The Apparent Nature of Relative Simultaneity

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Abstract. This paper presents the proof of the apparent nature of relative simultaneity originally derived from Einstein's Special Theory of Relativity (STR). The proof does not challenge the validity of the STR but uncovers fundamental and widespread error in understanding of practical implications of Lorentz transformations. It is demonstrated that more than a century long debates generally miss the point. This results in counterintuitive claims of coexisting multiple time realities by mere equivalence of equal clock indications and simultaneity. Such claims have little empirical significance but they are substantial in education and philosophy which has become utterly confused after universal acceptance of the STR and rejections of Henri Bergson's challenge. There is nothing more to "relative simultaneity" other than the effect of identical clocks being shifted by an offset which depends on synchronisation method.

Keywords Lorentz Transformation, Absolute Simultaneity, Relative Simultaneity, Relativity of Time, Temporal Coincidence

This publication is in draft version 0.2. The author welcomes any comment to improve the quality and factual correctness of this presentation by sending messages to email address shown above.

1 The Purpose and the Scope

The purpose of this paper is to convince readers that Lorentz Transformation and therefore the Special Theory of Relativity (STR) can be explained in such a way that the counterintuitive concept of relative simultaneity no longer needs to confuse natural temporal logic most people possess.

This is not as an attempt to invalidate the STR but to find the proper context in which this theory can be viewed and understood.

The STR originally introduced in 1905 by Einstein [1] as presented in English in [2, pp. 37-65] has changed the traditional concept of time which has been further extended by Minkowski in 1908 [2, pp. 73-91], followed by introduction of Einstein's General Theory of Relativity (GTR) published in 1916 as presented in [2, pp. 109-164].

The foundations of modern general relativity still lie in the classic STR. Understanding its concepts of time are essential for all of the subsequent derivatives. The scope of this paper therefore is limited to the STR only.

Having the STR explained to one's own common sense is a gratifying and an essential step to progress to more advanced theories. In the opinion of the author, all of the known education attempts to do so have failed to interpret relative simultaneity effect correctly.

There is probably no concept or theory that would not have had its opponents, however for majority of physical theories there is no disaccord between the leaders in the field and the recipients who learn and implement them in their work or everyday life. Would that be classical mechanics, fluid dynamics or thermodynamics, there is no cognitive problems in average student population. It is different with relativity.

Not only the general public fails to comprehend the relativity of time, but also the students who are accustomed to acquisition of new concepts in the process of their education.[i]

It is the hope of the author that relative simultaneity will no longer be a controversial subject and no more difficult to comprehend than different clock times at different time zone for simultaneous events on Earth.

This publication makes no attempt to disprove, redefine or stretch the STR to fit the stated purpose.

2 RELATIVE SIMULTANEITY

2.1 Introduction

If one assumes the world around us as the physical reality, one has to admit that the reality appears to human observers as models. Some models are built in the mind or more generally in the nervous system some are described on paper (e.g. physical theories in books), some are realised as systems of objects (e.g. simulators, experiments).

In the ideal world, the three kind of models should not contradict each other. For example, a logically constructed scenario that can be potentially realised as an experiment should not be contradicted by the relevant theory. This is the method of reasoning called "thought experiments" designed to test physical theories, which has become popular since Einstein [1]. If there is a discrepancy between the models, then either one or more models are not correct or the reasoning about those model is incorrect.

After about one hundred years of consensus about the validity of the STR, the resistance to the concept of relative simultaneity resulting from it still exists in various forms. The problem is frequently marginalised as being caused by the lack of proper education.

The innate temporal logic resists the notion of relative simultaneity, however using the STR does not always require this logic to be challenged, because one can stick to the mathematical representation, use the equations, make predictions and be satisfied they match some experiments. For example, finding the μ-Mesons' lifetime extended due to their relative motion in the atmosphere [3] is very convincing that particles behave as if time "has slowed down" for them, but there is nothing inherently illogical in the fact that some processes may run slower in different conditions. Relative simultaneity is not explicit at this level of STR practical application, but "slowing down time" is.

Relative simultaneity comes into play in another kind of scenarios. The most famous one is the Twin Paradox (TP) which has attracted attention since it has been conceived until present as interestingly presented by Shuler [4].

2.2 The Claim of Apparent Nature of Relative Simultaneity

The claim made in this paper can be expressed in many different forms on different level of formalism. This is not uncommon publish informal explanations after it has been done multiple times by Einstein with the help of his famous train. This paper is no exception.

2.2.1 Informal Presentation of the Apparent Relative Simultaneity

The apparent nature of relative simultaneity is exposed by the fact that:

1. Two or more material points representing idealised particles belonging to a static structure at rest, form an integral part of some inertial system. They have to be relatively at rest with each other and to that system for any moving inertial system K'.
2. Any two material points described above when they simultaneously depart from the structure at the same speed on parallel trajectories, are both moving away from the structure while retaining their relative rest.
3. It can be shown using Lorentz Transformation that under some conditions and using standard relativity interpretation of time, the two or more points at some stage of their motion as described above, will be moving relatively to each other thus violating obvious assertions 1 and 2.

   This can be illustrated on two drawings for a simple scenario involving abstract material points and rigid structures in an inertial system K and observations in the moving system K'.

2.2.1.1 Description to Figure 1:

1. Two material points 1 and 2 are resting on a fixed elongated structure (rail) designating line segment $\boldsymbol{A_1 B_2}$ at $y=-D_y$ where $D_y>0$;
2. The material points depart the fixed position towards X axis ($y=-D_y$) at time t=0.
3. The material points continue the motion maintaining the same distance from each other until they simultaneously pass another structure designating the line segment $\boldsymbol{C_1 D_2}$.
4. Finally the material points arrive at X axis where they stop reaching an inelastic obstacle.

