The Dirac Equation
is a Special Case of
the Maxwell-Cassano Equations

For vector $\boldsymbol{\psi}_D$ The Klein-Gordon equation may be written [4]:

$$\left(\nabla^2 - \frac{\partial^2}{\partial t^2} - m^2\right)\boldsymbol{\psi}_D = \mathbf{0} \ .$$

Whenever $\boldsymbol{\psi}_D \equiv \begin{pmatrix} \boldsymbol{\phi}_D^A \\ \boldsymbol{\phi}_D^B \end{pmatrix}$ is a 2^M-dimensional vector, via a matrix differential operator factorization, it may be written (in the Dirac representation):

$$\begin{pmatrix} \mathbf{I}_2\left(i\frac{\partial}{\partial t} + m\right) & i\boldsymbol{\sigma}\bullet\vec{\nabla} \\ -i\boldsymbol{\sigma}\bullet\vec{\nabla} & \mathbf{I}_2\left(-i\frac{\partial}{\partial t} + m\right) \end{pmatrix}\begin{pmatrix} \mathbf{I}_2\left(-i\frac{\partial}{\partial t} + m\right) & -i\boldsymbol{\sigma}\bullet\vec{\nabla} \\ i\boldsymbol{\sigma}\bullet\vec{\nabla} & \mathbf{I}_2\left(i\frac{\partial}{\partial t} + m\right) \end{pmatrix}\begin{pmatrix} \boldsymbol{\phi}_D^A \\ \boldsymbol{\phi}_D^B \end{pmatrix} = \mathbf{0}$$

and, since these matrix operators are commutative:

$$\begin{pmatrix} \mathbf{I}_2\left(-i\frac{\partial}{\partial t} + m\right) & -i\boldsymbol{\sigma}\bullet\vec{\nabla} \\ i\boldsymbol{\sigma}\bullet\vec{\nabla} & \mathbf{I}_2\left(i\frac{\partial}{\partial t} + m\right) \end{pmatrix}\begin{pmatrix} \mathbf{I}_2\left(i\frac{\partial}{\partial t} + m\right) & i\boldsymbol{\sigma}\bullet\vec{\nabla} \\ -i\boldsymbol{\sigma}\bullet\vec{\nabla} & \mathbf{I}_2\left(-i\frac{\partial}{\partial t} + m\right) \end{pmatrix}\begin{pmatrix} \boldsymbol{\phi}_D^A \\ \boldsymbol{\phi}_D^B \end{pmatrix} = \mathbf{0}$$

Where:

$$\boldsymbol{\sigma}\bullet\vec{\nabla} = \sum_{\nu=1}^{3}\boldsymbol{\sigma}^\nu\frac{\partial}{\partial x^\nu}$$

$$\boldsymbol{\sigma}^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathbf{I}_2$$

$$\boldsymbol{\sigma}^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad , \quad \boldsymbol{\sigma}^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad , \quad \boldsymbol{\sigma}^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\Rightarrow \boldsymbol{\sigma}\bullet\vec{\nabla} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\partial_1 + \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\partial_2 + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\partial_3$$

$$= \begin{pmatrix} \partial_3 & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & -\partial_3 \end{pmatrix}$$

$$\Rightarrow \left(\boldsymbol{\sigma}\bullet\vec{\nabla}\right)^2 = \begin{pmatrix} \partial_3 & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & -\partial_3 \end{pmatrix}\begin{pmatrix} \partial_3 & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & -\partial_3 \end{pmatrix} = (\partial_3^2 + \partial_1^2 + \partial_2^2)\mathbf{I}_2 = \nabla^2\mathbf{I}_2$$

Let:

$$\begin{pmatrix} \boldsymbol{\psi}_D^A \\ \boldsymbol{\psi}_D^B \end{pmatrix} = \begin{pmatrix} \mathbf{I}_2\left(-i\frac{\partial}{\partial t} + m\right) & -i\boldsymbol{\sigma}\bullet\vec{\nabla} \\ i\boldsymbol{\sigma}\bullet\vec{\nabla} & \mathbf{I}_2\left(i\frac{\partial}{\partial t} + m\right) \end{pmatrix}\begin{pmatrix} \boldsymbol{\phi}_D^A \\ \boldsymbol{\phi}_D^B \end{pmatrix}$$

$$\begin{pmatrix} \boldsymbol{\psi}_D^C \\ \boldsymbol{\psi}_D^D \end{pmatrix} = \begin{pmatrix} \mathbf{I}_2\left(i\frac{\partial}{\partial t} + m\right) & i\boldsymbol{\sigma}\bullet\vec{\nabla} \\ -i\boldsymbol{\sigma}\bullet\vec{\nabla} & \mathbf{I}_2\left(-i\frac{\partial}{\partial t} + m\right) \end{pmatrix}\begin{pmatrix} \boldsymbol{\phi}_D^A \\ \boldsymbol{\phi}_D^B \end{pmatrix}$$

