Static Process Algebra as Pre-arithmetical Content for School Arithmetic

Jan A. Bergstra
Minstroom Research,* Utrecht

April 8, 2015

Abstract

Parallel composition in a static setting introduces algebra, in the form of static process algebra, as a modelling tool at the level of primary school mathematics. Static process algebra may play the role of a prearithmetic algebra. Multi-dimensional counters can be used to measure the number of components in a static process expression.

*Minstroom Research BV, Utrecht, The Netherlands (hereafter called Minstroom Research), KvK nr. 59560347. Author's email address: info@minstroomreserch.org, janaldertb@gmail.com. Appendix A provides detailed statements concerning copyright protection of this document and about its status. This paper is a nopreprint in conformance with the definition given in Appendix B. Minstroom Research nopreprint series nr. 9 (Minstroom Research NPP#9, subseries I4PSM#3). The paper has Minstroom Research document class B (see Appendix C).

Contents

1 Introduction

This paper carries on with the work in [2, 3] with focus on concurrent systems. Following these papers I will take the remarkable preview of forthcoming educational material in DPII [15] as a source of inspiration. With that material in mind I will describe how to provide access from first principles to the concept of a concurrent system. The motivation behind this development is that expressions for denoting concurrents systems can be taught very early on in a primary school mathematics (PSM) course, and in my view potentially and productively, if not preferably in advance of any significant arithmetical teaching. I will call a theme in algebra pre-arithmetical (and a particular algebra pre-arithmetic), if it can be taught in advance of any arithmetic.[1] In more detail the following objectives wil be envisaged:

1. To illustrate that expressions and formulae are like drawings. One may design these at will and one may impose one's own perspective on meaning. This flexibility must be made aware to students as early as possible, very much like it is being done with drawing, music, and physical movement. The precision of mathematics is not a property of its immutable and rigid notation, on the contrary.

2. To demonstrate that it is meaningful to introduce a sort of systems (of some kind) and to define expressions for denoting elements of that sort.

3. To illustrate how functions on concurrent systems may be defined.

4. Establishing the central role of concurrent composition as a constructor for systems (system expressions).

5. To demonstrate the notion of a model as an abstraction from some form of reality, illustrating that the same reality may be modeled in different ways, and that the same model (static process expression) may describe different realities (real scenes).

6. To demonstrate that formal systems modelling may in principle precede the appearance of arithmetic and numbers.

7. To illustrate how numbers and operations on numbers arise within the concurrency modelat hand.

8. To equip counting at once with a dimensional version such that counts of different types may be added in a multidimensional setting. Indeed apple is a dimension just as meter, but I work under the hypothesis that, as a dimension, it can be taught at an earlier stage of development of arithmetical skills and competences.

[1] In terms of inclusion at a reference level I consider static process algebra to be a plausible candidate for inclusion in a forthcoming level 1/2F as indicated in [14].

2 Counting and scene modelling

In DPII we find pictures and photographs of real life scenes that provide an incentive for counting and for arithmetic. More specifically the idea is that if a set of k containers each contains l items of some kind K then in total there are (we have access to, one sees, one observes, one has etc.) $k \times l$ items of kind K. These matters are illustrated graphically very clearly. An underlying assumption is that a student is able to count the number of items of some kind in a picture independently of its modular structure.

In particular some scenes concern a plurality of transparant containers each containing an identical number (l) of some kind (K) of item. I consider the static mechanics of these pictures and their relation to numbers as a source of inspiration for the work below. Static process algebra is meant as a notation for a missing link in between of the pictures and the arithmetical notations that is descriptive of the same pictures.

2.1 Counting in the sums of unit notation

Although static process algebra may be introduced without any preliminary introduction to numbers and counting I will look at an order of presentation where a very limited presentation of arithmetic is presupposed.

A highly rudimentary presentation of a structure of natural numbers is found by means of a constant 1 and an operation $-+-$ on number expressions which is used in a repeated manner for all arities above 1. This leads to expressions like $1, 1+1, 1+1+1$, etc. This presentation for natural numbers may be called the SoUNNN (sums of unit natural number notation). It may be augmented with a zero (written 0) serving the role of an empty sum, thus obtaining SoUzNNN.