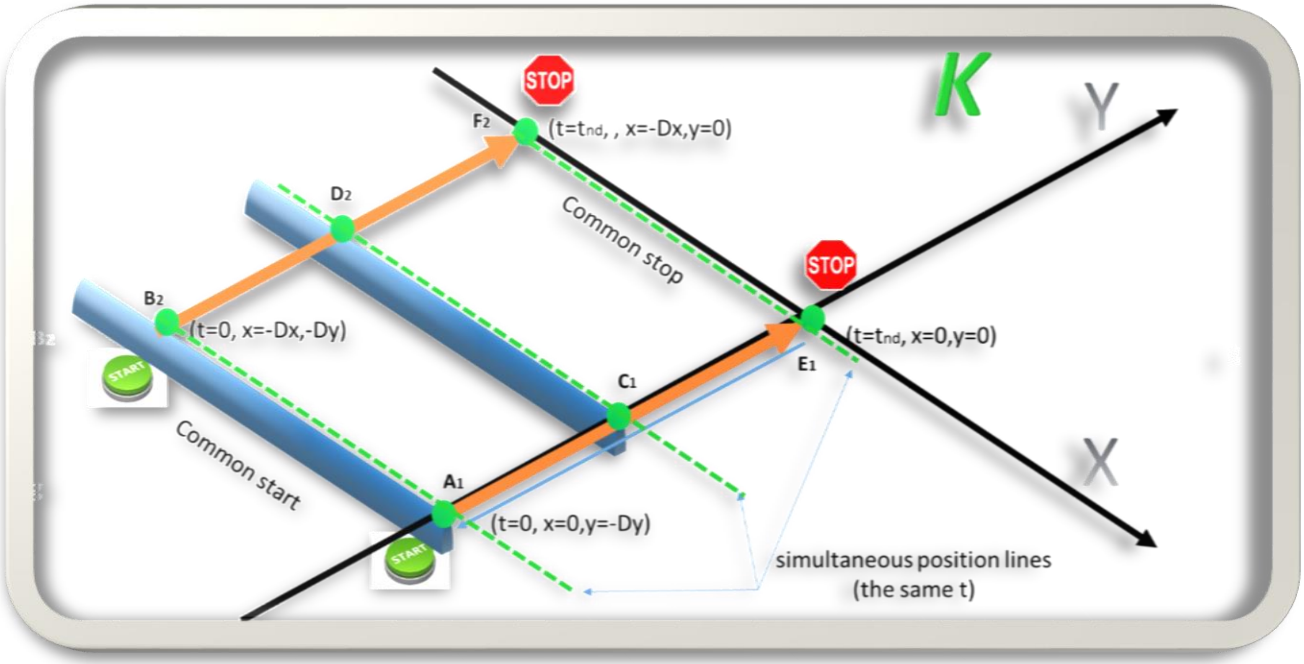

Figure 1 Transverse motion relative to X axis in the inertial system K

2.2.1.2 Description to Figure 2:

The Figure 2 has been constructed based on results of Lorentz transformation from an inertial system $\boldsymbol{K}$ to the inertial system $\boldsymbol{K'}$. Both have $\boldsymbol{X}$ and $\boldsymbol{X'}$ axes coincident and $\boldsymbol{Y}$ $\boldsymbol{Y'}$, $\boldsymbol{Z}$,$\boldsymbol{Z'}$ axes parallel to each respectively as chosen Einstein in [2, p. 43]. The interpretation of time t' is consistent with the view that equal clock times at distant locations designate simultaneous moments of a common temporal reality in $\boldsymbol{K'}$ in the same way as it is seen in $\boldsymbol{K}$.

5. Two material points 1 and 2 are resting on a fixed elongated structure (rail) designating the line segment $\boldsymbol{A_1}$ $\boldsymbol{B_2}$ at y=-D_y where D_y>0;
6. The material point 1 departs from the fixed position $\boldsymbol{A_1}$ at (y=-D_y) towards $\boldsymbol{X'}$ axis at time t'=0.
7. Due to relative simultaneity, the material point 2 lags behind by staying immobile at $\boldsymbol{B_2}$ while the other point continues to move until time $t' = v\gamma D_x/c^2$ when it joins the point 1 in a lead-wingman formation towards X′ axis.
8. At the time point 2 starts from the edge of the structure $\boldsymbol{A_1}$ $\boldsymbol{B_2}$ the other is in direct contact with another structure coinciding with $\boldsymbol{C_1}$ $\boldsymbol{D_2}$
9. The material points now continue the motion maintaining the same distance from each other however they no longer designate a line parallel to X′

10. Point 1 reaches X′ axes and stops while the other continues to move until they end up on X′ forming line segment E_1 F_2.

<u>It seems that once point 1 tears off the rail A_1 B_2 and the other is still there at rest, the inconsistency arises in K′ that has no place in system K</u>.

For a period of time, only one object is in motion and the system K structural integrity is violated. In K, the rail A_1 B_2 has either two material points attached it and nothing is in motion, or both points are away and moving. There is nothing in between.

This inconsistency conclusion rests on the correctness of relations presented in Figure 2 but one can already agree knowing the usual claim of special relativity: Events simultaneous in K are not simultaneous in K′.