then:

$$\begin{pmatrix} \mathbf{I}_2\left(i\frac{\partial}{\partial t} + m\right) & i\boldsymbol{\sigma}\bullet\vec{\nabla} \\ -i\boldsymbol{\sigma}\bullet\vec{\nabla} & \mathbf{I}_2\left(-i\frac{\partial}{\partial t} + m\right) \end{pmatrix}\begin{pmatrix} \boldsymbol{\psi}_D^A \\ \boldsymbol{\psi}_D^B \end{pmatrix} = \mathbf{0}$$

$$\begin{pmatrix} \mathrm{I}_2\left(-i\frac{\partial}{\partial t} + m\right) & -i\boldsymbol{\sigma}\bullet\vec{\nabla} \\ i\boldsymbol{\sigma}\bullet\vec{\nabla} & \mathrm{I}_2\left(i\frac{\partial}{\partial t} + m\right) \end{pmatrix}\begin{pmatrix} \boldsymbol{\psi}_D^C \\ \boldsymbol{\psi}_D^D \end{pmatrix} = \mathbf{0}$$

conformability of the matrices requires that:

$\boldsymbol{\phi}_D^A$. $\boldsymbol{\phi}_D^B$; $\boldsymbol{\psi}_D^A$. $\boldsymbol{\psi}_D^B$.$\boldsymbol{\psi}_D^C$.$\boldsymbol{\psi}_D^D$ are all 2×1 matrices; so setting:

Let:

$\boldsymbol{\phi}_D^A = \begin{pmatrix} \phi_D^0 \\ \phi_D^1 \end{pmatrix}$	$\boldsymbol{\phi}_D^B = \begin{pmatrix} \phi_D^2 \\ \phi_D^3 \end{pmatrix}$

$\boldsymbol{\psi}_D^A = \begin{pmatrix} \psi_D^0 \\ \psi_D^1 \end{pmatrix}$	$\boldsymbol{\psi}_D^B = \begin{pmatrix} \psi_D^2 \\ \psi_D^3 \end{pmatrix}$	$\boldsymbol{\psi}_D^C = \begin{pmatrix} \psi_D^4 \\ \psi_D^5 \end{pmatrix}$	$\boldsymbol{\psi}_D^D = \begin{pmatrix} \psi_D^6 \\ \psi_D^7 \end{pmatrix}$

then:

$$\begin{pmatrix} \begin{pmatrix} \left(i\frac{\partial}{\partial t}+m\right) & 0 \\ 0 & \left(i\frac{\partial}{\partial t}+m\right) \end{pmatrix} & i\begin{pmatrix} \partial_3 & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & -\partial_3 \end{pmatrix} \\ -i\begin{pmatrix} \partial_3 & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & -\partial_3 \end{pmatrix} & \begin{pmatrix} \left(-i\frac{\partial}{\partial t}+m\right) & 0 \\ 0 & \left(-i\frac{\partial}{\partial t}+m\right) \end{pmatrix} \end{pmatrix} \begin{pmatrix} \begin{pmatrix} \psi_D^0 \\ \psi_D^1 \end{pmatrix} \\ \begin{pmatrix} \psi_D^2 \\ \psi_D^3 \end{pmatrix} \end{pmatrix} = \mathbf{0}$$

$$= -\left(\nabla^2 - \frac{\partial^2}{\partial t^2} - m^2\right)\boldsymbol{\Psi}_D \ .$$

$$\begin{pmatrix} \begin{pmatrix} \left(-i\frac{\partial}{\partial t}+m\right) & 0 \\ 0 & \left(-i\frac{\partial}{\partial t}+m\right) \end{pmatrix} & -i\begin{pmatrix} \partial_3 & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & -\partial_3 \end{pmatrix} \\ i\begin{pmatrix} \partial_3 & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & -\partial_3 \end{pmatrix} & \begin{pmatrix} \left(i\frac{\partial}{\partial t}+m\right) & 0 \\ 0 & \left(i\frac{\partial}{\partial t}+m\right) \end{pmatrix} \end{pmatrix} \begin{pmatrix} \begin{pmatrix} \psi_D^4 \\ \psi_D^5 \end{pmatrix} \\ \begin{pmatrix} \psi_D^6 \\ \psi_D^7 \end{pmatrix} \end{pmatrix} = \mathbf{0}$$

$$= -\left(\nabla^2 - \frac{\partial^2}{\partial t^2} - m^2\right)\begin{pmatrix} \boldsymbol{\Psi}_D^C \\ \boldsymbol{\Psi}_D^D \end{pmatrix} \ .$$

