Physics of Elemental Space-Time and Cosmology

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Abstract We postulate that our space is filled by the "Gamma elements" having energy and mass with the size approximately given by the wavelength of the highest energy gamma rays and that time and distance are both discretized by the process of light propagation from one Gamma element to the next. These postulates provide us with a theoretical ground to explain why the speed of light, c, should remain constant to all observers regardless of their inertial frames of reference. In the cosmological scale, the energy of the Gamma elements filling space has been equated to the energy represented by the cosmological constant, i.e., the dark energy. When applied to the expanding universe, the EST model brings additional 25% of the total mass simply as the relativistic correction to the non-relativistic results; hence the dark matter is closely identified as merely the relativistic correction to the dark energy predicted by the Friedman equation. The agreement with the observed magnitudes convincingly supports this interpretation of dark energy and dark matter.

Keywords Space · Time · Element · Gravity · Cosmological Constant · Dark Energy· Dark Matter

1 ELEMENTAL SPACE-TIME

Our space is postulated to be filled with "Gamma elements" having extremely small mass, energy, and dimensions. It is further postulated that light propagates through the Gamma elements by energizing them, say by means of some process of relativistic boost of internal energy. Light cannot propagate without the medium of Gamma elements. The light propagation through Gamma elements defines the relativistic space and time.

We further postulate that each Gamma element occupies a cubicle of space with a linear dimension l_p and a volume l_p^3. Then the distance between two neighboring Gamma elements is also l_p. Furthermore, the time required for light energy to propagate from one Gamma element to the next is the elemental time interval, which we will denote t_p. Thus t_p and l_p are the elemental units of time and length, respectively, and $l_p/t_p \equiv c$ defines the speed of light propagation.

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The principle of relativity and that c is constant in all inertial frames leads to the theory of the special relativity. These two conditions of special relativity are both captured by the assumption that our space-time is discretized and that $l_p/t_p \equiv c$ by definition in all inertial frames. We will call the latter the principle of elemental space-time (EST), or simply the EST condition. This also asserts that the elemental length and time are the smallest units of length and time and that we merely count the number of l_p and t_p to perceive the distance and time, respectively. The magnitudes of l_p and t_p may change by the relativistic effect but their counts don't, hence the constancy of the speed of light.

We can build a bridge between the continuum physics and the elemental space-time as following. Now let n_s be the number of the Gamma elements energized per "second" by light, i.e., 1 second = $n_s t_p$. The distance traveled by light in one second then must be $n_s l_p$. The speed of the light propagation then is calculated to be

$$c = \frac{n_s l_p}{n_s t_p} = \frac{l_p}{t_p} \tag{1}$$

which is constant in all inertial frames.

Unfortunately we don't find a way to directly measure the values l_p or t_p. We note, however, l_p must be a true low limit for the wavelength of electromagnetic waves. We find the smallest electromagnetic wavelength that has been experimentally observed or projected comes from the ultrahigh energy gamma rays [1, 2, 3, 4] in the range,

$$\lambda_{\gamma\text{-}ray} \approx 1 \text{ x } 10^{-19} \text{ m} - 1 \text{ x } 10^{-25} \text{ m}.$$

We then deduce

$$l_p = 1 \text{ x } 10^{-19} \text{ m} - 1 \text{ x } 10^{-25} \text{ m}$$

hence

$$t_p = l_p/c = 3.34 \text{ x } 10^{-28} - 3.34 \text{ x } 10^{-34} \text{ s}.$$

We note that these elemental units, l_p and t_p, have the origins unrelated to the conventional Planck units.

2 THE COSMOLOGICAL CONSTANT

The stress energy tensor of the general relativity associated with the cosmological constant may be expressed as [5, 6].

$$T_{\mu\nu}^{(vac)} = -\frac{\Lambda c^4}{8\pi G} g_{\mu\nu} \quad (\mu, \nu = 0, 1, 2, 3) \tag{2}$$

where $T_{\mu\nu}^{(vac)}$ is the stress energy tensor of the vacuum, $g_{\mu\nu}$ the metric tensor, G the gravitational constant, and Λ the cosmological constant. The time-time component of the above is given by

$$\rho_{vac} \equiv \frac{\Lambda c^4}{8\pi G} \tag{3}$$

where ρ_{vac} is the vacuum energy density and have the same quality as the energy density of Gamma elements, $\rho_\gamma c^2$, with ρ_γ the mass density of the Gamma elements.

