The minimal non-realistic modification of Quantum Mechanics

Jiří Souček

Charles University in Prague
Faculty of Philosophy
U Kříže 8, Prague 5, 158 00
jiri.soucek@ff.cuni.cz

Abstract In this article we consider the variant of quantum mechanics (QM) which is based on the non-realism. There exists the theory of the modified QM introduced in [1] and [2] which is based on the non-realism, but it contains also other changes with respect to the standard QM (stQM). We introduce here the other non-realistic modification of QM (n-rQM) which contains the minimal changes with respect to stQM. The change consists in the replacement of the von Neumann‘s axiom (ensembles which are in the pure state are homogeneous) by the anti von Neumanns axiom (any two different individual states must be orthogonal). This introduces the non-realism into n-rQM. We shall show that experimental consequences of n-rQM are the same as in stQM, but these two theories are substantially different. In n-rQM it is not possible to derive (using locality) the Bell inequalities. Thus n-rQM *does not imply* the non-locality (in contrast with stQM). Because of this the locality in n-rQM can be restored. The main purpose of this article was to show what could be the minimal modification of QM based on the non-realism, i.e. that the realism of stQM is completely contained in the von Neumanns axiom.

1 Introduction

The derivation of Bell's inequalities (BI) implies the choice between non-locality and non-realism. The variant based on the non-locality was studied in many papers (see e.g. [5]). We have choosen the non realism in [1]. Then we have studied this variant in [3] and [2]. In [2] we have studied the axiomatic formulation of the so-called modified Quantum Mechanics (modQM) where we have made two changes with respect to the standard QM:

(i) to replace the von Neumann's axiom $\mathbf{Ax}_{vN}$ by the anti-von Neumann's axiom $\mathbf{Ax}_{avN}$

(ii) to replace the concept of the measurement by the concept of the observation.

In this paper we want to apply the change (i) without applying (ii) - i.e. to introduce only the anti-von Neumann's axiom $\mathbf{Ax}_{avN}$. In this way we introduce the minimal non-realistic modification of QM (n-rQM).

In the second part we shall describe the simple axiomatization for stQM. In part 3. we describe n-rQM. In part 4. we shall discuss the properties of n-rQM. In part 5. we shall formulate conclusions.

We shall show that experimental consequences of n-rQM are the same as in stQM. These two theories are experimentally indistinguishable. But theoretical consequences are different.

$$\text{locality} + \text{stQM} \Rightarrow \text{BI}$$

$$\text{locality} + \text{n-rQM} \nRightarrow \text{BI}.$$

Thus in n-rQM the locality can be restored (as in modQM). The n-rQM is closer to stQM than modQM - it is the minimal modification of stQM. Only one axiom is changed and this change has no experimental consequences.

Acknowledgments.
Many thanks to J. Richter (from Charles University, Prague) for helping me with the preparation of this paper.

2 The axiomatic description of stQM.

Here we shall sum up standard axioms but with two modifications

1. we shall reformulate axioms for ensembles, since this is necessary for our purpose
2. we shall explicitly discuss so-called von Neumann's axiom ($\mathbf{Ax}_{vN}$) - see [3] - since this is central for our considerations.

Definition 2.1. The ensemble is the set of systems

$$\mathbb{E} = \{S_1, \dots, S_N\}, \ N \to \infty$$

which are prepared by some preparation procedure.

Ax1 (Hilbert space.) To each system S it is associated its Hilbert space $\mathcal{H}_S$. We shall assume that $\mathcal{H}_S$ is the finite-dimensional complex Hilbert space. It is assumed that in the ensemble $\mathbb{E}$, the Hilbert spaces are same

$$\mathcal{H}_{S_1} = \dots = H_{S_N} = \mathcal{H}_{\mathbb{E}}.$$

Ax2 (States of an ensemble.) Possible states of an ensemble $\mathbb{E}$ are given by density operators in $\mathcal{H}_{\mathbb{E}}$,

$$St(\mathcal{H}_{\mathbb{E}}) = \{\varrho : \mathcal{H}_{\mathbb{E}} \to \mathcal{H}_{\mathbb{E}} \mid \varrho = \varrho^*, \ \varrho \geq 0, \ tr\varrho = 1\}.$$