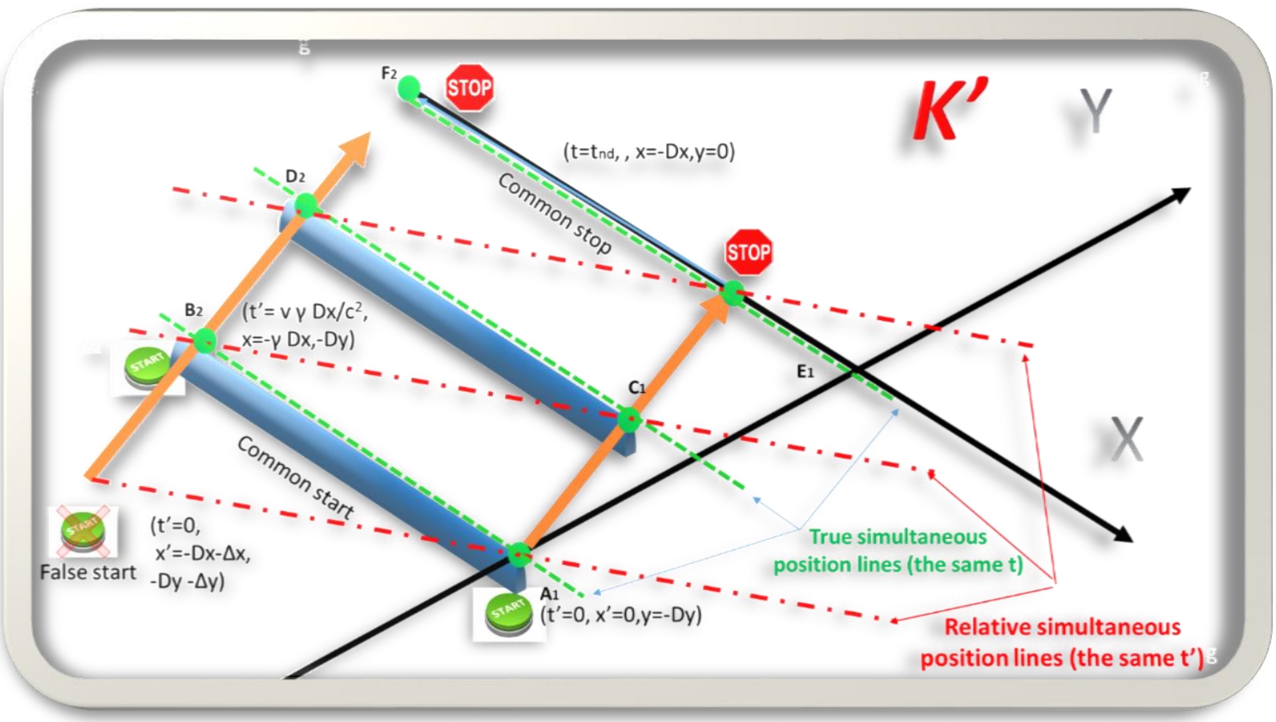

Figure 2 Transverse motion relative to X′ axis in the inertial system K′ as seen after Lorentz transfformations

More formalised approach to this inconsistency is required and one a bit more formal is given in the following sections.

2.2.2 Apparent Relative Simultaneity in Transverse Motion

Assuming that the STR is valid and defined as described by Einstein in his 1905 work [1] or as in the English translation [2, pp. 37-65], the following claim is put forward and justified within this work:

Assuming that:

1. $\boldsymbol{K}$ is a stationary inertial physical system (frame) and $\boldsymbol{K'}$ is a system moving relatively at constant velocity $\vec{\boldsymbol{v}}$.
2. The $\boldsymbol{\Sigma} = \{\boldsymbol{O}, \vec{\boldsymbol{w}}, \vec{\boldsymbol{x}}, \vec{\boldsymbol{y}}, \vec{\boldsymbol{z}}\}$ is a coordinate system associated with $\boldsymbol{K}$ and $\boldsymbol{\Sigma}' = \{\boldsymbol{O}', \vec{\boldsymbol{w}}', \vec{\boldsymbol{x}}', \vec{\boldsymbol{y}}', \vec{\boldsymbol{z}}'\}$ with $\boldsymbol{K'}$where $\boldsymbol{O}$ and $\boldsymbol{O'}$ are respective origins. The coordinates $\vec{\boldsymbol{w}}, \vec{\boldsymbol{x}}, \vec{\boldsymbol{y}}, \vec{\boldsymbol{z}}$ and $\vec{\boldsymbol{w}}', \vec{\boldsymbol{x}}', \vec{\boldsymbol{y}}', \vec{\boldsymbol{z}}'$are chosen such that $\boldsymbol{X}$ and $\boldsymbol{X'}$ axes coincide and they are parallel to velocity vector $\vec{\boldsymbol{v}}$ while the remaining axes remain parallel to their counterparts respectively.
3. Linear Lorenz transformation from $\boldsymbol{\Sigma}$ to $\boldsymbol{\Sigma}'$:

$$\overrightarrow{\boldsymbol{P}'} = \boldsymbol{L}\overrightarrow{\boldsymbol{P}} \tag{1}$$

describes the mapping of a 4D position vector $\overrightarrow{\boldsymbol{P}}$ of a physical object in $\boldsymbol{\Sigma}$ of the inertial system $\boldsymbol{K}$ to the corresponding true position vector of the same object $\overrightarrow{\boldsymbol{P}'}$ in $\boldsymbol{\Sigma'}$ of the system $\mathbf{K'}$, where:

$$\mathbf{L}=\begin{bmatrix} \gamma & \frac{-v}{c}\gamma & 0 & 0 \\ \frac{-v}{c}\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

   is the Lorentz transformation matrix.