Hence by equating $\rho_\gamma = \rho_{vac}/c^2$, we get

$$\Lambda = \frac{8\pi G \rho_\gamma}{c^2} \tag{4}$$

where Λ has a dimension of [1/m^2]. In this way, we now have established the equivalence of the energy of Gamma elements and the energy of the cosmological constant. In other words, the existence of Gamma elements means the existence of the cosmological constant and vice versa.

3 RELATIVISTIC EXTENSION FOR THE NEWTON'S LAW OF GRAVITY

We begin by writing the Newton's gravitational law for two bodies as following:

$$F = -G\frac{M_1 M_2}{R^2} \tag{5}$$

where M_1 and M_2 are the masses of the two bodies and R is the distance between them. If they move with the velocity υ_1 and υ_2, respectively, without losing generality we assume $\upsilon_1 = -\upsilon_2 \equiv \upsilon$. We can then rewrite the above to include the special relativistic effect, where the force transforms as following:

$$F = -G\frac{M_1 M_2}{R^2}\gamma^2 \tag{6}$$

with γ, the Lorentz factor, given by

$$\gamma = 1\Big/\sqrt{1-\frac{\upsilon^2}{c^2}}\ . \tag{7}$$

The need for γ^2 term is also seen by the length contraction of R, a special relativistic effect when υ is not too small compared to c. If $\upsilon << c$, taking the first order approximation we have

$$F \approx -G\frac{M_1 M_2}{R^2}(1+\frac{\upsilon^2}{c^2})\ . \tag{8}$$

For the planetary motion around the Sun, let us replace M_1 by M_0, the mass of the Sun, M_2 by M_m, the mass of Mercury, and R by R_C, the average distance (i.e., of a circular motion) between the Sun and Mercury. The velocity of Mercury around the Sun may be approximated by ignoring the eccentricity of the orbit,

$$\upsilon^2 \approx \frac{GM_0}{R_C}, \tag{9}$$

and we then have

$$f \equiv \frac{F}{M_m} \approx -G\frac{M_0}{R_C^2}(1+\frac{GM_0}{c^2 R_C}) \tag{10}$$

where f is the specific force (i.e., per unit mass) for Mercury. We recall $\mathcal{K}^2 \equiv GM_0R_C$, where $\mathcal{K}$ is the specific angular momentum of Mercury for the orbital motion and nearly constant, the above then becomes

$$f = -\frac{GM_0}{R_C^2} - \frac{GM_0\mathcal{K}^2}{c^2 R_C^4}\ . \tag{11}$$

The second term is recognized, except for a numerical factor three, as the general relativistic correction, $3GM_0\,\mathcal{K}^2/(c^2R_C^4)$, for the planet motion that famously leads to the correction to the perihelion motion of Mercury [7, 8]. This lends credence to Eq. **(6)** being a valid relativistic extension to the Newton's law of gravitation for moving bodies.

4 TENSILE PRESSURE IN THE GAMMA ELEMENT SPACE

4.1 Non-relativistic Derivation

Let us take a place in the Gamma element space where the gravity of a particular star or planet does not dominate. Here a Gamma element is influenced by the gravity of all masses of the universe at a particular time or epoch. (See Fig. 1.) We assert that the Gamma element space is isotropic and homogeneous and we further assume that it behaves as perfect fluid, i.e., its behavior can be characterized by the mass density ρ_γ and isotropic pressure p_γ. ($-p_\gamma$ is tensile pressure.)