For symmetry purposes, let:

$$t \equiv ix^0$$

then, combining into a single matrix equation,

Let:

$$\boldsymbol{\theta} = \begin{pmatrix} \begin{pmatrix} \theta_D^0 \\ \theta_D^1 \end{pmatrix} \\ \begin{pmatrix} \theta_D^2 \\ \theta_D^3 \end{pmatrix} \\ \begin{pmatrix} \theta_D^4 \\ \theta_D^5 \end{pmatrix} \\ \begin{pmatrix} \theta_D^6 \\ \theta_D^7 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} \begin{pmatrix} (\partial_0+m) & 0 \\ 0 & (\partial_0+m) \end{pmatrix} & i\begin{pmatrix} \partial_3 & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & -\partial_3 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ -i\begin{pmatrix} \partial_3 & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & -\partial_3 \end{pmatrix} & \begin{pmatrix} (-\partial_0+m) & 0 \\ 0 & (-\partial_0+m) \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} (-\partial_0+m) & 0 \\ 0 & (-\partial_0+m) \end{pmatrix} & -i\begin{pmatrix} \partial_3 & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & -\partial_3 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & i\begin{pmatrix} \partial_3 & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & -\partial_3 \end{pmatrix} & \begin{pmatrix} (\partial_0+m) & 0 \\ 0 & (\partial_0+m) \end{pmatrix} \end{pmatrix} \begin{pmatrix} \begin{pmatrix} \psi_D^0 \\ \psi_D^1 \end{pmatrix} \\ \begin{pmatrix} \psi_D^2 \\ \psi_D^3 \end{pmatrix} \\ \begin{pmatrix} \psi_D^4 \\ \psi_D^5 \end{pmatrix} \\ \begin{pmatrix} \psi_D^6 \\ \psi_D^7 \end{pmatrix} \end{pmatrix}$$

$$\begin{pmatrix} \begin{pmatrix} (-\partial_0+m) & 0 \\ 0 & (-\partial_0+m) \end{pmatrix} & -i\begin{pmatrix} \partial_3 & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & -\partial_3 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ i\begin{pmatrix} \partial_3 & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & -\partial_3 \end{pmatrix} & \begin{pmatrix} (\partial_0+m) & 0 \\ 0 & (\partial_0+m) \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} (\partial_0+m) & 0 \\ 0 & (\partial_0+m) \end{pmatrix} & i\begin{pmatrix} \partial_3 & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & -\partial_3 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & -i\begin{pmatrix} \partial_3 & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & -\partial_3 \end{pmatrix} & \begin{pmatrix} (-\partial_0+m) & 0 \\ 0 & (-\partial_0+m) \end{pmatrix} \end{pmatrix} \begin{pmatrix} \begin{pmatrix} \theta_D^0 \\ \theta_D^1 \end{pmatrix} \\ \begin{pmatrix} \theta_D^2 \\ \theta_D^3 \end{pmatrix} \\ \begin{pmatrix} \theta_D^4 \\ \theta_D^5 \end{pmatrix} \\ \begin{pmatrix} \theta_D^6 \\ \theta_D^7 \end{pmatrix} \end{pmatrix} =$$

$$= -(\Box - m^2)\boldsymbol{\theta} = \mathbf{0} \ .$$

Now,

Just as there are a number of representations of th Dirac equation, there is more than one matrix operator factorization of the Maxwell-Cassano equations [3].

One such, is:

$$
\begin{pmatrix} \mathbf{H}^1 \\ \mathbf{H}^2 \\ \mathbf{H}^3 \\ \mathbf{H}^4 \\ \mathbf{H}^5 \\ \mathbf{H}^6 \\ \mathbf{H}^7 \\ \mathbf{H}^8 \end{pmatrix}
=
\begin{pmatrix}
(\partial_1 - m_1) & (\partial_2 - m_2) & (\partial_3 - m_3) & (\partial_4 - m_4) & 0 & 0 & 0 & 0 \\
-(\partial_2 + m_2) & (\partial_1 + m_1) & 0 & 0 & 0 & 0 & (\partial_4 - m_4) & -(\partial_3 - m_3) \\
(\partial_3 + m_3) & 0 & -(\partial_1 + m_1) & 0 & 0 & (\partial_4 - m_4) & 0 & -(\partial_2 - m_2) \\
(\partial_4 + m_4) & 0 & 0 & -(\partial_1 + m_1) & 0 & -(\partial_3 - m_3) & (\partial_2 - m_2) & 0 \\
0 & 0 & 0 & 0 & (\partial_1 + m_1) & (\partial_2 + m_2) & (\partial_3 + m_3) & (\partial_4 + m_4) \\
0 & 0 & (\partial_4 + m_4) & -(\partial_3 + m_3) & -(\partial_2 - m_2) & (\partial_1 - m_1) & 0 & 0 \\
0 & (\partial_4 + m_4) & 0 & -(\partial_2 + m_2) & (\partial_3 - m_3) & 0 & -(\partial_1 - m_1) & 0 \\
0 & -(\partial_3 + m_3) & (\partial_2 + m_2) & 0 & (\partial_4 - m_4) & 0 & 0 & -(\partial_1 - m_1)
\end{pmatrix}
\begin{pmatrix} h^1 \\ h^2 \\ h^3 \\ h^4 \\ h^5 \\ h^6 \\ h^7 \\ h^8 \end{pmatrix}
$$

Then:

$$
\begin{pmatrix}
(\partial_1 + m_1) & -(\partial_2 - m_2) & (\partial_3 - m_3) & (\partial_4 - m_4) & 0 & 0 & 0 & 0 \\
(\partial_2 + m_2) & (\partial_1 - m_1) & 0 & 0 & 0 & 0 & (\partial_4 - m_4) & -(\partial_3 - m_3) \\
(\partial_3 + m_3) & 0 & -(\partial_1 - m_1) & 0 & 0 & (\partial_4 - m_4) & 0 & (\partial_2 - m_2) \\
(\partial_4 + m_4) & 0 & 0 & -(\partial_1 - m_1) & 0 & -(\partial_3 - m_3) & -(\partial_2 - m_2) & 0 \\
0 & 0 & 0 & 0 & (\partial_1 - m_1) & -(\partial_2 + m_2) & (\partial_3 + m_3) & (\partial_4 + m_4) \\
0 & 0 & (\partial_4 + m_4) & -(\partial_3 + m_3) & (\partial_2 - m_2) & (\partial_1 + m_1) & 0 & 0 \\
0 & (\partial_4 + m_4) & 0 & (\partial_2 + m_2) & (\partial_3 - m_3) & 0 & -(\partial_1 + m_1) & 0 \\
0 & -(\partial_3 + m_3) & -(\partial_2 + m_2) & 0 & (\partial_4 - m_4) & 0 & 0 & -(\partial_1 + m_1)
\end{pmatrix}
\begin{pmatrix} \mathbf{H}^1 \\ \mathbf{H}^2 \\ \mathbf{H}^3 \\ \mathbf{H}^4 \\ \mathbf{H}^5 \\ \mathbf{H}^6 \\ \mathbf{H}^7 \\ \mathbf{H}^8 \end{pmatrix}
=
$$

$$
\begin{pmatrix}
(\Box - |m|^2) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & (\Box - |m|^2) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & (\Box - |m|^2) & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & (\Box - |m|^2) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & (\Box - |m|^2) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & (\Box - |m|^2) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & (\Box - |m|^2) & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & (\Box - |m|^2)
\end{pmatrix}
\begin{pmatrix} h^1 \\ h^2 \\ h^3 \\ h^4 \\ h^5 \\ h^6 \\ h^7 \\ h^8 \end{pmatrix}
$$

Another matrix operator factorization of the Maxwell-Cassano equations may be compactly written, as follows [1][3]:

From the definitions:

$$
\mathbf{f} \equiv \mathbf{w}^{4;i} f^i \quad , \textit{where}: \quad f^i \equiv \begin{pmatrix} f^i_+ \\ f^i_- \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} f^i_+ \\ f^i_- \end{pmatrix} \quad , \quad f^i_+, f^i_- \in R
$$

$$
D_i^+ \equiv (\partial_i + m_i) \quad , \quad D_i^- \equiv (\partial_i - m_i)
$$

$$
D_i \equiv \begin{pmatrix} D_i^+ & 0 \\ 0 & D_i^- \end{pmatrix} \quad , \quad D_i^{\Updownarrow} \equiv \begin{pmatrix} D_i^- & 0 \\ 0 & D_i^+ \end{pmatrix} \quad ,
$$

$$
D_i^{\Leftrightarrow} \equiv \begin{pmatrix} 0 & D_i^- \\ D_i^+ & 0 \end{pmatrix} \quad , \quad D_i^{\Leftrightarrow\Updownarrow} \equiv \begin{pmatrix} 0 & D_i^+ \\ D_i^- & 0 \end{pmatrix}
$$