4. A set of two distinct 3D trajectories in $\boldsymbol{K}$: $\{ \boldsymbol{Ti} = [x_i(t), y_i(t), z_i(t))]^T \} \mid i \in \mathbb{N} : 1 \le i \le 2$ of two objects moving during the same time interval t: $t_0 \le t \le t_N$ from the established initial positions $\boldsymbol{P_{0i}} \equiv [ct_0, x_{0i}, y_{0i}, z_{0i}]$ to distinct terminal positions $\boldsymbol{P_i} \equiv [ct_N, x_i, y_i, z_i]$ such that $x_1(t) \neq x_2(t)$,

the following can be said about the two trajectories of the same objects $\{ \boldsymbol{T'_i} = [x'_i(t'), y'_i(t'), z'_i(t'))]^T \}$ in $\boldsymbol{K'}$:

PROPOSITION 1

There exist pairs of trajectories of moving objects in the inertial system $\boldsymbol{K}$ and mathematically represented in $\boldsymbol{\Sigma}$, which after Lorentz transformation to $\boldsymbol{\Sigma'}$ cannot both at the same time t' describe the true position of the associated objects in $\boldsymbol{K'}$.

Details of definitions, necessary concepts and terms used in the above proposition and its premises are deferred to sections 3 and 4.

Superficially, the claim seems to be against common understanding of coordinate systems and transformations between them. How can possibly one trajectory be correct and the other not? It looks like a complete nonsense. If the reader recalls the informal presentation in section 2.2.1, it is obvious the claim is not without merit. The apparent contradiction is not the mathematical one, but it belongs to the domain of physical semantics (as understood by C. Weizsäcker [5, pp. 80-81]).

The claim is a subject of thorough justification and it will soon be clear that it is fully justified. Proving the proposition 1 is equivalent to a statement that relative simultaneity effect is not real but it is just a mathematical artefact of the STR framework.

3 PRELIMINARIES

3.1 Basic Assumptions.

Further assumptions used in the process of proving the Proposition 1 and constraining the physical contexts are:

1. Special Theory of Relativity and its general mathematical framework as originally presented by Einstein and described in [2, pp. 37-65] are assumed correct.
2. Three dimensional physical realisations of abstract coordinate systems can be built in the stationary system $\boldsymbol{K}$ and in the moving system $\boldsymbol{K'}$ with the help of solid objects in the neighbourhoods of the selected origins to provide experimental framework. Each system also has reference clocks positioned exactly at respective origins. To distinguish them from the four dimensional coordinate systems $\boldsymbol{\Sigma}$ and $\underline{\boldsymbol{\Sigma}'}$ we use the same symbol $\boldsymbol{K}$ an $\boldsymbol{K'}$ namely: $\boldsymbol{K} = \{\boldsymbol{O}, \vec{\boldsymbol{x}}, \vec{\boldsymbol{y}}, \vec{\boldsymbol{z}}\}$,
$\boldsymbol{K'} = \{\boldsymbol{O'}, \vec{\boldsymbol{x}}', \vec{\boldsymbol{y}}', \vec{\boldsymbol{z}}'\}$.

All details and conventions regarding coordinate systems are understood as explained by Einstein [2, p. 43 §3]. This abstraction is thought to be representative to a high degree to an ensemble of physical objects away from gravitational sources. The symbology used here does not fully match that in used in [1].

3.2 Relative Simultaneity Concept

There exists extensive literature dedicated to the concept of simultaneity and multitude of definitions and approaches exist. The best collection of information and the wealth of referenced publications can be found in the work of Max Jammer [6].

The notorious circularity problem is that simultaneity is generally defined by time while you need simultaneity to define time.[ii]

The following definition of time as duration to be used here is derived from Einstein's Princeton Lectures [4, p. 28]:

DEFINITION 1

TIME AS DURATION

Time as duration represented by symbols t, t' etc. is defined as the ensemble of the indications of similar clocks at rest relatively to an inertial frame which register the same, assuming that the clocks are synchronised by Einstein's method in the relevant frames of reference[iii].

DEFINITION 2

SIMULTANEITY OF EVENTS

Simultaneity of two physical events in an inertial frame from the set of $\{\boldsymbol{E_i}\}$, *where* $i \in \mathbb{N} : i>0$, *is a relation by which for any pair* $\boldsymbol{E_m}$ *and* $\boldsymbol{E_n}$ where $n,m \in \mathbb{N} : n,m>0$, for which times of occurrence $t_n(\boldsymbol{E_n}) = t_m(\boldsymbol{E_m})$, *where* t_m *and* t_m *represent times registered by clocks in respective locations when and where the events have occurred.*

To ensure there is no confusion about the central concept of an event, the following definition is proposed:

DEFINITION 4

PHYSICAL EVENT

A physical event is a physically discernible change of state of zero duration exhibited by a Physical Object.

An event is an empirical fact not just a point in a four dimensional coordinate system as commonly presented. A physical object is located in space where the said object exists when the event happens. Therefore it can be associated with relative coordinates of space and time pertinent to the object and its state. Zero duration highlights the fact of the uniqueness and distinction from common notions of events such as a conferences or celebrations, which have beginning and duration in common language.

Examples of physical events can be: collisions (or intersections of trajectories or trajectories with lines in mathematical models), lightning strikes of trains and train platforms, emissions of photons etc. Under this definition there is no event without a physical object existing and changing state.

DEFINITION 5

PHYSICAL OBJECT

Physical Object is anything physically discernible that can be located in the physical space.

Generally any known particle or any sets of particles are physical objects. In the context of physical models in a coordinate system, physical objects may be reduced to the abstraction of a material points, or rigid bodies.