For simplicity, instead of single Gamma element we now consider a small spherical volume of radius Δa of Gamma elements located in the center of the Cartesian coordinate system. For the gravity force per the unit volume of Gamma elements, f_i, exerted by all other masses in the universe, the stress equilibrium requires

$$\sigma_{ji,j} + f_i = 0 \quad (i, j = 1,2,3; \text{ summed over repeated indices}). \tag{12}$$

But

$$\sigma_{ij} = -p_\gamma \delta_{ij} \quad (i, j = 1,2,3) \tag{13}$$

where δ_{ij} is the Kronecker delta. Hence we get

$$f_i = \nabla_i p_\gamma \quad (i = 1,2,3). \tag{14}$$

Applying the spherical symmetry, the only non-trivial component is that in the radial direction, r,

$$f_r = \partial_r p_\gamma . \tag{15}$$

If we define the gravitational potential

$$\Phi(r) = -\frac{G\rho_\gamma M}{r} \tag{16}$$

where G is the gravity constant, ρ_γ the mass density of the Gamma elements, M the mass of a mass element of the universe, and r the distance between the Gamma element and the mass element being considered.

We have

$$f_r = -\nabla\Phi = -\partial_r \Phi . \tag{17}$$

From Eqs. **(15)** and **(17)**, we then get

$$\nabla(p_\gamma + \Phi) = 0 , \tag{18}$$

hence

$$p_\gamma + \Phi = constant . \tag{19}$$

By combining Eqs. **(16)** and **(19)**, we get

$$p_\gamma = \frac{G\rho_\gamma M}{r} + constant . \tag{20}$$

We note that while the potential function Eq. **(16)** is a vector in the r-direction, the pressure p_γ is a scalar. To obtain the total pressure exerted by the gravity of the universe, we will consider, in particular, the pressure $p_{\gamma n}$ exerted by the mass M_n at a distance r_n,

$$p_{\gamma n} = \frac{G\rho_\gamma M_n}{r_n} + constant \,. \tag{21}$$

Now since p_γ is a scalar, we have the total pressure

$$p_\gamma = \sum_n p_{\gamma n} \tag{22}$$

where the summation is for all the masses M_n in the Universe (other than the small volume of the Gamma elements itself being considered). The summation may then be performed as following:

$$\sum_n \frac{M_n}{r_n} = \int_{\Delta a}^{r} \rho_u \frac{4\pi r^2}{r} dr = 2\pi\rho_u r^2$$

where ρ_u is the mass density and a the radius of the Universe.

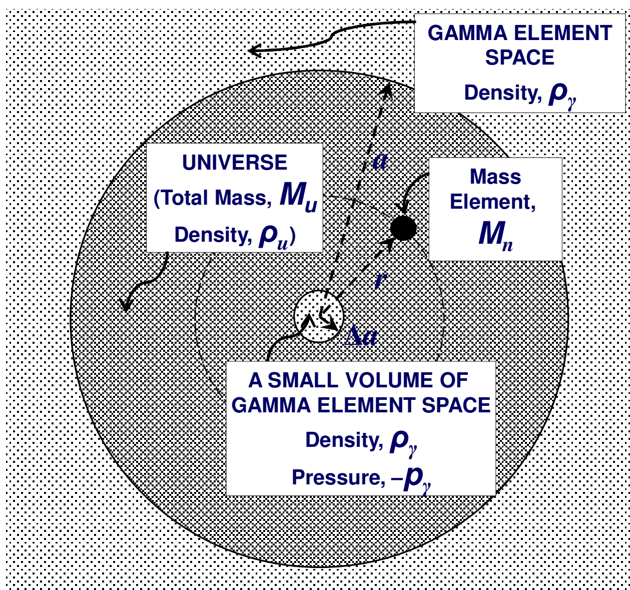

FIG. 1 An Elemental Space-Time Model for the Universe
(Not to scale)

The lower bound of the integration, Δa, may be replaced with zero without losing accuracy as long as Δa is sufficiently small compared to the upper bound, r. Hence we get

$$p_\gamma - C = \sum_n p_{\gamma n} = \sum_n \frac{G\rho_\gamma M_n}{r_n} = G\rho_\gamma \int_0^a \rho_u \frac{4\pi r^2}{r} dr = 2\pi G\rho_\gamma \rho_u a^2 \tag{23}$$

where C is constant. ρ_u is the average density of the Universe (from the radius Δa to a) that is the sum of the density of the Gamma elements, ρ_γ, which is constant and independent of a, and the density of all other matters that is proportional to a^{-3}.