In spite of its very basic outlook, from a logical point of view SoUNNN is not entirely trivial given that addition works as follows: in order to add say $1 + 1$ and $1+1+1$ we need to understand that a binary and a ternary use of addition glue into a quintenary use of addition: $(1+1)+(1+1+1) = 1+1+1+1+1$. One might claim as an objection to this presentation that counting in SoUNNN presupposes the ability to count the number of arguments of repeating versions of addition, but I consider that objection unproblematic for the given purposes.

Use of SoUNNN in teaching depends on the possibility to find manners to work with the notation and to exercise those processes in a meaningful manner. Examples of such exercises are these:

1. To inspect a picture of say 5 apples and to write $1 + 1 + 1 + 1 + 1$ as a model for it.

   Here the idea is that one uses the natural number expressions (NNE's) to write what one sees. That is done for instance by writing that "the picture contains the following NNE" of apples. This seems to be the essence of counting: finding a natural number expression which "expresses" the quantity of some class of entities or events.[2]

[2] The correspondence between a scene and its count needs to be made explicit informally

2. A similar question for a drawing of 4 pears; this exercise points to the fundamental abstraction of counting: abstraction from what is counted.

3. and for 7 oranges, further abstraction from what is counted.

4. The request to compose a scene with $1+1+1+1+1+1$ many apples (given sufficiently many apples and an originally empty scene). Uncounting requires the adittional information of what was counted and a supply to those items.

5. Exercises that make use of counting in inter-agent communication. This works the other way around. One uses the NE's to communicatie how many apples one wishes to obtain. Can you give me $1+1+1+1$ apples? And so on.

2.2 Introducing 2, 3, etc.

Next one may introduce 2 for $1+1$, thus $2 = 1+1$. This allows simplifications: $1+1+1 = 2+1 = 1+2$, and $1+1+1+1 = 2+2$. We also need $1+2 = 2+1$ (and $2+1 = 1+2$?) It can now be shown that every expression with a number of 1's can be written as an expression with at most one 1. This statement can be demonstrated by means of examples, its proof with induction on the structure of expressions is probably not a matter of PSM.

A systematic name for this notation is SoU2NNN without and with zero: SoUz2NNN. Now similar questions are posed and one is supposed to find number expressions with at most one 1 as models for the scenes.

As a further step we may write $2+1 = 3$, and use $x+y = y+x$ in calculations. Again it can be asked to write numbers is a simplest form: either a repeated sum of 3's, or a repeated sum of 3's with a 2 added to it, or with a 1 added to it, or an empty sum. A systematic name for this notation is SoU23NNN. [3]

2.3 Multiplication

Multiplication of two NNE's may be understood as follows: $1 \times X = X$, $(1+1) \times X = X + X$, $(1+1+1) \times X = X + X + X$, and so on. As an example we find: $(1+1) \times (1+1) = 1+1+1+1$. Obviously in this context multiplication requires very little calculation. Repeated substitution suffices, as in the context of SoUNNN multiplication is merely repeated substitution.

---

by means of some reference to the notion of a 1-1 correspondence.

In my view counting is not connected to decimal notation, nor does it have a bias towards an ordinal use (counting as enumeration) of numbers over a cardinal use (counting as size determination) of numbers. In fact I see some preference for a cardinal view of small natural numbers as being the more important one and the more easily accessible one.

[3] This notation applies in the absence of zero, while the presence of zero may be indicated with an additional "z": SoUz23NNN.

2.4 Using brackets

The story of the sums of unit notation becomes conceptually simpler if only a binary addition is used and brackets are allowed. In that case there is no underlying count of arities which is begging the question of where counting begins. Technically that is not simpler, however, as the proper role of commutativity and associativity of addition must be asserted. There are at least two ways to go: (i) associativity is considered a justification for omitting brackets, and (ii) omitting brackets is a mere abbreviation and brackets must be reintroduced in a specific manner (e.g. association to the left) when reasoning about natural number expressions is done.

Now both conventions come with specific complications and I feel that it is critical for the idea of pre-arithmetical algebra not to make use of either convention and to allow repeated use of infix operations such as $- + -$ without any hesitation. The way in which a plurality of such multi-argument operations might be construed in terms of a single two-place function is a topic in post-arithmetical algebra rather than in pre-arithmetical algebra.

3 Scene modelling with static process algebra

We now imagine teaching material featuring sentences such as: "in figure X (*a photograph*) you find two baskets containing 4 apples each; together you see 8 apples."