DEFINITION 6

RELATIVE SIMULTANEITY EFFECT

Relative simultaneity effect occurs when the equality $t_m(\boldsymbol{E_m}) = t_n(\boldsymbol{E_n})$ for two simultaneous physical events $\boldsymbol{E_m}$ and $\boldsymbol{E_n}$ located at different spatial coordinates (x_m, y_m, z_m) and (x_n, y_n, z_n) in the inertial system $\boldsymbol{K}$ results in the inequality $t'_m(\boldsymbol{E_m}) \neq t'_n(\boldsymbol{E_n})$ for (x'_m, y'_m, z'_m) and (x'_n, y'_n, z'_n) after Lorentz Transformation to the moving system $\boldsymbol{K'}$ using four dimensional framework of $\boldsymbol{\Sigma}$ and $\boldsymbol{\Sigma'}$.

4 THE APPARENT NATURE OF RELATIVE SIMULTANEITY EFFECT EXPLANATION

It is quite problematic to give a reliable definition of what is real and what is apparent, however individual cases can be demonstrated. For example if one wishes to say that two events were simultaneous because they happened at the same clock time in two different time zones, one can easily prove the apparent nature of such poorly defined simultaneity. The case of relative simultaneity effect is less trivial.

The apparent nature of relative simultaneity is uncovered by proving the proposition 1 in the following sections.

4.1 Introduction to the Proof of the Proposition 1

We will use Lorentz transformation and equations of motion (EOMs). An equation of motion can be seen as a generator of successive coordinates of events relative to a stationary frame or any frame where the moving object is not at rest. Of particular interest is the initial event starting the motion such as creation and ejection of a particle high speed and the termination event when the said particle may cease to exist. An event represented by an EOM abstraction in $\boldsymbol{K}$ is a departure or an arrival of a material point from/to a given static position at a particular time. Most of controversies associated with the STR are in the context of EOMs.

The meaning of time variables t, t' is of critical importance here. The coordinate systems $\boldsymbol{\Sigma}$ and $\boldsymbol{\Sigma'}$ or $\boldsymbol{K}$ and $\boldsymbol{K'}$ are unaware of physical semantics so t and t' are just scalar parameters as understood in Newtonian physics applications. Time is a scalar in spatial coordinates because it is not derived from space but it is said to exist everywhere. Despite Newton's claims that time "flows equably without regard to anything external" [7], there is no definition of it, nor identifiable direction in the 3D space where it flows.

The claims made in the name of relativity on non-existence of "Now" (or universal present moment) such as that of Eddington in [8, p. 59][iv], are inconsistent with the fact that the universe happily coexists with one instance of *t* and we can extend the meaning of *t* to infinity and consider it being representative for all the universe, which - as we are told - has a definite age.

Extension to the fourth dimension to make time related coordinate is possible, however by adding the new dimension one should respect the physical nature of the three existing dimensions in terms of units of measure, therefore a formal spatial coordinate w was introduced, which all values are agreed to be covered by ct or ct' respectively, where c is the speed of light and t and t' as usual, scalar parameters somewhat representing clock time in respective frames.

1. Time as duration represented by a scalar parameter t can be linked to the reference clock indication at $\boldsymbol{O}(x,y,z)$. By the same token t' represents the reference clock time indication at $\boldsymbol{O}(x',y,'z')$ and moving relatively to the rest clock. Variables t and t' apply to the same indicated value of all properly synchronised clocks within respective inertial frames.
2. A trajectory of an arbitrary material point in the 3 dimensions in $\boldsymbol{K}$ is

$$\boldsymbol{T_{3D}} = \begin{bmatrix} \boldsymbol{x(t)} \\ \boldsymbol{y(t)} \\ \boldsymbol{z(t)} \end{bmatrix}, \tag{3}$$

   where $x \equiv T[1] = x(t)$, $y \equiv T[2] = y(t)$, $x \equiv T[3] = z(t)$ are values of coordinates in the Cartesian system fixed to $\boldsymbol{K}$ and coordinates are scalar functions of the scalar parameter t modifying the magnitude of respective unit vectors. Needless to say, we are operating in Euclidean 3D spaces which is consistent with Einstein's interpretation presented in [1].[v]

   We introduce the fourth abstract orthogonal axis $\boldsymbol{W}$ to create the coordinate system $\boldsymbol{\Sigma}$. In this system the trajectory is defined as:

$$T = \begin{bmatrix} ct \\ x(t) \\ y(t) \\ z(t) \end{bmatrix} \tag{4}$$

3. The $\boldsymbol{W}$ axis is called an abstract axis because it cannot be physically realised and calibrated by rigid standards as the other axes. There is no possibility to use a wire to trace $\boldsymbol{W}$ axis while it can be done with $\boldsymbol{X}$, $\boldsymbol{Y}$ and $\boldsymbol{Z}$. When referring to x, y, z or w without any index or bracket we mean: $w \equiv P[1]=w(t)$, $x \equiv P[2]=x(t)$, $y \equiv P[3]=y(t)$, $z \equiv P[4]=z(t)$.
4. In such defined system one can transform the trajectory $\boldsymbol{T}$ to $\boldsymbol{\Sigma'}$ system using Lorentz transformation as follows:

$$T' = LT \tag{5}$$

Where $\boldsymbol{L}$ is the Lorentz transformation matrix (2).
Similar can be done for a hypothetical Galilean system which is a good approximation for small relative velocities.

$$T' = GT \tag{6}$$

Where $\boldsymbol{G}$ is Galilean transformation matrix defined as:

$$\mathbf{G}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{-v}{c} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

AXIOM 1

THE SAME EVENT

A unique event is the same for two different inertial frames if it happens at the same location designated by a persistent extended rigid physical object that is at rest within one of the frames.