Interpreting the above as a barotropic equation of state, the above equation then presents a local (inside the small volume considered) sound wave speed c_g [9, 10],

$$c_g^2 = \frac{\partial p_\gamma}{\partial \rho_\gamma} = 2\pi G \rho_u a^2 , \tag{24}$$

which may be recognized as the speed of the gravitational tensile pressure waves. One can, therefore, express the radius of the Universe,

$$a(\text{ NR }) = \frac{c_g}{\sqrt{2\pi G \rho_u}} \tag{25}$$

where (NR) indicates this is a non-relativistic calculation.

Let, $c_g \equiv qc$, where q is a dimensionless constant, we then have

$$G = zc^2 \tag{26}$$

where we define $z \equiv \frac{q^2}{2\pi\rho_u a}$, a constant.

One can obtain a similar relationship by simply quantizing G,

$$G = G_0 \frac{m^3}{kg \cdot s^2} = \frac{G_0}{c_0^2} \frac{m}{kg} c^2 . \tag{27}$$

Comparing the two, we get

$$z = \frac{q^2}{2\pi\rho_u a} = \frac{G_0}{c_0^2} \frac{m}{kg} . \tag{28}$$

According to the Hubble's law,

$$H = \frac{\dot{a}}{a} , \tag{29}$$

the velocity of the expansion would begin to exceed the speed of light, $\dot{a} = c$, at the critical radius,

$$a_c = \frac{c}{H} . \tag{30}$$

With the current value of the Hubble constant reported by NASA [11, 12],

H_0 = 2.25 x 10^{-18} s^{-1},

we get

a_c = 1.33 x 10^{26} m (or 1.41 x 10^{10} light-years.)

If $c_g = c$, the speed of light, then Eqs. **(23)** and **(24)** with $C = 0$ yields

$$\rho_\gamma = p_\gamma / c^2 . \tag{31}$$

Wilczek [13] calls the above (except for the sign) the well-tempered equation. Also if $c_g = c$, Eq. **(25)** is the same as the Einstein's static Universe obtained by the use of the Friedman equation but with a factor of √ 2 difference. NASA [11, 12] reports the current density of the Universe,

ρ_{u0} (NASA reported) = 9.90 x 10^{-27} kg/m^3.

If we use this value, Eq. **(25)** yields the current radius of the Universe

a_0 (NR, calculated) = 1.47 x 10^{26} m (or 1.56 x 10^{10} light years)

and Eq. **(23)** with $C = 0$ the tensile pressure

$-p_\gamma = -\rho_\gamma c^2 =$ –6.42 x 10^{-10} kg m s^{-2} m^{-2}.

4.2 Relativistic (R) Correction

Since this radius is greater than the critical value, thus according to this non-relativistic calculation, the outer edge the Universe is currently expanding at the speed greater than the speed of light. This is unsatisfactory since according to the special relativity (and EST), nothing will travel at the velocity greater than the speed of light. To include the special relativistic effect, we add the Lorentz factor to account for the length contraction of r in the gravity potential to get

$$\Phi(r) = -\frac{G\rho_\gamma M}{r}\gamma \tag{32}$$

where

$$\gamma = \frac{1}{\sqrt{1-(\frac{\dot{r}}{c})^2}}, \tag{33}$$

$\dot{r}$ denoting the time derivative of r or the radial velocity. Then Eq. **(23)** becomes

$$p_\gamma - C = \sum_n p_{\gamma n} = \sum_n \frac{G\rho_\gamma M_n}{r_n}\gamma_n = G\rho_\gamma \int_0^a \rho_u \frac{4\pi r^2}{r} \frac{1}{\sqrt{1-(\frac{\dot{r}}{c})^2}} dr. \tag{34}$$

Now we may apply the Hubble's empirical law $\dot{r} = Hr$ to the above equation assuming H is constant throughout the Universe at a particular time or epoch, and integrate over the Universe (Δa to a) to get

$$p_\gamma - C == 4\pi G\rho_\gamma \rho_u \frac{c^2}{H^2}\left(1-\sqrt{1-\frac{H^2}{c^2}a^2}\right). \tag{35}$$

The above equation Eq. **(35)** presents a gravitational tensile pressure wave characterized by the sound wave speed

$$c_g^2 = \frac{\partial p_\gamma}{\partial \rho_\gamma} = 4\pi G\rho_u \frac{c^2}{H^2}\left(1-\sqrt{1-\frac{H^2}{c^2}a^2}\right). \tag{36}$$

If $\dot{a} = Ha << c$, Eqs. **(35)** and **(36)** are approximated to Eqs. **(23)** and **(24)**, respectively, as expected.