I claim that such figures and texts speak of systems an that the word system may be used right from the start of mathematical teaching.

3.1 Preliminary issues: reality versus its pictures

Some preliminary preliminary questions require attention, not so much because clearcut answers to these questions must be provided in order to make progress but because each path of development will be confronted with such questions in its own manner:

- Can we subsume counting of items in a scene description under modelling? (I would say yes.)
- Are pictures (an in particular photos) a part of a realistic approach to be distinguished from a real approach unmediated by any pictures or images or other descriptions? (I vote positive on this question.)
- Do we see apples or merely pictures of apples? (We see pictures of apples, not apples.)
- And does it matter if we make that sort of distinction? (Yes, in tricky cases it does.)

- Are we counting apples or are we counting pictures of apples. (When following DPII as I intend to do, we are not counting apples, we only speak as if we are counting apples.)
- In the latter case, is counting apples a case of applying the ability to count pictures of apples (in a realistic context) to the counting of certain kinds of entities in a real context? (Yes, that's how an application works in principle. Nevertheless the generalisation of assertions about counting items in a picture to the counting of depicted items in a depicted scene may be wrong.)
- Can it be the case, if only in principle, that (a) there is an underlying theory of counting which applies to models like pictures, and (b) that this application of that theory to the real world reveals some kind of mistake in the theoretical work? (I hold that this form of mismatch is impossible in principle.[4])

3.2 Reasoning principles in a realistic setting

In a realistic setting we talk and reason about pictures and other descriptions (models) as if these are the underlying reality. The following considerations apply to that:

1. The photo of a display with baskets and apples can be considered a model of a reality with baskets and apples. By reasoning about the picture we may obtain information about the underlying reality. Then it must be taken into account that the underlying reality must be understood at the time of producing the picture, and not at the time of rendering it.
2. If the photo is turned into a picture providing much less information but providing a highlighted subset of it that facilitates a particular form of interpretation, once more a model is found of the original scene.
3. It may be useful to use formulae and expressions as models for a scene, rather than pictures (drawings or photo's) or sentential text.
4. When contemplating scenes it is a great simplification (which must be applied in practice with great care, however) to take a photo for the underlying reality (thus obtaining a realistic setting), and to speak of the items realistically displayed on the photo in terms of instances of the entity types that these are images of.

[4]But one must consider the case that the picture of a scene provides multiple images of the same item which may erroneously be taken for images of different items. The theory that may turn out to be refuted in such a case includes both the mathematics of counting and its application to models (pictures) of a scene, as well as a generalisation of that application to the analysis of true scenes.

3.3 Static Process algebra

Instead of counting items in a scene a more detailed modelling in terms of expressions can be imagined. Let A represent an apple (that is a single apple), P a pear, and O an orange.

Now it is plausible to think in terms of systems composed with $- \mid\mid -$, an operator representing the parallel (concurrent, simultaneous) composition of two systems. Then $A \mid\mid A$ represents (models) the concurrent presence of two apples, and $A \mid\mid A \mid\mid A$ represents the concurrent presence of three apples, $A \mid\mid A \mid\mid A \mid\mid A$ models the simultaneous presence of 4 apples, and so on. The expression $A \mid\mid P \mid\mid A \mid\mid O$ models the simultaneous presence of two apples, a pear, and an orange.

Just as addition can be repeated, concurrent composition can be repeated and for each number of items there is a repetition of $- \mid\mid -$ which allows the concurrent composition precisely that number of items. One may understand this as a generalised form of counting but that's not quite right as an intuition, because in counting the core idea of simultaneity gets lost.

An important property of parallel composition is commutativity: $x \mid\mid y = y \mid\mid x$. Thus e.g.: $A \mid\mid P \mid\mid A \mid\mid O = A \mid\mid O \mid\mid A \mid\mid P$, an equation that follows after two applications of commutativity.

The static aspect of static process algebra relates to the idea that components have a single state only, whereas in process algebra in general components may perform state transitions. However unusual this notation may seem at first sight, using $- \mid\mid -$ (or variations thereof) for the concurrent composition of systems has a long tradition in informatics.[5] I hold the view that in a period of less than 40 years parallel composition has become the most important operator in that part of theoretical computer science which deals with concurrent system design, and in addition parallel composition constitutes the most manifest novelty that theoretical informatics has brought to pre-existing logic and mathematics.