A simple example of such location for the event in question is a corner of a cube that can be given precise and time independent coordinate in one of the inertial systems. There is very little doubt that a designated cube with one corner in the coordinate system origin is unique, so are the remaining corners. An event that happens once on the corner of the cube cannot happen elsewhere for any other inertial system.

The axiom does not claim that only such events can be regarded as the same but it assumes that such class exists and this is sufficient for the purpose of this analysis. Time of the event is not important as every event is unique. The fact that some real two events may be difficult to discriminate is of no concern. One has to realise the level of abstraction here. When it comes to real objects at atomic level it is hard to tell what location really means, we are however operating in the idealised world of ideally rigid bodies as generally considered in relativity debates.

4.2 The Proof

To prove the proposition 1 it is sufficient to demonstrate a single example. If successful, the whole problem should be re-defined in the future in more general terms, and formally proven.

1. A set of two distinct trajectories of two objects in $\boldsymbol{K}$ moving during the same time interval $t: t_0 \leq t \leq t_N$ starting precisely from the initial points $\boldsymbol{P_{0i}} \equiv [ct_0, x_{0i}, y_{0i}, z_{0i}]$ to distinct terminal locations $\boldsymbol{P_i} \equiv [ct_N\ x_i, y_i, z_i]$ in the Σ system of $\boldsymbol{K}$ can be chosen as follows:

$$\boldsymbol{T_1} = \begin{bmatrix} \boldsymbol{ct} \\ \boldsymbol{0} \\ -\boldsymbol{D_y} + \boldsymbol{v_y t} \\ \boldsymbol{0} \end{bmatrix} \text{ and } \boldsymbol{T_2} = \begin{bmatrix} \boldsymbol{ct} \\ -\boldsymbol{D_x} \\ -\boldsymbol{D_y} + \boldsymbol{v_y t} \\ \boldsymbol{0} \end{bmatrix} \tag{8}$$

$\boldsymbol{P_{01}} \equiv [0, 0, -Dy, 0]$, $\boldsymbol{P_{02}} \equiv [0, -Dx, -Dy, 0]$,
$\boldsymbol{P_1} \equiv [cD_y/v_y, 0, 0, 0]$, $\boldsymbol{P_2} \equiv [cD_y/v_y, -D_x, 0, 0]$ and
$D_y>0, v_y>0, D_x>0$.

Transformation of the trajectories to $\boldsymbol{\Sigma'}$ in accordance with equations:

$$\boldsymbol{T'_1} = \boldsymbol{LT_1} \text{ and } \boldsymbol{T_2}' = \boldsymbol{LT_2} \tag{9}$$

yields:

$$\boldsymbol{T'}_1 = \begin{bmatrix} \boldsymbol{\gamma ct} \\ -\boldsymbol{v\gamma t} \\ -\boldsymbol{D_y} + \boldsymbol{v_y t} \\ \boldsymbol{0} \end{bmatrix} \text{ and } \boldsymbol{T'}_2 = \begin{bmatrix} \boldsymbol{\gamma ct} + \boldsymbol{v\gamma D_x/c} \\ -\boldsymbol{v\gamma t} - \boldsymbol{\gamma D_x} \\ -\boldsymbol{D_y} + \boldsymbol{v_y t} \\ \boldsymbol{0} \end{bmatrix} \tag{10}$$

It is <u>taken without a proof</u> based on universal acceptance of the STR, that each trajectory designated by the subset of 3 coordinates x′, y′ z′ namely $\mathbf{T'_1}$ and $\mathbf{T'_2}$, is a faithful representation of motions described in $\boldsymbol{K}$ as perceived by $\boldsymbol{K'}$. Therefore we assume $\boldsymbol{T'_1}$ is correct. Bringing $\boldsymbol{T'_1}$ to a covariant form with $\boldsymbol{T_1}$ yields:

$$\boldsymbol{T'}_1 = \begin{bmatrix} \boldsymbol{ct'} \\ -\boldsymbol{vt'} \\ -\boldsymbol{D_y} + \boldsymbol{v_y t'/\gamma} \\ \boldsymbol{0} \end{bmatrix} \tag{11}$$

At $t'=0$ the start event of the motion of the object 1 takes place on the line $y' = -D_y$. This lline naturally coincides with $y = -D_y$ so we see the location of the start event in y′ direction matches exactly where it originates in K as given by $\boldsymbol{P_{01}}$. In order for the two trajectories to be used in the context of the same local time t', both coordinates w should be equal. The equality:

$$\gamma ct = \gamma ct + v\gamma D_x/c \tag{12}$$

can only be true for $D_x=0$ which is only when $\boldsymbol{T_1}$ is identical with $\boldsymbol{T_2}$.

Therefore there is no common coordinate w′ for the two trajectories so there is no common time variable t' value for which the trajectory $\boldsymbol{T'_2}$ could be compared with $\boldsymbol{T'_1}$, hence $\boldsymbol{T'_2}$ cannot describe the correct position of the object 2 at the same time t'.

This should not be a surprise noting that the respective coordinates w' of the $\boldsymbol{T'_1}$ and $\boldsymbol{T'_2}$ are always at variance by $v\gamma D_x/c$, or in terms of clock time:

$$\Delta \mathrm{t}' = v\gamma D_x/c^2 \tag{13}$$

The proof could almost stop there but not the whole aspect of the problem would be clear.

