-composite", I conjectured that these numbers have the property mentioned in title. In further papers I shall check to what extent the Fermat pseudoprimes (Poulet numbers and Carmichael numbers) are g/s-integers (notion defined for the first time in this paper). Another thing to be checked: the formula n + q^2 – 1 can lead sometimes to Poulet numbers (it is the case 6553 + 7^1 – 1 = 6601).

ADDENDA 1. C/M-INTEGERS

Definition of a c-prime:

We name a c-prime a positive odd integer which is either prime either semiprime of the form p(1)*q(1), p(1) < q(1), with the property that the number q(1) – p(1) + 1 is either prime either semiprime p(2)*q(2) with the property that the number q(2) – p(2) + 1 is either prime either semiprime with the property showed above... (until, eventualy, is obtained a prime).

Example: 4979 is a c-prime because 4979 = 13*383, where 383 – 13 + 1 = 371 = 7*53, where 53 – 7 + 1 = 47, a prime.

Definition of a m-prime:

We name a m-prime a positive odd integer which is either prime either semiprime of the form p(1)*q(1), with the property that the number p(1) + q(1) - 1 is either prime either semiprime p(2)*q(2) with the property that the number p(2) + q(2) - 1 is either prime either semiprime with the property showed above... (until, eventualy, is obtained a prime).

Example: 5411 is a m-prime because 5411 = 7*773, where 7 + 773 - 1 = 779 = 19*41, where 19 + 41 - 1 = 59, a prime.

Definition of a cm-prime:

We name a cm-prime a number which is both c-prime and m-prime (not to be confused with the notation c/m-primes which I use to express "c-primes or m-primes").

Definition of a c-composite:

We name a c-composite the composite number with three or more prime factors n = p(1)*p(2)*...*p(m), where p(1), p(2), ..., p(m) are the prime factors of n, which has the following property: there exist p(k) and p(h), where p(k) is the product of some distinct prime factors of n and p(h) the product of the other distinct prime factors such that the number p(k) - p(h) + 1 is a c-prime.

Definition of a m-composite:

We name a m-composite the composite number with three or more prime factors n = p(1)*p(2)*...*p(m), where p(1), p(2), ..., p(m) are the prime factors of n, which has the following property: there exist p(k) and p(h), where p(k) is the product of some distinct prime factors of n and p(h) the product of the other distinct prime factors such that the number p(k) + p(h) - 1 is a m-prime.

Definition of a cm-composite:

We name a cm-composite a number which is both c-composite and m-composite (not to be confused with the notation c/m-composites which I use to express "c-composites or m-composites").

Definition of a c/m-integer:

We name a c/m-integer a number which is either c-prime, m-prime, cm-prime, c-composite, m-composite or cm-composite.

ADDENDA 2. G/S-INTEGERS

Definition of a g-prime:

We name g-primes the semiprimes of the form p*q, p < q, with the property that q can be written as k*p + k – 1, where k is positive integer (it can be seen that, for k = 2, p is a Sophie Germain prime because q = 2*p + 1 is also prime).

Examples: n = 1081 = 23*47 is a g-prime because 47 = 23*2 + 1 and also n = 1513 = 17*89 is a g-prime because 89 = 17*5 + 4.

Definition of a s-prime:

We name s-primes the semiprimes of the form p*q, p < q, with the property that q can be written as k*p - k + 1, where k is positive integer.

Examples: n = 91 = 7*13 is a s-prime because 13 = 7*2 - 1 and also n = 4681 = 31*151 is a s-prime because 151 = 31*5 - 4.

Definition of a g-composite:

We name a g-composite the composite number with three or more prime factors n = p(1)*p(2)*...*p(m), where p(1), p(2), ..., p(m) are the prime factors of n, which has the following property: there exist p(k) and p(h), where p(k) is the product of some distinct prime factors of n and p(h) the product of the other distinct prime factors, also there exist the number m, positive integer, such that p(h) can be written as m*p(k) + m - 1.

Example: n = 8569 = 11*19*41 is a g-composite because 11*65 + 65 - 1 = 19*41.

Definition of a s-composite:

We name a s-composite the composite number with three or more prime factors n = p(1)*p(2)*...*p(m), where p(1), p(2), ..., p(m) are the prime factors of n, which has the following property: there exist p(k) and p(h), where p(k) is the product of some distinct prime factors of n and p(h) the product of the other distinct prime factors, also there exist the number m, positive integer, such that p(h) can be written as m*p(k) - m + 1.

Example: n = 8569 = 11*19*41 is a s-composite because 19*25 – 25 + 1 = 11*41.

Definition of a gs-composite:

We name a gs-composite a number which is both g-composite and s-composite (not to be confused with the notation g/m-composites which I use to express "g-composites or s-composites").

Definition of a g/s-integer:

We name a g/s-integer a number which is either g-prime, s-prime, g-composite, s-composite or gs-composite.