4 Working with dimensions: applecount as a dimension

The specific aspect of A in a process expression is the indication of the present existence of an object of type apple. One may, however, think about a plurality of apples that need not coexist in time but that may be have successive existences. With $\underline{\mathsf{A}}$ I will denote the unit of "appleness". This is more abstract than A, that is if we see an A we may also observe $\underline{\mathsf{A}}$, that is one unit of $\underline{\mathsf{A}}$. I will write $\underline{\mathsf{A}} = 1 \cdot \underline{\mathsf{A}}$. Now $\underline{\mathsf{A}} + \underline{\mathsf{A}}$ represents two units of appleness in whatever modality. Different modalities are: concurrently existent, subsequently existent in future and or past, being required by another agent, being expected to be delivered, being needed for some purpose, being for sale, being for rent, having

[5] For my own experience with the use of notations for concurrent/parallel composition I refer to work on concurrent processes in [10] (and papers cited there) and on multi-threading in [6, 7] and to work on the composition of sequential services in [11, 8].

been bought, being in the possession of an agent, having been lost by an agent, etc.

At this stage an interesting simplification is possible: $\underline{\mathsf{A}} + \underline{\mathsf{A}} = 1 \cdot \underline{\mathsf{A}} + 1 \cdot \underline{\mathsf{A}} = (1{+}1)\cdot\underline{\mathsf{A}} = (2)\cdot\underline{\mathsf{A}} = 2\cdot\underline{\mathsf{A}}$. An expression $t\cdot\underline{\mathsf{A}}$ with t a natural number expression is a specific quantity expression (alternatively a dimensional quantity expression).

For pears a similar simplification reads: for instance $\underline{\mathsf{P}} + \underline{\mathsf{P}} = 3 \cdot \underline{\mathsf{P}}$. It follows that natural number expressions are shared by quantities for different types.

Further it makes perfect sense to add specific quantity expressions for different types of quantity: $2 \cdot \underline{\mathsf{A}} + 3 \cdot \underline{\mathsf{P}}$ (when featuring in connection with describing some scene) indicates that some mix of two specific quantities plays a role (in that scene).

4.1 Applecount extraction

We introduce $\#_A(-)$ as a function, named applecount extraction, which counts the number of apples in a system description. We find these defining equations:

$$\begin{aligned} \#_A(A) &= 1, \\ \#_A(P) &= 0, \\ \#_A(O) &= 0, \\ \#_A(S_1 \mid\mid S_2) &= \#_A(S_1) + \#_A(S_2). \end{aligned}$$

It follows that $\#_A(A \mid\mid A) = 2$. For other component types a similar counting operator can be introduced. We find: $\#_A(A \mid\mid O \mid\mid A) = 2$, $\#_O(A \mid\mid O \mid\mid A) = 1$.

Dimensional applecount $\underline{\#}_A(-)$ is defined by $\underline{\#}_A(S) = \#_A \cdot \underline{\mathsf{A}}$. A similar definition works for other item classes.

Dimensional counts can be added without leading to confusion, which is the main argument in favour of the use of dimensional counts. Component count takes a finite collection C of component types and reads thus:

$$\underline{\#}_C(S) = \sum_{c \in C} \underline{\#}_c(S).$$

As an example, with $C = \{\underline{\mathsf{A}}, \underline{\mathsf{P}}, \underline{\mathsf{O}}\}$ one finds:

$$\underline{\#}_{\{\underline{\mathsf{A}},\underline{\mathsf{P}},\underline{\mathsf{O}}\}}(A \mid\mid O \mid\mid A \mid\mid P) = 2 \cdot \underline{\mathsf{A}} + \underline{\mathsf{P}} + \underline{\mathsf{O}}.$$

4.2 Abstraction from rotation symmetry

Being liberal about what constitutes a system has become quite common in informatics, and I consider that to be of great value. Having, and permitting, some freedom in the design of expressions for systems is nowadays the rule rather than the exception. If we denote a box with 2 items x and y with $b(x, y)$ then we find the following system expression for two as system consisting of two of such boxes both containing 2 apples $b(A, A) \mid\mid b(A, A)$. An appreciation of the model of this depicted scene may precede becoming aware that $2 + 2 = 4$.

If one assumes that the box is symmetric in that it is invariant under a rotation of 180 degrees, one may express that state of affairs with the equation $b(x, y) = b(y, x)$. This equation allows models to be somewhat more abstract under the mentioned invariance assumption.