The conversion of $\boldsymbol{T'_2}$ into a covariant form using parameter t' by solving $\gamma ct + v\gamma D_x/c = ct'$ from which $t = -vD_x/c^2 + t'/\gamma$ yields:

$$\boldsymbol{T'}_2 = \begin{bmatrix} \boldsymbol{ct'} \\ \boldsymbol{v^2\, \gamma D_x/c^2 - \gamma D_x - vt'} \\ \boldsymbol{-D_y - vv_y\, D_x/c^2 + v_y t'/\gamma} \\ \boldsymbol{0} \end{bmatrix} \tag{14}$$

When we examine the initial position at t`$=0$ the y coordinate is below the well-known pre-defined and unique start event location lying exactly on line $y' = y = -D_y$, common to $\boldsymbol{K}$ and $\boldsymbol{K'}$ by an offset of $-vv_y\, D_x/c^2$. <u>This is not a position the object 2 could have started from, it has never been there, and never will according to the EOM</u>. Trajectory $\boldsymbol{T'_1}$ on the contrary, has started at $t'=0$ exactly where expected at $y = y' = -D_y$. Additionally in the system K, there is no instant where two points 1 and 2 are ever at different relative distance to the line $y = -D_y$ or to each other. When after some time $t' = v\gamma D_x/c^2$ $\boldsymbol{T'_2}$ leaves the line $y' = -D_y$, $\boldsymbol{T'_1}$ indicates position closer towards $y' = 0$. There is never the case in K when the two points 1 and 2 belong to two distinct lines parallel to X′ a is it would indicate they belong to two different physical structures. Trajectories $\boldsymbol{T'_1}$ and $\boldsymbol{T'_2}$ taken literally, violate structural integrity of the stationary system K. From the above we can say about trajectories $\boldsymbol{T'_1}$ and $\boldsymbol{T'_2}$**:**

<u>There exist a pair of trajectories of moving objects in ***K*** and mathematically represented in ***Σ***, which after Lorentz transformation to ***Σ′*** </u>they <u>cannot both at the same time t' describe the true position of the associated objects in ***K′***</u>. Q.E.D.

5 DISCUSSION

5.1 Implications

The immediate implications of the above proof are:

1. Both EOMs $\boldsymbol{T'_1}$ and $\boldsymbol{T_2'}$ correctly describe the motion in $\boldsymbol{K'}$ separately in their own right. At the moment of simultaneous start while $y' = -D_y$, the local clock at object 1 shows 0 and that of the object 2 shows $v\gamma D_x/c^2$. This is not much different in principle from the scenario when the engine of a train at the border of a time zone starts at 00h:00m while a carriage left behind in a different zone starts at 01:00h am. Equal time on clocks in general, do not necessarily designate simultaneous moments.
2. By the virtue of the Axiom 1 unique events 1 and 2 originating at the edge of a hypothetical linear physical object in $\boldsymbol{K}$ that might have the edge co-linear with $y = y' = -D_y$, cannot happen anywhere else so placing the object 2 at t'=0 at $y = -D_y - vv_y\, D_x/c^2$ is against the reality of object's existence in constrained locations. This supports our initial suspicion about the apparent nature of relative simultaneity effect.
3. Negative implication of these findings may not be catastrophic to the STR because what is really impacted are the informal statements about simultaneity mostly of little empirical significance. Serious doubts need to be raised about parallel time reality related fantasies. Another casualty might be the teaching methodologies which were trying to convince people to acknowledge something that has no place in reality.
4. The impact on certain areas of philosophy of time is expected. After the challenge to relativity of time by H. Bergson, due to its negative reception and public humiliation he was forced to withdraw the support to his own writing published under the title *Durée et simultanéité* in 1922 ([9] translation published against his will). Since then philosophers are not keen of this kind of challenge and they have evolved to accept science based on overwhelming consensus.
5. Positive implication is the likely resolution of all relativity paradoxes to the satisfaction of all the interested parties with the prominent Twin Paradox leading the pack. Subsequently the mysteries of Gödel closed time-like loops may find rational explanation as well.

5.2 Case Study – Rigid Rod in Transverse Motion

The example chosen for the proof may be linked to the description of motions of two opposite ends of a rigid rod parallel to $\boldsymbol{X}$ axis starting its motion at *t=0* from line $y = -D_y$ towards $y = 0$ with velocity v_y pointing up towards the $\boldsymbol{X}$.

The conventional relativistic wisdom maintains that due to relativity of simultaneity the rod is perceived in $\boldsymbol{K'}$ as being at an angle to the $\boldsymbol{X'}$ axis even though the $\boldsymbol{X'}$ axis is physically coincident with $\boldsymbol{X}$ by definition and to which the rod is parallel at all times in $\boldsymbol{K}$ during its travel . The author found this very odd, and has engaged in multiple discussions on various internet forums with all the same effect: It must be traveling at an angle to $\boldsymbol{X'}$. The figure below shows the scenario.