Definition 2.2.. $\varrho \in St(\mathcal{H}_{\mathbb{E}})$ is a pure state iff there exists $\psi \in \mathcal{H}_{\mathbb{E}}$ such that

$$\varrho = P_\psi = \psi \otimes \psi^*, \ ||\psi|| = 1.$$

Pure states can be parametrized by rays

$$\vec{\psi} = \{\alpha\psi \mid \alpha \in \mathbb{C}, \ |\alpha| = 1\}, \ ||\psi|| = 1.$$

The space of pure states is the projective Hilbert space

$$\mathcal{PH}_{\mathbb{E}} = \{\vec{\psi} \mid \psi \in \mathcal{H}_{\mathbb{E}}, \ ||\psi|| = 1\}.$$

Ax3 (Evolution.) The evolution of the state of an ensemble is given by

$$\varrho(t) = \mathcal{U}_t \varrho(0) \mathcal{U}_t^*, \ t \in \mathbb{R}$$

where $\mathcal{U}_t = exp(-iHt)$ is one-parameter unitary group in $\mathcal{H}_{\mathbb{E}}$.

Ax4 (Composition.) For the composition of ensembles $\mathbb{E} = \mathbb{E}_1 \oplus \mathbb{E}_2$ we have

$$\mathcal{H}_{\mathbb{E}} = \mathcal{H}_{\mathbb{E}_1} \otimes \mathcal{H}_{\mathbb{E}_2}$$

and if systems in $\mathbb{E}_1$ are independent form systems from $\mathbb{E}_2$, then

$$\varrho = \varrho_1 \otimes \varrho_2.$$

The next three axioms describe the measurement. Let A be an observable, i.e. the self adjoint operator in $\mathcal{H}_{\mathbb{E}}$ with the spectral decomposition

$$A = \sum_{i=1}^{n} a_i P_i, \;\; P_i = \phi_i \otimes \phi_i^*.$$

Ax6 (Possible outputs.) The output value belongs to the set

$$spA = \{a_1, \dots, a_n\}.$$

Ax7 (Born's rule.) The probability of obtaining the output a_k is given by

$$prob(a_k \mid \varrho, A) = tr(\varrho P_k)$$

This means the following. Let us define the ensemble

$$\mathbb{E}_{a_k} = \{S \in \mathbb{E} \mid \text{ output of } S = a_k\}.$$

Then the relative frequency of the output a_k is

$$\frac{1}{N} \cdot |\mathbb{E}_{a_k}| \to \; prob(a_k \mid \varrho, A) \; as \; N \to \infty.$$

Ax8 (Collapse rule.)Let A be a non degenerate observable (i.e. $a_i \neq a_j, \;\forall i \neq j$). Then after the measurement od A the state of a sub-ensemble $\mathbb{E}_{a_k}$ will be $P_k = \phi_k \otimes \phi_k^*$.

Up to now all axioms describe ensembles, their states and their evolutions and measurement processes. In the last axiom we shall specify the concept of a state of an individual system (=individual state).

The concept of an individual state was defined by von Neumann in his classical monograph [6] in the following way. The ensemble $\mathbb{E}$ is homogeneous, if all systems $S \in \mathbb{E}$ are in the same (individual) state. It is equivalent to say that the state $\varrho \in St(\mathcal{H}_{\mathbb{E}})$ is an individual state if the ensemble $\mathbb{E}$ in the state ϱ is homogeneous. We postulate that for each ensemble $\mathbb{E}$ there exists a subset of individual states

$$\tilde{D_{\mathbb{E}}} \subset St(\mathcal{H}_{\mathbb{E}}).$$

In the axiom 5 we shall describe the basic properties of the set $\tilde{D_{\mathbb{E}}}$ of individual states (discussed by von Neumann)

Ax5 The set $\tilde{D_{\mathbb{E}}}$ of individual states must satisfy

(i) $\tilde{D_{\mathbb{E}}} \subseteq \mathcal{PH}_{\mathbb{E}}$, i.e. only pure states could be individual states

(ii) $\tilde{D_{\mathbb{E}}}$ generate $\mathcal{PH}_{\mathbb{E}}$, i.e. each ray $\vec{\psi} \in \mathcal{PH}_{\mathbb{E}}$ can be written as a linear combination of rays from $\tilde{D_{\mathbb{E}}}$.

