An analysis of seven Smarandache concatenated sequences using the notion of cm-integers

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Abstract. In this paper I show that many Smarandache concatenated sequences, well known for the common feature that contain very few terms which are primes (I present here The concatenated square sequence, The concatenated cubic sequence, The sequence of triangular numbers, The symmetric numbers sequence, The antisymmetric numbers sequence, The mirror sequence, The "n concatenated n times" sequence) contain (or conduct to, through basic operations between terms) very many numbers which are cm-integers (c-primes, m-primes, c-composites, m-composites).

Observation:

Many Smarandache concatenated sequences, well known for the common feature that contain very few terms which are primes, contain (or conduct to, through basic operations between terms) very many numbers which are cm-integers (c-primes, m-primes, c-composites, m-composites).

Note:

In the following analysis I will not show how I calculated the c-reached primes and the m-reached primes, see for that the paper "The notions of c-reached prime and m-reached prime".

Verifying the observation for the following Smarandache concatenated sequences:

(1) The concatenated square sequence

S_n is defined as the sequence obtained through the concatenation of the first n squares. The first ten terms of the sequence (A019521 in OEIS) are 1, 14, 149, 14916, 1491625, 149162536, 14916253649, 1491625364964, 149162536496481, 149162536496481100.

This sequence seems to have the property that the value of the number a(n+1) – a(n), where a(n) and a(n+1) are two consecutive terms and n is odd, is often a c-prime or a c-composite.

The first few such values are:

: 14 – 1 = 13. This number is prime, so by definition c-prime;
: 14916 – 149 = 14767. This number is prime, so by definition c-prime;
: 149162536 – 1491625 = 147670911 = 3^3*109*50177. This number is c-composite, having a c-reached prime equal to 149551;
: 1491625364964 – 14916253649 = 1476709111315 = 5*449*657776887. This number is c-composite, having a c-reached prime equal to 3288883987;
: 149162536496481100 – 149162536496481 = 149013373959984619 = 29*5138392205516711. This number is c-composite, having a c-reached prime equal to 11922023678263.

(2) The concatenated cubic sequence

S_n is defined as the sequence obtained through the concatenation of the first n cubes. The first ten terms of the sequence (A019521 in OEIS) are 1, 18, 1827, 182764, 182764125, 182764125216, 182764125216343, 182764125216343512, 182764125216343512729, 1827641252163435127291000.

This sequence seems to have the property that the value of the number a(n) + a(n+2) – a(n+1), where n is even, is often a mc-integer.

The first few such values are:

: 18 + 182764 – 1827 = 180955 = 5*36191. This number is c-prime, having a c-reached prime equal to 36187.
: 182764 + 182764125216 – 182764125 = 182581543855 = 5*17*2148018163. This number is c-composite, having a c-reached prime equal to 36516308767.
: 182764125216 + 182764125216343512 – 182764125216343 = 182581543855252385 = 5*1249*29236436165773. This number is m-composite, having a m-reached prime equal to 36516308771050481 and a m-reached prime equal to 146182180830113.
: 182764125216343512 + 1827641252163435127291000 – 182764125216343512729 = 1827458670802344000121783. This number is prime, so c-prime and m-prime (cm-prime) by definition.

(3) The sequence of triangular numbers

S_n is defined as the sequence obtained through the concatenation of the first n triangular numbers. The triangular numbers are a subset of the polygonal numbers (which are a subset of figurate numbers) constructed with the formula T(n) = (n*(n + 1))/2 = 1 + 2 + 3 +… + n. The first ten terms of the sequence (A078795 in OEIS) are 1, 13, 136, 13610, 1361015, 136101521, 13610152128, 1361015212836, 136101521283645, 13610152128364555.

There are only two terms of this sequence that are primes (among the first 5000 terms, i.e. 13 and 136101521); on the other side, seems that relatively easy can be constructed primes using basic operations between the terms of the sequence, like for instance a(n) + a(n+1) – 1, for n and n + 1 even, and a(n) + a(n+1) + 1, for n and n + 1 odd.

Two such values are:

: 1361015 + 136101521 + 1 = 137462537, a prime number;
: 13610152128 + 1361015212836 – 1 = 1374625364963, a prime number.