Counting operations can be extended to boxes by requiring that: $\#_A(b(x, y)) = \#_A(x) + \#_A(y)$, $\underline{\#}_A(b(x, y)) = \#_A(b(x, y)) \cdot \underline{\mathsf{A}}$, and $\underline{\#}_C(b(x, y)) = \underline{\#}_C(x) + \underline{\#}_C(y)$.

A box with four items may be written $b(-,-,-,-)$. Rotation invariance of the four item box over 90 degrees is modeled by the equation $b(x, y, u, v) = b(u, x, v, y)$. Counting operators are extended in an obvious manner to this case. Other boxes may be introduced, such as the 6 items box which is often used for apples and oranges (though less for pears).

At this stage a non-trivial equivalence can be defined on system expressions: being models of the same scene. I will write $S_1 =_{mss} S_2$ for this equivalence. It turns out that $=_{mss}$ is completely axiomatised by equations that have been listed above.

Exercises regarding this theme may ask to build scenes from system expressions and to use manipulation of the scenes in order to evaluate $=_{mss}$-equivalence. Another way to look at the matter is that scenes are equivalent if corresponding models are $=_{mss}$-equivalent, a property which can be demonstrated by means of the given axioms.

4.3 An alternative: flexible position boxes

Instead of writing $b(x, y, z, u)$ for a box with 4 items one may write alternatively $b_4(x \mid\mid y \mid\mid z \mid\mid u)$. This notation has the advantage that it insists in using $- \mid\mid -$ for concurrent composition, whereas $b(x, y, z, u)$ uses $-,-$ as a non-commutative version of concurrent composition in the context of a box. I will speak of flexible position boxes (of a given maximum capacity) thus marking a contrast with the rigid position box $b(x, y, z, u)$.

Now it must be secured that a box with capacity 4 will not have more than 4 item in it (while it may contain fewer objects). Working with a as an additional value that indicates an error, just as in the common meadows of [12], the equation $b_4(x \mid\mid y \mid\mid z \mid\mid u \mid\mid v) = a$ expresses that a container (box) with too many items in it is erroneous. An additional equation required for this setup reads: $a \mid\mid x = a$.[6]

Modelling containers in terms of flexible position boxes works at a higher level of abstraction than with rigid position boxes, and by consequence dealing with rotational symmetries of boxes is made redundant.

[6]The use of a may be avoided if the overflow of a box is represented by, say an empty box, e.g. written $b_0()$. This way of dealing with design errors is similar to the choice of 0 for 1/0 in the (involutive) meadow of rational numbers in [13], or the choice of other rational values for 1/0 in non-involutive meadows in [9]. The need to deal with expressions having problematic values or at least to discuss such expressions has been discussed in the PSM literature, for instance in [16] and in [17].

4.4 Nested containers with variable sizes

One may imagine that containers have a size in addition to a capacity. Now the context is different from that of boxes with apples or oranges. Assuming that A, P, O represent atomic items, one may imagine that a box, say $b_4^3(-)$ of (maximum) capacity 4 and size 3 size 3 may contain atomic items as well as boxes of capacity 4 or below with size less than 3. Counting functions may be extended hierarchically through layered boxes. Counting functions may take the number an type of boxes into account as well, and boxes may be considered components in their own right.

One may imagine a construction kit allowing the construction of such hierarchical structures and the modelling of such structures in a corresponding static process algebra. In addition septic process algebra expressions may be taken as descriptions of constructions that are to be realized.

5 Concluding remarks

Static process algebra is a very simple from of process algebra which I expect to be amenable to being transformed into teachable material for students at a most introductory level. This algebra may be taught in advance of arithmetic and in particular in advance of calculating with natural numbers in decimal notation.

Rather than installing the intuition that expressions and formulae a fixed, determined, and rigid, one may convey the idea of a world of expressions which may serve a s a carrier for the student's wish to design his or her own tools of expression.

References

[1] J.A. Bergstra. A nopreprint on the pragmatic logic of fractions. Minstroom Research NPP#3, `http://vixra.org/abs/1501.0231`, (2015).

[2] J.A. Bergstra. "Rekenen-informatica": informatics for primary school mathematics. Minstroom Research NPP#7, I4PSM#1, `http://vixra.org/abs/1503.0136`, (2015).