Then von Neumann postulated that

Ax$_{vN}$ Each pure state is an individual state, i.e.

$$\tilde{D_{\mathbb{E}}} = \mathcal{PH}_{\mathbb{E}}.$$

This axiom has no experimental consequences since the predictions of QM are probabilistic and the probability can be associated only to ensembles and not to individual systems.

On the other hand there are important theoretical consequences: together with the locality this implies BI. The axiom **Ax**$_{vN}$ is the exact expression of the realism in QM: the pure state can be associated with the individual system.

The axiom **Ax**$_{vN}$ together with **Ax8** imply the Collapse rule. Let the $\varrho = P_\psi = \psi \otimes \psi^*$ be a pure state of an ensemble $\mathbb{E}$. After the measurement (with the output a_k) the corresponding sub-ensemble $\mathbb{E}_{a_k}$ will be in the state $P_{\psi_k} = \psi_k \otimes \psi_k^*$. Using **Ax**$_{vN}$ we can assert that the individual system $S \in \mathbb{E}$ was (before measurement) in the individual state $\vec{\psi}$ and after measurement in the individual state $\vec{\psi}_k$. Thus the Collapse Rule is the direct consequences of **Ax**$_{vN}$.

3 The minimal non-realistic modification of QM

In the paper [3] (and [2]) we have proposed the anti-von Neumann axiom:

$\mathbf{Ax}_{avN}$: $\vec{\psi}_1, \vec{\psi}_2 \in \tilde{D_{\mathbb{E}}}$, $\vec{\psi}_1 \neq \vec{\psi}_2 \Rightarrow \vec{\psi}_1 \perp \vec{\psi}_2$, i.e. $\vec{\psi}_1, \vec{\psi}_2$ are orthogonal

$\mathbf{Ax}_{avN}$ together with **Ax5** imply that the set $\tilde{D_{\mathbb{E}}}$ of individual states is the orthogonal bases $\{\vec{\psi}_1, \dots, \vec{\psi}_n\}$ of $\mathcal{H}_{\mathbb{E}}$ (if $n = dim\mathcal{H}_{\mathbb{E}}$).

There is a question of the choice of representants $\psi_1, \dots, \psi_n$ of rays $\vec{\psi}_1, \dots, \vec{\psi}_n$.

We choose one set of representants

$$D_{\mathbb{E}} = \{\psi_1, \dots, \psi_n\}.$$

(For the dependence on this choice see the discussion in [2].)

Thus we define the non-realistic QM by

Definition . n-r QM=$\{$**Ax1-Ax8**,$\mathbf{Ax}_{avN}\}$.

This is clearly the rather full form of non-realism: only a finite set of individual states are individual states. Also we obtain simply that no individual state can be a non-trivial superposition of pure states (the anti-superposition principle from [1]).

It is clear (see above) that $\mathbf{Ax}_{avN}$ will not have any experimental consequences (like $\mathbf{Ax}_{vN}$). But $\mathbf{Ax}_{avN}$ has many theoretical consequences.

(i) There is no Collapse rule for individual systems in n-rQM. In fact the phenomenon of collapse requires that both the initial state ψ and the final state ψ_k be individual states. In this situation either $\vec{\psi}=\vec{\psi}_k$ or $\vec{\psi} \perp \vec{\psi}_k$. But if $\vec{\psi}_k \perp \vec{\psi}$ then the probability of this outcome is zero, i.e. this event never happens.

But see the Remark in the following section implying that the problem of Collapse cannot be solved in n-rQM. In fact, the problem of Collapse can be solved only in modQM.

(ii) It is not possible to derive BI in n-rQM. Let us consider the well-known Mermin's paper [7]. At the page 43, right column, author introduces the concept of the "instruction set" which must exist in every run of experiment. Exactly this "instruction set" cannot exist in n-rQM since the state of a particle is not an individual state. In general the non-realism represented by $\mathbf{Ax}_{avN}$ prevents any possibility to prove BI.

(iii) In stQM we have the rule that the individual state of measuring system $\Rightarrow$ the individual state of the measured system. In n-rQM we have the opposite situation (in general) the individual state of the measuring system $\nRightarrow$ the individual state of the measured system.