We can also express the radius of the Universe,

$$a = \frac{c_g}{\sqrt{2\pi G\rho_u}}\sqrt{1-\frac{c_g^2}{c^2}\frac{H^2}{8\pi G\rho_u}}. \tag{37}$$

If $c_g = c$, this reduces to

$$a(\mathrm{R}) = \frac{c}{\sqrt{2\pi G\rho_u}}\sqrt{1-\frac{H^2}{8\pi G\rho_u}} \tag{38}$$

where (R) indicates this is a relativistic calculation.

If $c_g = c$, the speed of light, then Eqs. **(35)** and **(36)** with $C = 0$ again yields Eq. **(31)**, the well-tempered equation. The well-tempered equation is, therefore, relativistic.

Again, by the use of NASA [11, 12] - reported current values, H_0 and ρ_{u0}, we get

$$a_0 \text{ (R, calculated)} = 1.23 \times 10^{26} \text{ m } (1.30 \times 10^{10} \text{ light years,})$$

a significant reduction from the non-relativistic value of a_0 (NR). The tensile pressure $-p_\gamma = -\rho_\gamma c^2 = -6.42 \times 10^{-10}$ kg m s^{-2} m^{-2}, remains the same as the non-relativistic value. Thus, the radius of the Universe calculated with the relativistic effect is smaller than the critical radius, Eq. **(30)** and the outer edge of the Universe is currently expanding at the speed slightly lower than the speed of light. In this way, we have shown that the Gamma element space is consistent with the presence of negative vacuum pressure and presence of the gravitational waves with a sound wave speed, c_g. There is, however, currently no firm theoretical ground to assert that c_g is equal to the speed of light, c.

5 THE EXPANDING UNIVERSE

We use Newton's shell theorem to calculate the total energy, the sum of the kinetic energy (*K.E.*) and the potential energy (*P.E.*), of the n^{th} particle having a mass, M_n, at the radius a from the center. Note that Newton's shell theorem ignores the gravitational pressure derived in the previous section. It is justified since such pressure is in equilibrium and does not contribute to the motion of the particle. Hence we write an equation for the total energy,

$$K.E. + P.E. = \gamma M_n c^2 - M_n c^2 - \gamma \frac{GM_n M_u}{a} = const \equiv \pm M_n \kappa^2 c^2 \tag{39}$$

where

$$\gamma = \frac{1}{\sqrt{1 - (\frac{\dot{a}}{c})^2}} \tag{40}$$

and M_u is the total mass within the sphere of the radius $a(t)$, a function of time. We have set the constant arbitrarily to be $\pm M_n c^2 \kappa^2$ where κ is a constant without losing generality. If $\kappa > 0$, the kinetic energy provides more than the escape velocity; if $\kappa = 0$, the kinetic energy provides just enough an escape velocity; and if $\kappa < 0$, the kinetic energy is insufficient for M_n to overcome the gravity of M_u and escape. This leads to

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{(1 \pm \kappa^2)^2}\left[\frac{2GM_u}{a^3} - \frac{1}{c^2}\frac{G^2 M_u^2}{a^4} + \frac{\kappa^2 c^2 (\pm 2 + \kappa^2)}{a^2}\right]. \tag{41}$$

For sufficiently small κ, i.e., $\kappa << 1$, the above approximates to

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{2GM_u}{a^3} - \frac{G^2 M_u^2}{c^2 a^4} \pm \frac{2\kappa^2 c^2}{a^2}. \tag{42}$$

The non-relativistic version of the above, i.e., without the second term, is in the same form as the Friedman equation. A term which is proportional to a^{-4} is interpreted in the literature as radiation dependent case of the first term as opposed to matter dependent first term. Here the second term, which is also proportional to a^{-4}, is simply a relativistic correction to the first term.