(4) The symmetric numbers sequence

S_n is defined as the sequence obtained through concatenation in the following way: if n is odd, the n-th term of the sequence is obtained through concatenation 123…(m-1)m(m-1)…321, where m = (n + 1)/2; if n is even, the n-th term of the sequence is obtained through concatenation 123…(m-1)mm(m-1)…321, unde m = n/2. The first ten terms of the sequence (A007907 in OEIS) are 1, 11, 121, 1221, 12321, 123321, 1234321, 12344321, 123454321, 1234554321, 12345654321.

This sequence seems to have the following property: the terms of the form 12…(n-1)n(n-1)…21, where n is odd, are often cm-integers.

Few such values are:

: 1234567654321 = 239^2*4649^2. This number is m-composite, having a m-reached prime equal to 21670321.
: 12345678987654321 = 3^4*37^2*333667^2. This number is m-composite, having a m-reached prime equal to 457247369913149;

: 123456789101110987654321 = 7*17636684157301569664903. This number is c-composite, having a c-reached prime equal to 17636684157301569664897.

(5) The antisymmetric numbers sequence

S_n is defined as the sequence obtained through the concatenation in the following way: 12…(n)12…(n). The first ten terms of the sequence (A019524 in OEIS) are 11, 1212, 123123, 12341234, 1234512345, 123456123456, 12345671234567, 1234567812345678, 123456789123456789.

This sequence seems to have the following property: the values of the numbers 2*a(n) + 1, where a(n) are the terms corresponding to n odd, are often cm-integers.

Few such values are:

: 2*123123 + 1 = 246247. This number is prime, so c-prime and m-prime (cm-prime) by definition;
: 2*1234512345 + 1 = 2469024691 = 7^2*50388259. This number is c-composite, having a c-reached prime equal to 50388211;
: 2*123456789123456789 + 1 = 246913578246913579 = 17*14524328132171387. This number is c-composite, having a c-reached prime equal to 14524328132171371.

(6) The mirror sequence

S_n is defined as the sequence obtained through concatenation in the following way: n(n – 1)…32123…(n – 1)n. The first ten terms of the sequence (A007942 in OEIS) are 1, 212, 32123, 4321234, 543212345, 65432123456, 7654321234567, 876543212345678, 98765432123456789, 109876543212345678910.

This sequence seems to have the following property: the values of the numbers obtained deconcatenating to the right with the last digit the even terms are often cm-integers.

The first few such values are:

: 21 = 3*7. This number is cm-prime, having the c-reached prime equal to 5 and the m-reached prime equal to 5;

: 432123 = 3*17*37*229. This number is cm-composite, having c-reached primes equal to 59 and 8423 and m-reached primes equal to 19, 1699, 4003;
: 6543212345 = 5*71*271*117779. This number is m-composite, having a m-reached prime equal to 371573;
: 87654321234567 = 3^4*229*239*4253*4649. This number is m-composite, having a m-reached prime equal to 87654321234567;
: 1098765432123456789 = 3^2*17*37*333667*581699347. This number is c-composite, having a c-reached prime equal to 36815221.

(7) The "n concatenated n times" sequence

S_n is defined as the sequence of the numbers obtained concatenating n times the number n. The first ten terms of the sequence (A000461 in OEIS) are 1, 22, 333, 4444, 55555, 666666, 7777777, 88888888, 999999999, 10101010101010101010.

This sequence seems to have the property that the value of the number a(n+1) – a(n) is often a m-prime or a m-composite.

The first few such values are:

: 22 – 1 = 21 = 3*7. This number is m-prime, having the m-reached prime equal to 5;
: 333 – 22 = 311. This number is prime, so m-prime by definition;
: 4444 – 333 = 4111. This number is prime, so m-prime by definition;
: 55555 – 4444 = 51111 = 3^4*631. This number is m-composite, having two m-reached primes, equal to 23 and 1559;
: 666666 – 55555 = 611111. This number is prime, so m-prime by definition;
: 7777777 – 666666 = 7111111 = 7*19*127*421. This number is m-composite, having three m-reached primes, equal to 103, 8887 and 374287;
: 88888888 – 7777777 = 81111111 = 3*27037037. This number is m-prime, having the m-reached prime equal to 342319.