On the Relativistic Length Contraction and Special Relativity: Twisted Conceptions

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Abstract

For relatively moving inertial frames, the constancy of the speed of light principle physically leads to time dilation in the transverse direction. This time dilation is irreconcilable in the longitudinal direction unless a length contraction in the relative motion direction is postulated. However, time dilation is contradictorily coupled with length expansion, a fact erroneously twisted in the special relativity and related text books, as demonstrated in this paper. The typical physical demonstration of the length contraction is shown to be inconsistent and contradicts its derivation from the Lorentz transformation. The misinterpretation of the Lorentz Transformation in predicting the length contraction is revealed. The constancy of the speed of light is consequently unviable.

Keywords: *Special Relativity, time dilation, length contraction, speed of light postulate*

1. Introduction

Virtually, all textbooks on Special Relativity[1-3] explain the relativistic time dilation through considering a bouncing light pulse in a traveling inertial frame (e.g., a train traveling at a uniform speed) and calculating its round trip duration, under the constancy of the speed of light assumption, from the perspective of an observer in a stationary frame (e.g., a train station platform). This duration is compared to the respective one, all under the same assumption of the light speed invariance, from the perspective of an observer in the traveling frame, to find it's dilated by the relativistic factor γ. Then, the length contraction is deduced from the obtained time dilation. This is constantly done through calculating the distance traveled by the moving frame between the two fixed ends of a certain object (e.g., a bench or a parked train) in the stationary frame, using the travel time from the perspective of each observer, to conclude that the distance measured in the stationary frame is larger than that measured in the traveling frame, due to the time dilation. This is interpreted as the object length in the stationary frame being contracted from the perspective of the moving observer, since the object in the stationary frame is moving with respect to them. In this paper, the misleading and erroneous interpretation of length contraction as a consequence of time dilation is simply revealed.

2. Light Speed Principle, Time Dilation and Length Contraction

A typical text book demonstration of the relativistic time dilation is carried out a follows. A traveling frame (e.g., train wagon) going at a uniform speed v relative to a stationary frame (e.g., bench) is considered (Fig. 1). The travel time of a light pulse to go back and forth (by reflection) along a transverse path in the traveling frame shall be determined from the perspective of both an observer, O', in the traveling frame, and another observer, O, in the stationary frame. Using the constancy of the speed of light principle (i.e., the light pulse travels at the same speed c relative to both observers), the light path from the perspective of each observer is shown in Fig. 1.

From the perspective of O', we have

$$t' = \frac{2h'}{c}. \tag{1}$$

Whereas, with respect to O, we can write

$$c^2t^2 = v^2t^2 + c^2t'^2;$$

$$t = \frac{1}{\sqrt{1 - v^2/c^2}}t' = \gamma t' = \gamma\frac{2h'}{c}, \tag{2}$$

where $\gamma = \left(1 - v^2/c^2\right)^{-1/2}$ is the time dilation factor.

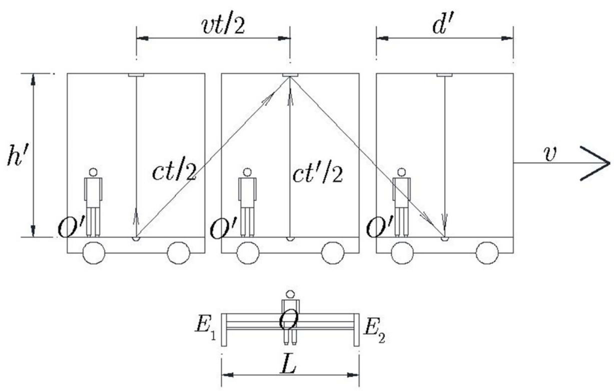

Fig. 1 Typical illustration of the time dilation

The time t', being the time interval measured in O' frame between two events occurring at a point at rest relative to O' is referred to as a proper time with respect to O'. Whereas, t is the perceived (dilated) time relative to O.

Conversely, if the light pulse was traveling back and forth transversally in the "O" frame, its travel time will be dilated with respect to the "O'" frame (in fact, the stationary frame becomes the traveling one and vice versa). In this case, we would have

$$t' = \gamma t, \tag{3}$$

where, in this case, t is the proper time with respect to O, and t' is the perceived (dilated) time relative to O'.

On the other hand, if the light pulse was traveling back and forth in the longitudinal direction over a fixed length l' in the O' frame (i.e., a proper length), the "proper" travel time with respect to O' will be $t' = 2l'/c$, but with respect to O, it will be perceived as

$$t = \frac{l'}{c - v} + \frac{l'}{c + v};$$

$$t = \frac{2l'/c}{1 - v^2/c^2} = \gamma^2\frac{2l'}{c} = \gamma^2 t'. \tag{4}$$

Equation (4) is irreconcilable with Eq. (2), having a different time dilation factor, γ^2. To overcome this inconsistency, the "proper" length l' is hypothesized to be transformed to l with respect to O, such that

$$l = \frac{l'}{\gamma}, \tag{5}$$

which, after replacing l' with l for the observer O, modifies Eq. (4) to

$$t = \gamma^2\frac{2l'}{\gamma c} = \gamma t', \tag{6}$$

in line with Eq. (2). Hence, the length contraction is called for to conserve the time dilation emerged from the constancy of the speed of light postulate. Yet, according to the special relativity, the speed of light is a universal constant, c. Accordingly, the space (ct) and time (t) become equivalent. Therefore, this length contraction is nothing but a time contraction from $\gamma^2\,2l'/c$ to $\gamma\,2l'/c$.