We note

$$M_u = M_\gamma + M_a$$

where M_a is the mass of all matters other than M_γ, the mass of the Gamma elements, within the sphere of the radius a. Note that M_γ is proportional to a^3 ($M_\gamma \propto a^3$) while M_a is constant according to the model shown in Fig. 1. Also note that ρ_γ is constant while ρ_a is proportional to a^{-3} ($\rho_a \propto 1/a^3$.) Some useful relationships are listed below where the subscript $_0$ denotes the value at the present epoch:

$$\rho_\gamma = \frac{M_\gamma}{\frac{4\pi}{3}a^3} = \frac{M_{\gamma 0}}{\frac{4\pi}{3}a_0^3} = Constant,$$

$$M_a = \frac{4\pi}{3}a^3\rho_a = \frac{4\pi}{3}a_0^3\rho_{a0} = Constant,$$

$$\rho_a = \rho_{a0}\frac{a_0^3}{a^3},$$

$$\rho_u = \frac{M_u}{\frac{4\pi}{3}a^3} = \rho_\gamma + \rho_a = \rho_\gamma + \rho_{a0}\frac{a_0^3}{a^3},$$

and

$$\frac{M_\gamma}{M_a} = \frac{\rho_\gamma}{\rho_a}.$$

Now we get from Eq. **(42)**,

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G\rho_u}{3} - \frac{16\pi^2 G^2\rho_u^2}{9}\frac{a^2}{c^2} \pm \frac{2\kappa^2 c^2}{a^2}. \tag{43}$$

Eq. **(43)** may be also be written as

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}(\rho_\gamma + \rho_a) - \frac{16\pi^2 G^2}{9}\frac{a^2}{c^2}(\rho_\gamma + \rho_a)^2 \pm \frac{2\kappa^2 c^2}{a^2} \tag{44}$$

or as

$$\left(\frac{\dot a}{a}\right)^2 = \frac{\Lambda c^2}{3}\left(1 + \frac{\delta^3}{a^3}\right) - \frac{\Lambda^2 c^2}{36}a^2\left(1 + \frac{2\delta^3}{a^3} + \frac{\delta^6}{a^6}\right) \pm \frac{2\kappa^2 c^2}{a^2} \tag{45}$$

where $\delta^3 \equiv \rho_{a0}\, a_0^3/\rho_\gamma$ and $\Lambda \equiv 8\pi G\rho_\gamma/c^2$.

If $\kappa = 0$, the above yields some useful results as following.

5.1 Relativistic (R) Treatment

The relativistic (R) case is obtained from Eq. **(43)** (with $\kappa = 0$,)

$$\left(\frac{\dot a}{a}\right)^2 \equiv H^2 = \frac{8\pi G\rho_u}{3} - \frac{16\pi^2 G^2\rho_u^2}{9}\frac{a^2}{c^2}. \tag{46}$$

This, together with the relativistic equation of state Eq. **(36)** and $c_g = c$, are solvable for ρ_u and a to yield,

$$\rho_u(\mathrm{R}) = \frac{H^2}{2\pi G} \tag{47}$$

and

$$a(\mathrm{R}) = \frac{c}{\sqrt{(8/3)\pi G\rho_u}} = \frac{\sqrt{3}}{2}\frac{c}{H}. \tag{48}$$

With H_0 = 2.25 x 10^{-18} s^{-1}, we get

ρ_{u0} (R, calculated) =12.04 x 10^{-27} kg/m^3,

a_0(R, calculated) =1.16 x 10^{26} m (or 1.22 x 10^{10} light years)

and $a_0 < a_c$, a_c being given by Eq. **(30)**.

