

Non-Euclidean Mathematics

$$m \quad n$$

$$m^2 - n^2 \quad 2mn \quad m^2 + n^2$$

form a Pythagorean triangle (triple)

$$A^2 + B^2 = C^2$$

Example $m = 2 \quad n = 1$

$$m^2 - n^2 \quad 2mn \quad m^2 + n^2$$

$$2^2 - 1^2 \quad 2*2*1 \quad 2^2 + 1^2$$

$$3^{(2)} + 4^{(2)} = 5^{(2)}$$

Formula Expansion

C fix

$$(mA) - (nB) = A' \quad (mB) + (nA) = B'$$

$$3^{(2)} + 4^{(2)} = 5^2 \quad 6 - 4 = 2 \quad 8 + 3 = 11$$

$$2^{(2)} + 11^{(2)} = 5^3 \quad 4 - 11 = -7 \quad 22 + 2 = 24$$

$$-7^{(2)} + 24^{(2)} = 5^4 \quad -14 - 24 = -38 \quad 48 + -7 = 41$$

$$-38^{(2)} + 41^{(2)} = 5^5 \quad -76 - 41 = -117 \quad 82 + -38 = 44$$

$$-117^{(2)} + 44^{(2)} = 5^6 \quad \dots \text{infinite...}$$

Formula Expansion

A fix

$$\begin{array}{rcl}
 & (mB) + (nC) = B' & (mC) + (nB) = C' \\
 \\
 3^2 + 4^{(2)} = 5^{(2)} & 8 + 5 = 13 & 10 + 4 = 14 \\
 \\
 3^3 + 13^{(2)} = 14^{(2)} & 26 + 14 = 40 & 28 + 13 = 41 \\
 \\
 3^4 + 40^{(2)} = 41^{(2)} & 80 + 41 = 121 & 82 + 40 = 122 \\
 \\
 3^5 + 121^{(2)} = 122^{(2)} & 242 + 122 = 364 & 244 + 121 = 365 \\
 \\
 3^6 + 364^{(2)} = 365^{(2)} & \dots\text{infinite...} &
 \end{array}$$

Formula Expansion

B fix

$$\begin{array}{rcl}
 m^2 - n^2 & 2mn & m^2 + n^2 \\
 \\
 3^2 + 4^2 = 5^2 & & \\
 \\
 2m^3 - n^3 & 2mn & 2m^3 + n^3 \\
 \\
 15^2 + 4^3 = 17^2 & & \\
 \\
 4m^4 - n^4 & 2mn & 4m^4 + n^4 \\
 \\
 63^2 + 4^4 = 65^2 & & \\
 \\
 \text{general formula} & m^z * 2^{(z-2)} - n^z & 2mn & m^z * 2^{(z-2)} + n^z
 \end{array}$$

Trivial solutions (elsewhere) C fix

$$\begin{array}{rcl}
 & (mA) - (nB) = A' & (mB) + (nA) = B' \\
 \begin{matrix} 3^{(2)} \\ 2^{(2)} \end{matrix} + \begin{matrix} 4^{(2)} \\ 11^{(2)} \end{matrix} = \begin{matrix} 5^2 \\ 5^3 \end{matrix} & \begin{matrix} 6 \\ 4 \end{matrix} - \begin{matrix} 4 \\ 11 \end{matrix} = \begin{matrix} 2 \\ -7 \end{matrix} & \begin{matrix} 8 \\ 22 \end{matrix} + \begin{matrix} 3 \\ 2 \end{matrix} = \begin{matrix} 11 \\ 24 \end{matrix} \\
 & (mA) + (nB) = A' & (mB) - (nA) = B' \\
 -7^{(2)} + 24^{(2)} = 5^4 & -14 + 24 = 10 & 48 - -7 = 55 \\
 & (mA) - (nB) = A' & (mB) + (nA) = B' \\
 10^{(2)} + 55^{(2)} = 5^5 & 20 - 55 = -35 & 110 + 10 = 120 \\
 -35^{(2)} + 120^{(2)} = 5^6 & \dots &
 \end{array}$$

A fix

$$\begin{array}{rcl}
 & (mB) + (nC) = B' & (mC) + (nB) = C' \\
 3^2 + 4^{(2)} = 5^{(2)} & 8 + 5 = 13 & 10 + 4 = 14 \\
 3^3 + 13^{(2)} = 14^{(2)} & 26 + 14 = 40 & 28 + 13 = 41 \\
 & (mB) - (nC) = B' & (mC) - (nB) = C' \\
 3^4 + 40^{(2)} = 41^{(2)} & 80 - 41 = 39 & 82 - 40 = 42 \\
 & (mB) + (nC) = B' & (mC) + (nB) = C' \\
 3^5 + 39^{(2)} = 42^{(2)} & 78 + 42 = 120 & 84 + 39 = 123 \\
 3^6 + 120^{(2)} = 123^{(2)} & \dots &
 \end{array}$$

note it's not necessary to change again immediately

Trivial solutions for B fix

because we extend m with 2

$$m = 1 \quad n = 2$$

$$2m^3 - n^3 \quad 2mn \quad 2m^3 + n^3$$

$$-6^2 \quad + \quad 4^3 \quad = \quad 10^2$$

We have multiplication, addition and subtraction

so it's naturally to make also division

with $m = 2$, $n = 1$ and C fix we have a simple primality test

Simpliest:

Every prime number $x \equiv 1 \pmod{4}$ divides "the $x - 1$ step"

Every prime number $y \equiv 3 \pmod{4}$ divides "the $y + 1$ step"

in B

Simpler:

Every prime number $x \equiv 1 \pmod{4}$ divides "the $(x - 1) / 2$ step"

Every prime number $y \equiv 3 \pmod{4}$ divides "the $(y + 1) / 2$ step"

in A or B

Example $x \equiv 1 \pmod{4} = 13$ $13 - 1 = 12$ $12 / 2 = 6$

$y \equiv 3 \pmod{4} = 11$ $11 + 1 = 12$ $12 / 2 = 6$

step

$$2 \quad 3^{(2)} + 4^{(2)} = 5^2$$

$$3 \quad 2^{(2)} + 11^{(2)} = 5^3$$

$$4 \quad -7^{(2)} + 24^{(2)} = 5^4$$

$$5 \quad -38^{(2)} + 41^{(2)} = 5^5$$

$$6 \quad -117^{(2)} + 44^{(2)} = 5^6 \quad (117 = 3^2, 13 \quad 44 = 2^2, 11)$$

...

$$9 - 718^{(2)} + - 1199^{(2)} = 5^9$$

...

$$12 \quad 11753^{(2)} + - 10296^{(2)} = 5^{12} \quad (10296 = 2^3, 3^2, 11, 13)$$

also a simple Primality Test with $m = 2$, $n = 1$ and A fix

$$3^{(x-1)} = x \text{ divides } B \text{ if prime}$$

$$3^{((x-1)/2)} = x \text{ divides } B \text{ or } C \text{ if prime}$$

$$\text{Example} \quad x = 13 \quad 3^{(13-1)} = 3^{12}$$

$$3^{12} + 265720^2 = 265721^2 \quad (2^3, 5, 7, 13, 73)$$

$$3^{((13-1)/2)} = 3^6$$

$$3^6 + 364^{(2)} = 365^{(2)} \quad (2^2, 7, 13) \quad (5, 73)$$

and a simple Primality Test for

$$2^{(x-1)} - 1 = x \text{ divides if prime}$$

$$2^{((x-1)/2)} + - 1 = x \text{ divides if prime}$$

$$\text{Example} \quad x = 13 \quad 2^{(13-1)} - 1 = 2^{12} - 1 = 4095 \quad (3^2, 5, 7, 13)$$

$$2^{((13-1)/2)} - 1 = 2^6 + - 1 = 65 \quad (5, 13) \quad 63 \quad (3^2, 7)$$

With *Some Number Examples* we see that we can give all possible even even odd

$$A \text{ fix} \quad m = 4 \quad n = 1 \quad \text{we get} \quad A^3 + B^4 = C^2$$

$$15^2 + 8^{(2)} = 17^{(2)} \quad 32 + 17 = 49 \quad 68 + 8 = 76$$

$$15^3 + 49^{(2)} = 76^{(2)} \quad \dots$$

$$(15^3 + 7^4 = 76^2)$$

$$A \text{ fix} \quad m = 12 \quad n = 1 \quad \text{we get} \quad A^3 + B^2 = C^4$$

$$143^2 + 24^{(2)} = 145^{(2)} \quad 288 + 145 = 433 \quad 1740 + 24 = 1764$$

$$143^3 + 433^{(2)} = 1764^{(2)} \quad \dots$$

$$(143^3 + 433^2 = 42^4)$$

$$A \text{ fix} \quad m = 2 \quad n = 1 \quad \dots \text{we get...} \quad A^5 + B^4 = C^2$$

$$3^5 + 121^{(2)} = 122^{(2)}$$

$$(3^5 + 11^4 = 122^2)$$

$$C \text{ fix} \quad m = 3 \quad n = 2 \quad \text{we get} \quad A^4 + B^2 = C^3$$

$$5^{(2)} + 12^{(2)} = 13^2 \quad 15 - 24 = -9 \quad 36 + 10 = 46$$

$$-9^{(2)} + 46^{(2)} = 13^3 \quad \dots$$

$$(-3^4 + 46^2 = 13^3)$$

C fix $m = 122$ $n = 27$ we get $A^2 + B^8 = C^3$

$$14155^{(2)} + 6588^{(2)} = 15613^2 \quad 1726910 - 177876 = \textcolor{red}{1549034} \quad 803736 + 382185 = \textcolor{green}{1185921}$$

$$\textcolor{red}{1549034}^{(2)} + \textcolor{green}{1185921}^{(2)} = 15613^3 \quad \dots$$

$$(1549034^2 + 33^8 = 15613^3)$$

With B $m = 1$ $n = 3$ $2m^3 - n^3, 2mn, 2m^3 + n^3$ we get $A^4 + B^3 = C^2$

$$-25^2 + 6^3 = 29^2$$

$$(-5^4 + 6^3 = 29^2)$$

With B $m = 203$ $n = 237$ $2m^3 - n^3, 2mn, 2m^3 + n^3$ we get $A^8 + B^3 = C^2$

$$3418801^2 + 96222^3 = 30042907^2$$

$$(43^8 + 96222^3 = 30042907^2)$$