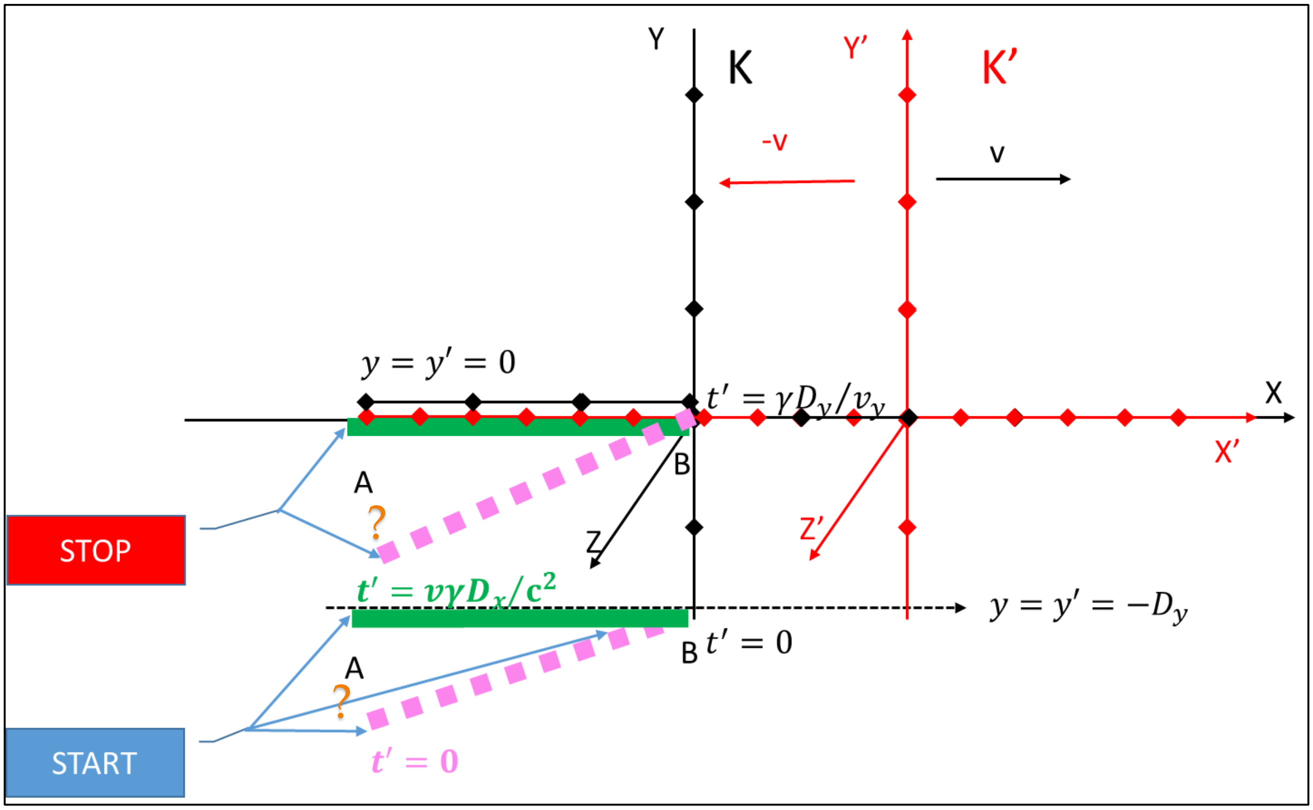

Figure 3 Transverse motion of the rigid rod

Only a handful of people supported the idea that the rod travels parallel to $\boldsymbol{X'}$ no matter what the clock at its end says, in the same way as it does relative to the $\boldsymbol{X}$.

The apparent nature of the relative simultaneity vindicates this point of view. The rod $\boldsymbol{AB}$ shown as solid left off $\boldsymbol{Y}$ as seen in $\boldsymbol{K}$ and in $\boldsymbol{K'}$ when each end is positioned as per direct Lorentz transformation. The rod shown dashed line is when we force the transformed trajectories to common time $\boldsymbol{t'}$. One can clearly see in the diagram the rod representation marked by dashed line according to the conventional interpretation has one end where it really starts, and the other where it has never been. Alternatively we could say the rod starts rotating at t'=0 until it reaches full tilt at $t' = v\gamma D_x/c^2$. That is at odds with the reality.

6 REFERENCES AND NOTES

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[i] Difficulties in comprehension of the STR by students of various grades are reported in literature. One study [8] finds that:

> *The results indicate that after standard instruction students at all academic levels have serious difficulties with the relativity of simultaneity and with the role of observers in inertial reference frames. Evidence is presented that suggests many students construct a conceptual framework in which the ideas of absolute simultaneity and the relativity of simultaneity harmoniously co-exist […] After instruction, more than two-thirds of physics undergraduates and one-third of graduate students in physics are unable to apply the construct of a reference frame in determining whether or not two events are simultaneous.*

[ii] For example in Merriam-Webster on line dictionary [10] definition of simultaneous is:

> *Existing or occurring at the same time: exactly coincident*

But according to Einstein [2, p. §1]

> *We have to take into account that all our judgments in which time plays a part are always judgments of simultaneous events.*

Einstein overcomes circularity by means of defining clocks which are synchronised by a given procedure under the assumption of constant speed of light in all directions. Simultaneity is then the result of this synchronisation and defined as the same clock indication. This corresponds to Webster's definition above and no obvious circularity can be seen.

[iii] The word "simultaneously" has been dropped from the original text and replaced by reference to Einstein's synchronisation which is equivalent, but removes apparent circularity that would arise in the next definition.

iv :Eddington has famously declared[8]:

> *[…] a difficulty has arisen because we have had to abolish Now. There is no absolute Now, but only the various relative Nows differing according to the reckoning of different observers […],*

This posture remains to this date.

[v] There is often a debate about whether Euclidean system is appropriate when motion is involved but in the opinion of the author there is little doubt the 3 dimensional spaces of inertial systems are Euclidean. This can be deduced rom the following passage [2]:

If a material point is at rest relatively to this system of co-ordinates, its position can be defined relatively thereto by the employment of rigid standards of measurement and the methods of Euclidean geometry, and can be expressed in Cartesian co-ordinates.

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https://www.researchgate.net/post/When_it_comes_to_simultaneity_is_Einstein_correct_or_is_Dingle_correct#5521dde4f15bc7bb5f8b45d4
and
https://www.researchgate.net/post/Is_a_rigid_rod_moving_perpendicularly_to_x_axis_in_the_standard_Special_Relativity_axes_convention_parallel_to_the_moving_system_x_axis
Last but not least, to my wife Cristy who is faster than light and had to put up with my slow reaction to daily challenges at home for a considerable period of time.