From Eq. **(45)** we get (with $\kappa = 0$,)

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\Lambda c^2}{3}\left(1+\frac{\delta^3}{a^3}\right) - \frac{\Lambda^2 c^2}{36}a^2\left(1+\frac{2\delta^3}{a^3}+\frac{\delta^6}{a^6}\right). \tag{49}$$

Eq. **(49)** may be written in an integral form to calculate the age of the Universe

$$\int \frac{a\,da}{\sqrt{a^3+\delta^3}\sqrt{\frac{12}{\Lambda}a-(a^3+\delta^3)}} = \int \frac{\Lambda c}{6}dt. \tag{50}$$

Unfortunately, a closed form solution is not easily available for the above equation. Some solutions are available for limiting cases; we will not, however, pursue those here.

5.2 Non-Relativistic (NR) Limit

A non-relativistic (NR) case is obtained by ignoring the second term of Eq. **(43)**. If also $\kappa = 0$, the same equation becomes

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8\pi G\rho_u}{3}. \tag{51}$$

This, together with the non-relativistic current equation of state Eq. **(24)** and with $c_g = c$, are solvable for ρ_u and a to yield

$$\rho_u(\mathrm{NR}) = \frac{3H^2}{8\pi G} \tag{52}$$

and

$$a(\mathrm{NR}) = \frac{c}{\sqrt{2\pi G\rho_u}} = \frac{2}{\sqrt{3}}\frac{c}{H}. \tag{53}$$

With H_0 = 2.25 x 10^{-18} s^{-1}, we get

ρ_{u0} (NR, calculated) = 9.03 x 10^{-27} kg/m^3,

a_0 (NR, calculated) = 1.54 x 10^{26} m or 1.63 x 10^{10} light years,

and a_0 (NR) > a_c, a_c being given by Eq. **(30)**.

Eq. **(51)** may also be written as,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\rho_\gamma + \rho_{a0}\frac{a_0^3}{a^3}\right). \tag{54}$$

The above equation is identified as the Friedman equation for the flat Universe, $\kappa = 0$, and may be integrated for time to yield (cf. [14]),

$$t(\mathrm{NR}) = \frac{1}{\sqrt{6\pi G\rho_\gamma}} \sinh^{-1}\left(\frac{\rho_\gamma}{\rho_a}\right)^{1/2} . \qquad (55)$$

We can invert Eq. **(55)** to get

$$\rho_a = \frac{\rho_\gamma}{\left[\sinh(\sqrt{6\pi G\rho_\gamma}\, t)\right]^2} . \qquad (56)$$

If we use the NASA - reported values, $\rho_{\gamma 0}$ = 7.14 x 10^{-27} kg/m^3 and t_0 = 1.37 x 10^{10} years, then we get

ρ_{a0} (calculated)= 2.51 x 10^{-27} kg/m^3.

Since we measure only 0.46 x 10^{-27} kg/m^3 for the observable matter density, the rest of the matter density, (2.51-0.46) x 10^{-27} kg/m^3 = 2.05 x 10^{-27} kg/m^3, must be attributed to something unknown, i.e., dark matter. This is in good agreement with the NASA estimated dark matter density, 2.31 x 10^{-27} kg/m^3.

Clearly, the dark matter postulate is necessary to account for the measured values of the density and the age of the Universe so long as we take the non-relativistic Friedman equation to model our Universe as in the ΛCDM model.

3.5 DARK ENERGY AND DARK MATTER

An inventory of the cosmic mass densities is shown in Fig. 2, divided into the observable matter, dark energy, and dark matter. Those obtained by the present EST model are compared with those obtained by the use of the standard ΛCDM model as reported from both WMAP [11, 12] and Planck projects [15, 16]. WMAP reports the densities with real SI units while Planck only with the ratios over total.

The mass densities of the Universe according to NASA measurements [11, 12] (as we used some of them in the above) are summarized below:

ρ_{u0} (Universe) ≈ 9.90 x 10^{-27} kg/m^3

ρ_{a0} (observable matter-baryonic and all others)

≈ 0.46 x 10^{-27} kg/m3 (4.6%)

ρ_{dm} (dark matter) ≈ 2.31 x 10^{-27} kg/m3 (23.3%)

ρ_{de} (dark energy) ≈ 7.14 x 10^{-27} kg/m3 (72.1%)

Observable matter includes baryonic matter and radiation including the cosmic microwave background (CMB) radiation [17].

