Formula based on squares of primes having the same digital sum that leads to primes and cm-primes

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Abstract. In this paper I present the observation that the formula p^2 – q^2 + 1, where p and q are primes with the special property that the sums of their digits are equal, leads often to primes (of course, having only the digital root equal to 1 due to the property of p and q to have same digital sum implicitly same digital root) or to special kinds of semiprimes: some of them named by me, in few previous papers, c/m-primes, and some of them named by me, in this paper, g-primes respectively s-primes. Note that I chose the names "g/s-primes" instead "g/s-semiprimes" not to exist confusion with the names "g/s-composites", which I intend to define and use in further papers.

Definition 1:

We name g-primes the semiprimes of the form p*q, p < q, with the property that q can be written as k*p + k – 1, where k is positive integer (it can be seen that, for k = 2, p is a Sophie Germain prime because q = 2*p + 1 is also prime).

Examples: n = 1081 = 23*47 is a g-prime because 47 = 23*2 + 1 and also n = 1513 = 17*89 is a g-prime because 89 = 17*5 + 4.

Definition 2:

We name s-primes the semiprimes of the form p*q, p < q, with the property that q can be written as k*p - k + 1, where k is positive integer.

Examples: n = 91 = 7*13 is a s-prime because 13 = 7*2 - 1 and also n = 4681 = 31*151 is a s-prime because 151 = 31*5 - 4.

Observation:

The formula p^2 – q^2 + 1, where p and q are primes with the special property that the sums of their digits are equal, leads often to primes (of course, having only the digital root equal to 1) or to c/m-primes or g/s-primes.

SQUARES OF PRIMES WITH THE DIGITAL SUM 4

The sequence of this squares is:
: 121(=11^2), 10201(=101^2).

SQUARES OF PRIMES WITH THE DIGITAL SUM 10

The sequence of this squares is:
: 361(=19^2), 5041(=71^2).

SQUARES OF PRIMES WITH THE DIGITAL SUM 13

The sequence of this squares is:
: 49(=7^2), 841(=29^2), 2209(=47^2), 3721(=61^2), 6241(=79^2).

SQUARES OF PRIMES WITH THE DIGITAL SUM 16

The sequence of this squares is:
: 169(=13^2), 529(=23^2), 961(=31^2), 1681(=41^2), 3481(=59^2).

SQUARES OF PRIMES WITH THE DIGITAL SUM 19

The sequence of this squares is:
: 289(=17^2), 1369(=37^2), 2809(=53^2), 5329(=19^2).

SQUARES OF PRIMES WITH THE DIGITAL SUM 22

The sequence of this squares is:
: 1849(=43^2), 9409(=97^2).

Verifying the observation:
(up to the square of 101)

: 5041 – 361 + 1 = 4681 = 31*151,
a s-prime because 151 = 31*5 – 4, a c-prime because 151 – 31 + 1 = 121, a square of prime, and also a m-prime because 151 + 31 – 1 = 181, prime;

: 841 – 49 + 1 = 793 = 13*61,
a s-prime because 61 = 13*5 – 4, a c-prime because 61 – 13 + 1 = 49, a square of prime and a m-prime because 13 + 61 – 1 = 73, prime;
: 2209 – 49 + 1 = 2161, prime;
: 3721 – 49 + 1 = 3673, prime;
6241 – 49 + 1 = 6193 = 11*563,

a g-prime because 11*47 + 46 = 563 and a c-prime because 563 - 11 + 1 = 553 = 7*79 and 79 - 7 + 1 = 73, prime;

: 2209 - 841 + 1 = 1369, square of prime (37^2);

: 3721 - 841 + 1 = 2881 = 43*67,
a c-prime because 67 - 43 + 1 = 25, square of prime, and a m-prime because 43 + 67 - 1 = 109, prime;

: 6241 - 841 + 1 = 5401 = 11*491,
a g-prime because 491 = 11*41 + 40 and a c-prime because 491 - 11 + 1 = 481 = 13*37 and 37 - 13 + 1 = 25, square of prime;

: 3721 - 2209 + 1 = 1513 = 17*89,
a g-prime because 89 = 17*5 + 4 and a c-prime because 89 - 17 + 1 = 73, prime;

: 6241 - 2209 + 1 = 4033 = 37*109,
s-prime because 109 = 37*3 - 2, also a c-prime and m-prime;

: 6241 - 3721 + 1 = 2521, prime;

: 529 - 169 + 1 = 361, square of prime (19^2);

: 961 - 169 + 1 = 793 = 13*61 (see above);

: 1681 - 169 + 1 = 1513 = 17*89 (see above);

: 3481 - 169 + 1 = 3313, prime;

: 1681 - 529 + 1 = 1153, prime;

: 3481 - 529 + 1 = 2953, prime;

: 1681 - 961 + 1 = 721 = 7*103,
a q-prime because 103 = 7*13 + 12, a c-prime because 103 - 7 + 1 = 97, prime, and a m-prime because 103 + 7 - 1 = 109, prime;

: 3481 - 961 + 1 = 2521, prime;

: 3481 - 1681 + 1 = 1801, prime;

: 1369 - 289 + 1 = 1081 = 23*47,
g-prime because 47 = 23*2 + 1 and c-prime because 47 - 23 + 1 = 25, square of prime;

: 2809 - 289 + 1 = 2521, prime;

: 5329 - 289 + 1 = 5041, square of prime (71^2);

: 2809 - 1369 + 1 = 1441 = 11*131,
g-prime because 131 = 11*11 + 10 and c-prime because 131 - 11 + 1 = 121, square of prime;

: 5329 - 1369 + 1 = 3961 = 17*233,
g-prime because 233 = 17*13 + 12 and c-prime because 233 - 17 + 1 = 217 = 7*31 and 31 - 7 + 1 = 25, square of prime;

: 5329 - 2808 = 2521, prime;

: 9409 - 1849 + 1 = 7561, prime.

Comment:

One of the semiprimes obtained above, 4033(=37*109), is also a 2-Poulet number; many such numbers are g-primes or s-primes; to give to these semiprimes a name is justified at least in the study of Fermat pseudoprimes to base two with two prime factors (see the sequence A214305 in OEIS).