Indeed, suppose that observer O' wishes to measure the "proper" length d' of his wagon by sending a light pulse from one wagon end, reflected on the other end, and returned to its emission point. By measuring the pulse two way travel time $\Delta t'$, it can conclude the wagon length to be

$$d' = c\Delta t'/2.$$

But, according to the time dilation, this "proper" travel time $\Delta t'$ will be perceived as $\gamma \Delta t'$ by the observer O, and the respective "perceived" wagon length with respect to O becomes

$$d = \gamma\, c\Delta t' / 2 = \gamma d',$$

exhibiting a length expansion, in contradiction with the postulated Eq.(5).

3. Length Contraction: a Twisted Concept

In relativity textbooks, the following is the general scenario commonly used to [falsely] demonstrate how the length contraction physically results from the time dilation.

As the wagon is traveling by the bench, O', relative to whom the bench is moving, sends out a signal at the two time instants when each of the edge points E_1 and E_2 of the bench appears to go over a fixed point in the wagon, say a tag marked on the wagon window. The "perceived" bench length is then calculated by O' to be the "proper" time interval $\Delta t'$ between the two signals, multiplied by the relative speed v :

$$\left(L'\right)_{perceived} = v\left(\Delta t'\right)_{proper}. \tag{7}$$

From the perspective of O, this proper time interval $(\Delta t')$ between the two signals emanated by O' will be perceived as Δt, dilated by the relativistic factor γ. Hence, the corresponding "proper" bench length with respect to O would be

$$\begin{aligned}\left(L\right)_{proper} &= v\left(\Delta t\right)_{perceived} = \\ &= \gamma v\left(\Delta t'\right)_{proper} = \gamma\left(L'\right)_{perceived}.\end{aligned} \tag{8}$$

It follows from the above expressions (7) and (8) that the "perceived" bench length L' is contracted by the relativistic factor γ with respect to O' when compared to its "proper" length L (i.e., relative to O).

The trick in the above scenario is done through calculating: (1) the O' "perceived" length of the moving bench "as" an O' "proper" time interval, and (2) the O "proper" length of the bench "as" the corresponding "perceived" (dilated) time interval. So, the "proper" length L is consequently dilated with the "perceived" time Δt when compared to the "perceived" length L' associated with the "proper" time $\Delta t'$.

Now, we will demonstrate how the above scenario can be used to contradictorily result in perceived length expansion. This is done through calculating the "proper" length using the "proper" time, and the "perceived" length using the "perceived" time, as it should be. In fact, as the wagon is traveling by the bench, O who is sitting in the middle of the bench, sends out a signal at the two time instants when a fixed point in the wagon, say the front corner, appears to pass by each of the edge points E_1 and E_2 of the bench. The "proper" bench length is then calculated by O to be the "proper" time interval Δt between the two signals (emanated from the bench midpoint by O), multiplied by the relative speed v :

$$\left(L\right)_{proper} = v\left(\Delta t\right)_{proper}. \tag{9}$$

From the perspective of O', this "proper" time interval (Δt) between the two signals emanated by O will be perceived as $\Delta t'$, dilated by the relativistic factor γ. Hence, the corresponding "perceived" bench length with respect to O' would be

$$\begin{aligned}\left(L'\right)_{perceived} &= v\left(\Delta t'\right)_{perceived} = \\ &= \gamma v\left(\Delta t\right)_{proper} = \gamma\left(L\right)_{proper}.\end{aligned} \tag{10}$$

It follows from the above expressions (9) and (10) that the "perceived" bench length L' is expanded by the relativistic factor γ with respect to O' when compared to its "proper" length L (i.e., relative to O). This is contradicting the special relativity length contraction prediction.

The falsification of the above scenario to demonstrate the length contraction can also be deduced from the Lorentz transformation spatial equation[1]:

$$x' = \gamma(x - vt). \tag{11}$$

In fact, the necessary condition for the length contraction to be implied by Eq. (11) is the events simultaneity. i.e., the time interval t must be set to zero in order for the above spatial transformation to yield the length contraction. However, this necessary condition does not hold for the above scenario, and is not satisfied by Eqs. (7) and (8) through which the length contraction is erroneously deduced.

4. Misinterpretation of the Lorentz Transformation

So far, it's been highlighted that for the time dilation physically resulting from the speed of light principle to hold, length contraction in the relative motion direction must take place. This has erroneously been validated in the Special Relativity. The fact is, as demonstrated above, the time dilation and the length contraction are physically incompatible.

On the other hand, one might argue that the Lorentz transformation (LT) equations are derived on the basis of the light speed principle,[1,2] and they exhibit time dilation and length contraction. The reply to this argument is that the LT equations have been misinterpreted in the Special Relativity, as demonstrated below.