The key numbers from the EST calculation are:

the relativistic total density

ρ_{u0}(R, calculated) =12.04 x 10^{-27} kg/m^3, Eq. **(47)**,

vs. non-relativistic total density

ρ_{u0}(NR, calculated) = 9.03 x 10^{-27} kg/m^3, Eq. **(52)**.

The difference is 3.01 x 10^{-27} kg/m^3 and shows that the correction due to the relativistic effect amounts to 25% of the relativistic total density. The non-relativistically calculated density, 9.03 x 10^{-27} kg/m^3, is reasonably close to the NASA-estimated density of the Universe, 9.90 x 10^{-27} kg/m^3.

		ΛCDM WMAP 10^{-27} kg/m^3	ΛCDM Planck (% of ρ_u)	EST 10^{-27} kg/m^3
ρ_{a0}	Measured	0.46	4.9	0.46
ρ_γ(NR) or Dark Energy	Calculated	7.14	68.3	8.57
ρ_{dm} Dark Matter	Calculated	2.31	26.8	N/A
ρ_{u0}(NR)	Calculated	9.90	100	9.03
ρ_R Relativistic Correction	Calculated	N/A	N/A	3.01
ρ_γ(R) or Dark Energy	Calculated	N/A	N/A	11.58
ρ_{u0} (R)	Calculated	N/A	N/A	12.04
ρ_{dm}/ρ_{u0}(NR)		23.3%	26.8%	N/A
ρ_R/ρ_{u0}(R)		N/A	N/A	25.0%

FIG. 2 Cosmic Energy Inventory
(Unit: 10^{-27} kg/m^3 or % of ρ_{u0})
(ρ_a scales with a^{-3} and is the energy density of all matters excluding ρ_γ. The latter, the energy density of Gamma elements, is identified as the vacuum energy density corresponding to the cosmological constant.)

A direct measurement of the densities is possible only for the observable matter which from WMAP measurement is 0.46 x 10^{-27} kg/m^3. With the relativistic EST result, therefore, we would simply interpret the Universe being composed of 0.46 x 10^{-27} kg/m^3 observable matter and the remaining (12.04 - 0.46) x10^{-27} kg/m^3 = 11.58 x10^{-27} kg/m^3 the energy of Gamma elements or the dark energy. If we used the non-relativistic limit of the EST calculation, this would be 0.46 x10^{-27} kg/m^3 observable matter and (9.03 - 0.46) x 10^{-27} kg/m^3 = 8.57 x10^{-27} kg/m^3 the energy of Gamma elements or the dark energy. We have 3.01 kg/m^3 simply as the relativistic correction.

The Friedman equation can be derived from the non-relativistic Newtonian gravity equation hence is a non-relativistic special solution of the general relativity despite the fact that it originates from the general relativity. This means that Friedman equation would miss the amount corresponding to the relativistic correction. Indeed, the results of the ΛCDM model reports the unexplained dark matter which is approximately 23.3% of the total by WMAP and 26.8% by the Planck.

The relativistic EST model brings additional 25% of the total mass simply as the relativistic correction to the non-relativistic results calculated by the Friedman equation. The agreement among the magnitudes is exceedingly convincing that this interpretation of the dark matter is correct. In this way, the equivalence of the Gamma elements, the

Einstein's cosmological constant, and the dark energy has also been convincingly demonstrated.

4 SUMMARY AND CONCLUDING REMARKS

The EST model may be viewed as bringing the aether back, only this time the aether is not absolute, but comprised of material elements called Gamma elements having energy and corresponding mass. The hypothesis of space-filling materials is shown to be in line with the existence of the cosmological constant and the corresponding energy, i.e., the dark energy. The space filled with Gamma elements is theoretically shown to be subject to negative gravitational pressure and the gravitational pressure waves. By this cosmological model combined with a relativistic extension to the Newton's law of gravity hence to the Friedman equation, we show the predicted dark energy to be in close agreement with the observed magnitudes but also the dark matter most likely to be mere relativistic correction to the non-relativistic estimate of the dark energy.

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