[3] J.A. Bergstra. School algebra as a surrounding container for school arithmetic. Minstroom Research NPP#8, I4PSM#2 `http://vixra.org/abs/1501.0246`, (2015).

[4] J.A. Bergstra and I. Bethke. Note on paraconsistency and the logic of fractions. `arXiv:1410.8692v2[cs.LO]`, (2015).

[5] J.A. Bergstra, I. Bethke and A. Ponse. Rekenen-Informatica. *UvA ReportTCS14-12*, `https://ivi.fnwi.uva.nl/tcs/pub/tcsreports/TCS1412.pdf`, (2014).

[6] J.A. Bergstra and C.A. Middelburg. Thread algebra for strategic interleaving. *Formal Aspects of Computing*, 19 (4) pp. 445–474, (2007).

[7] J.A. Bergstra and C.A. Middelburg. Distributed strategic interleaving with load balancing. *Future Generation Computer Systems*, 24 (6), pp. 530-548, (2008).

[8] J.A. Bergstra and C.A. Middelburg. Instruction sequence processing operators. *Acta Informatica*, 49: 139-172, (2012).

[9] J.A. Bergstra and C.A. Middelburg. Division by zero in non-involutive meadows. *Journal of Applied Logic*, 13(1): 1–12 (2015).

[10] J.A. Bergstra and A. Ponse. Register machine based processes. *J. ACM*, 48 (6): 1207–1241, (2001).

[11] J.A. Bergstra and A. Ponse. Combining programs and state machines. *Journal of Logic and Algebraic Programming*, 51 (2): 175–192, (2002).

[12] J.A. Bergstra and A. Ponse. Division by zero in common meadows. `arXiv:1406.6878 [math.RA]`, (2014).

[13] J.A. Bergstra and J.V. Tucker. The rational numbers as an abstract data type. *Journal of the ACM*, 54 (2), Article 7 (2007). Zeer gebruikelijke zaken omwerken naar universele algebra.

[14] Jet Jeene. Towards Testing at Intermediate Levels and the Definition of Intermediate Competences in the Learning Path for Arithmetic Between the Basis Level and the Reference Level 2F. `http://vixra.org/abs/1503.0036`, (2015).

[15] Irene Lugten, Sari Wolters, Sarah Brusell, Manon Keuenhof, Kim Klappe, Maartje van Middelaar, Martine Knijnenberg, Rob Lagendijk, Marloes Kramer, Jelte Folkertsma, Cyriel Kluiters, Jasper van Abswoude, and Rieke Wynia. Proefkatern. Startrekenen Vooraf, Op weg naar 1F/Startrekenen Instap, Rekenen tot 100. *Uitgeverij Deviant, Amersfoort*, (2015).

[16] J. Nelissen and A. Treffers. Onderwijskader. *In:van den Heuvel-Panhuizen,M.,Buijs, K., Treffers, A. (eds.) Kinderen Leren Rekenen (in Dutch)*, pp. 147–162. Noordhoff Uitgevers, Groningen (2000).

[17] A. Treffers. Weg van het cijferen, rekenmethodes vanaf 1800 tot heden. *ISBN 978909028793, Uitgeverij Reni Casoli*, (2015).

A Properties of this particular paper

The first Appendix contains information which is specific for this paper, the subsequent Appendices provide the necessary explanation. Frequently a section of paragraph merely contains a pointer to the corresponding section of paragraph of a previous document in the MR nopreprint series. These Appendices have not been designed as independently readable text and have, with the exception of the defensive novelty analysis, no bearing on the content of reports.

A.1 Licencing

A.2 Minstroom Research Nopreprint Series Number

This is #9 from the Minstroom Research Nopreprint Series (in brief Minstroom Research NPP#9). It is #3 in the subseries I4PSM (Informatics for Primary School Mathematics) The previous papers in this series are listed below.

1. Minstroom Research NPP#1: "Decision taking avoiding agency", http://vixra.org/abs/1501.0088 (2015); this paper is not explicitly labeled as a nopreprint but it sufficiently meets the criteria as listed below, though it lacks a defensive novelty analysis which admittedly is a deficiency,

2. Minstroom Research NPP#2: "A nopreprint on algebraic algorithmics: paraconsistency as an afterthought", http://vixra.org/abs/1501.0203 (2015),

3. Minstroom Research NPP#3: "A nopreprint on the pragmatic logic of fractions", http://vixra.org/abs/1501.0231 (2015),

4. Minstroom Research NPP#4: "Personal multithreading, account snippet proposals and missing account indications", http://vixra.org/abs/1502.0204 (2015).