4 Discussion

It is clear that stQM and n-rQM are different theories: $\mathbf{Ax}_{vN}$ excludes $\mathbf{Ax}_{avN}$ and vice versa. But the experimental consequences of both theories are the same. They are experimentally indistinguishable. The choice between them must be based on the theoretical considerations.

We have already noted that BI cannot be derived in n-rQM. This implies that

$$\text{n-rQM} \nRightarrow \text{non-locality}$$

while

$$\text{stQM} \Rightarrow \text{non-locality.}$$

The great advantages of n-rQM consists in the fact that it allows the restoration of locality (the same is true for modifiedQM - see [2]).

The original motivation of Bohr, Heisenberg, Dirac and von Neumann in postulating $\mathbf{Ax}_{vN}$ (or the superposition principle) was rather strange: the purpose was the effort to ensure that the wave function gives the complete description of the individual state. But this effort was purely ideological since $\mathbf{Ax}_{vN}$ has no experimental consequences. But the consequences of the acceptance of $\mathbf{Ax}_{vN}$ are rather heavy: the non-locality and the collapse - both are un-solved problems of quantum foundations.

Today the question of the so-called completeness of QM is un-important, if not completely irrelevant.

Remark. The problem of collapse is more complicated. Let the initial state of $\mathbb{E}$ be $\vec{\psi_1} \in \tilde{D_{\mathbb{E}}}$. This means that the individual state of any system $S \in \mathbb{E}$ is ψ_1. Let us assume that the measurement bases $\{\phi_1, \ldots, \phi_n\}$ is in the general position with respect to $\{\psi_1, \ldots, \psi_n\} = D_{\mathbb{E}}$, i.e. $\langle \phi_i, \psi_j \rangle \neq 0, \ \forall i, j$.

Having $St(\mathbb{E}) = \vec{\psi_1}$ we know that for each sub-ensemble $\mathbb{E}' \subset \mathbb{E}$ we still know that the individual state of any $S \in \mathbb{E}'$ is ψ_1. But after measurement with outcome a_1 (corresponding to ϕ_1) we obtain from **Ax8** that the state of $\mathbb{E}_{a_1} \subset \mathbb{E}$ will be ϕ_1, i.e. the state ψ_1 of $\mathbb{E}$ collapsed to ϕ_1 (the state of $\mathbb{E}_{a_1}$). This is the collapse rule. The full solution of the Collapse problem can be done only in modQM.

5 Conclusions

After describing the axiomatic structure of the standard QM we have clarified the role of the von Neumann's axiom (the ensemble in the pure state is homogeneous). The minimal non-realistic modification of QM consists in the replacement of $\mathbf{Ax}_{vN}$ by the anti-von Neumann axiom (different individual states are orthogonal).

StQM and n-rQM are different theories with the same experimental consequences.. N-rQM has important advantages: no BI, no non-locality.

We suggest that n-rQM should be preferable against stQM. (But modQM is still better.)

We have shown that $\mathbf{Ax}_{vN}$ has the purely ideological content related also to the so-called completness of QM. Thus the real physics does not depend on $\mathbf{Ax}_{vN}$.

References.

[1] Jiri Soucek, The principle of anti-superposition in QM and the local solution of the Bell's inequality problem, http://www.nusl.cz/ntk/nusl-177617, available also at http://vixra.org/abs/1502.0088

[2] J. Soucek, The restoration of locality: the axiomatic formulation of the modified Quantum Mechanics, viXra:1503.0109

[3] J. Soucek, The Logical Error in the Derivation of the Non-Locality, http://vixra.org/abs/1502.0169

[4] N. Harrigan, R. Spekkens, Einstein, incompleteness, and the epistemic view of quantum states, arXiv:0706.2661v1 (2007)

[5] Marek Zukowski, Bell's theorem tells us not what quantum mechanics is, but what quantum mechanics is not , arXiv: 1501.0564v1

[6] J. von Neumann, Mathematical Foundations of Quantum Mechanics (Princeton University Press, 1955), translated by R. T. Beyer from the German Mathematische Grundlagen der Quantenmechanik (Springer, 1932)

[7] N. David Mermin, Is the moon there when nobody looks ? Reality and the quantum theory, Physics Today, april 1985