Let $K(x,y,z,t)$ and $K'(x',y',z',t')$ be two coordinate systems attached to the frames of our observers O and O', respectively. The LT equations for the x and t coordinates are:

$$x' = \gamma\left(x - vt\right) \tag{12}$$

$$t' = \gamma\left(t - vx/c^2\right) \tag{13}$$

Equations (12) and (13) show how the coordinates x and t from the perspective of K are transformed in K'. For two co-local events[a] at K' origin (e.g., a light pulse traveling back and forth), with t' being the proper time interval between them, we have $x' = 0$, or according to Eq. (12) $x = vt$, which, when plugged in Eq. (13) yields the perceived time dilation:

$$t = \gamma t'. \tag{14}$$

Now, consider the fixed "proper" length L of the bench in K. Two simultaneous events[a] $(\Delta t = 0)$ take place at the two ends of the bench $(\Delta x = L)$. Therefore, the transformed (perceived) length $(\Delta x' = L')$ in K' would be obtained from Eq. (12) as

$$\left(L'\right)_{perceived} = \gamma\left(L\right)_{proper}, \tag{15}$$

which implies that the perceived length L' of the "moving" bench with respect to K' is expanded relative to its proper length in K. In Special Relativity, the length L' is erroneously interpreted as the proper length.

If L' is taken as the proper length, then the bench would have K' as its rest frame (i.e., fixed in K'). In this case, two simultaneous events in K' at the end points of the bench would have $\Delta x' = L'$ and $\Delta t' = 0$, or according to Eq. (13) $\Delta t = vL/c^2$, which when plugged in Eq. (12) yields the length expansion

$$\left(L\right)_{perceivrd} = \gamma\left(L'\right)_{proper}. \tag{16}$$

The error in the Special Relativity in predicting the length contraction is committed through the assumption that while O perceives the length of the bench moving with K' (proper length in K'), it perceives two simultaneous events (i.e., having zero proper time interval; $t = 0$) separated by the distance equal to the perceived bench length L in K, leading to the length contraction $L' = \gamma L$ from Eq. (12). For this assumption to be consistent, however, we can argue that, while O perceives the time interval between two events in K' (proper time in K'), it perceives two co-local events (i.e., having zero space interval; $x = 0$) separated by the time equals to the perceived time interval t in K,

[a] Co-local and simultaneous events are considered here for the argument sake only, as it's been shown in earlier works[4-6] that the LT transformation equations exclude zero coordinates.

leading to the time contraction $t' = \gamma t$ from Eq. (13).

For the consistent interpretation of the Lorentz transformation, the coordinates on the right hand side of Eqs. (12) and (13) should be maintained as the event proper coordinates, as intended by the transformation. So, for two simultaneous events in K (proper $t = 0$), separated by a proper distance $L\ (x = L)$, we get from Eq. (12) the perceived "moving" length $L' = \gamma L$ in K' as a length expansion. And, for two co-local events in K (proper $x = 0$), separated by a proper time interval $T\ (t = T)$, we get from Eq. (13) the perceived time interval $T' = \gamma T$, as a time dilation.

The inverse of the Lorentz transformation equations for the x' and t' coordinates are given by

$$x = \gamma\left(x' + vt'\right) \tag{17}$$

$$t = \gamma\left(t' + vx'/c^2\right) \tag{18}$$

Maintaining the coordinates on the right hand side of the inverse LT equations as the event proper coordinates, as intended by the transformation, the same conclusion as to the length expansion and time dilation will be obtained, namely

$$\left(L\right)_{perceived} = \gamma\left(L'\right)_{proper} \tag{19}$$

$$\left(T\right)_{perceived} = \gamma\left(T'\right)_{proper}, \tag{20}$$

contradicting the Special Relativity prediction of the length contraction.

Conclusion

The relativistic length contraction is a necessary condition for the validity of the time dilation emerging from the constancy of the speed of light principle. Length contraction, however, is shown to be incompatible with the time dilation. The customary physical demonstration of the length contraction is inconsistent and contradicts its derivation from the Lorentz transformation. The Lorentz Transformation is misinterpreted in predicting the length contraction. The constancy of the speed of light is consequently unviable.

References

1 Einstein, A. Zur elektrodynamik bewegter Körper. *Annalen der Physik* **322**, 891–921 (1905).

2 Einstein, A. Einstein's comprehensive 1907 essay on relativity, part I. *English translations in Am. Jour. Phys. 45 (1977), Jahrbuch der Radioaktivitat und Elektronik* **4** (1907).

3 Einstein, A. *Relativity: The Special and the General Theory* (Routledge, 1916).

4 Kassir, R. M. On Lorentz Transformation and Special Relativity: Critical Mathematical Analyses and Findings. *Physics Essays* **27**, 16 (2014).

5 Kassir, R. M. On Special Relativity: Root cause of the problems with Lorentz transformation. *Physics Essays* **27**, 198-203 (2014).

6 Kassir, R. M. The Critical Error in the Formulation of the Special Relativity. *International Journal of Physics* **2**, 197-201, doi:10.12691/ijp-2-6-3 (2014).