5. Minstroom Research NPP#5: "Terminology for instruction sequencing", http://vixra.org/abs/1502.0204 (2015).

6. Minstroom Research NPP#6: "A SWOT analysis for Instruction Sequence Theory", http://vixra.org/abs/1502.0231 (2015).

7. Minstroom Research NPP#7 (I4PSM#1) "Rekenen-informatica": Informatics for Primary School Mathematics, http://vixra.org/abs/1503.0136 (2015).

8. Minstroom Research NPP#8 (I4PSM#2) School algebra as a surrounding container for school arithmetic.http://vixra.org/abs/1503.0246 (2015).

A.2.1 NPP Subseries on I4PSM

"Informatics for primary school mathematics (I4PSM)" is coined as the name for a theme within Minstroom Research. For that theme a subseries of the NPP series is maintained. The paper is the second entry in that subseries, which is reflected in its extended code: Minstroom Research NPP#9 I4PSM#3.

A.2.2 Rationale for I4PSM as a theme within Minstroom Research

For this rationale I refer to the corresponding subsection of NPP#7 (I4PSM#1) [2].

A.2.3 Subseries rationale

For this rationale I refer to the corresponding subsection of NPP#7 (I4PSM#1) [2].

A.3 Minstroom Research Document Class

This paper has document class B in the Minstroom Research Document classification scheme. This scheme is detailed in Appendix C. This classification refers to the body of the paper with the exclusion of the Appendices.

A.3.1 Justification of this particular Minstroom Research document classification

In this particular case the classification in class B has the following motivation:

1. The nopreprint status is intentional, submission to a (selectively) peer reviewed publication outlet is not intended. (This indicates Minstroom Research as an appropriate affiliation bringing with the need for classification in A B, C, or D). Forthcoming agreement of any peer review system with the design decisions in the paper is not sought. Striving for peer reviewed publication makes much more sense after classroom testing of teachable material on static process algebra has been done.

2. Subsequent academic research on the basis of the content of this work is not foreseen by the author. Subsequent research, whether academic or non-academic, on static process algebra per se is not foreseen either. This paper brings static process algebra to a point from where valorisation is conceivable. In particular subsequent development into teachable material is supposed to be doable from this stage and potentially rewarding. Working this way within Minstroom Research towards material that can be used in practice is also intended.

3. The paper contains no novelty claims and does not (intend to) contradict existing literature either.

A.4 Defensive novelty analysis

A nopreprint ought to be equipped with a so-called defensive novelty analysis. An explanation of this this notion as well as an explanation of why it is needed in the case of a nopreprint is given in Appendix B below). For this paper I put forward the following arguments:

- The work proposes so-called static process algebra, a minuscule fragment of known process algebras, and expresses the expectation (or rather the hope) that static process algebra can be taught in primary school before the first principles of arithmetic have been taught.
- There is no technical content that might be wrong. No novelty of any technical aspect is claimed.
- The only potential novelty of the paper lies in the very idea that parallel composition might be prior in teaching to all operators from arithmetic. As such it constitutes an invitation for experimentation with novel content.
- The paper does not formulate proposals on how other persons ought to work or on how they may understand certain concepts.
- The proposed relation between static process algebra and arithmetic seems to be new. Yet no claim of novelty can be made given the limited exposure the author had to literature that surveys options for teaching non-arithmetic mathematical content before arithmetic is taught.

B Formalities and policy statements I: about nopreprints

This Appendix begins with brief historical remarks concerning the possibly novel ideas that are put forward in this Appendix as well as and in the following Appendix. The remaining part of this Appendix spells out the details an rational of nopreprints as a novel class of papers and publications.

B.1 Remarks on micro-history

For these remarks see the corresponding section in [2].

B.2 Nopreprints and micro-institutions

This Paragraph and subsequent Paragraphs with are identical (modulo the renaming of MRbv into Minstroom Research) to the corresponding Paragraphs of MR NPP#6, and are not repeated here for that reason.

C Formalities and policy statements 2: using a private micro-institution as an affiliation

This Appendix is identical to Appendix C (modulo the renaming of MRbv into Minstroom Research) of MR NPP#6, it will not be repeated here for that reason.