$$
\widehat{\mathbf{f}} \equiv \begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix}
$$

such a factorization may be

$$
\begin{pmatrix}
D_0 & D_3^{\Leftrightarrow} & -D_2^{\Leftrightarrow} & D_1 \\
-D_3^{\Leftrightarrow} & D_0 & D_1^{\Leftrightarrow} & D_2 \\
D_2^{\Leftrightarrow} & -D_1^{\Leftrightarrow} & D_0 & D_3 \\
D_1^{\Updownarrow} & D_2^{\Updownarrow} & D_3^{\Updownarrow} & -D_0^{\Updownarrow}
\end{pmatrix}
\begin{pmatrix}
D_0^{\Updownarrow} & -D_3^{\Leftrightarrow} & D_2^{\Leftrightarrow} & D_1 \\
D_3^{\Leftrightarrow} & D_0^{\Updownarrow} & -D_1^{\Leftrightarrow} & D_2 \\
-D_2^{\Leftrightarrow} & D_1^{\Leftrightarrow} & D_0^{\Updownarrow} & D_3 \\
D_1^{\Updownarrow} & D_2^{\Updownarrow} & D_3^{\Updownarrow} & -D_0
\end{pmatrix}
\begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix}
$$

$$
=
\begin{pmatrix}
-D_0 & D_3^{\Leftrightarrow} & -D_2^{\Leftrightarrow} & -D_1 \\
-D_3^{\Leftrightarrow} & -D_0 & D_1^{\Leftrightarrow} & -D_2 \\
D_2^{\Leftrightarrow} & -D_1^{\Leftrightarrow} & -D_0 & -D_3 \\
-D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0^{\Updownarrow}
\end{pmatrix}
\begin{pmatrix}
-D_0^{\Updownarrow} & -D_3^{\Leftrightarrow} & D_2^{\Leftrightarrow} & -D_1 \\
D_3^{\Leftrightarrow} & -D_0^{\Updownarrow} & -D_1^{\Leftrightarrow} & -D_2 \\
-D_2^{\Leftrightarrow} & D_1^{\Leftrightarrow} & -D_0^{\Updownarrow} & -D_3 \\
-D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0
\end{pmatrix}
\begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix}
$$

$$
=
\begin{pmatrix}
D_0 & -D_3^{\Leftrightarrow} & D_2^{\Leftrightarrow} & D_1 \\
D_3^{\Leftrightarrow} & D_0 & -D_1^{\Leftrightarrow} & D_2 \\
-D_2^{\Leftrightarrow} & D_1^{\Leftrightarrow} & D_0 & D_3 \\
D_1^{\Updownarrow} & D_2^{\Updownarrow} & D_3^{\Updownarrow} & -D_0^{\Updownarrow}
\end{pmatrix}
\begin{pmatrix}
D_0^{\Updownarrow} & D_3^{\Leftrightarrow} & -D_2^{\Leftrightarrow} & D_1 \\
-D_3^{\Leftrightarrow} & D_0^{\Updownarrow} & D_1^{\Leftrightarrow} & D_2 \\
D_2^{\Leftrightarrow} & -D_1^{\Leftrightarrow} & D_0^{\Updownarrow} & D_3 \\
D_1^{\Updownarrow} & D_2^{\Updownarrow} & D_3^{\Updownarrow} & -D_0
\end{pmatrix}
\begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix}
$$

$$= \begin{pmatrix} (\Box - |m|^2)f^1 \\ (\Box - |m|^2)f^2 \\ (\Box - |m|^2)f^3 \\ (\Box - |m|^2)f^0 \end{pmatrix} = (\Box - |m|^2)\,\widehat{\mathbf{f}}$$

For the stationary state the source/sink density term vanishes in the Maxwell-Cassano equations, which allows an equating of the Maxwell-Cassano equation & Dirac equation factorizations.
These may imply correlations between the Dirac equation and the Maxwell-Cassano equations as the correspondences/mappings:
$m \Leftrightarrow |m|$ & $-\theta_D^j \Leftrightarrow f^h$.

The Dirac equation may be expanded with the above notation into:

$$\begin{aligned}
&(-\partial_0 + m)\theta_D^0 - i\partial_3\theta_D^2 - i(\partial_1 - i\partial_2)\theta_D^3 = 0\\
&(-\partial_0 + m)\theta_D^1 - i(\partial_1 + i\partial_2)\theta_D^2 + i\partial_3\theta_D^3 = 0\\
&i\partial_3\theta_D^0 + i(\partial_1 - i\partial_2)\theta_D^1 + (\partial_0 + m)\theta_D^2 = 0\\
&i(\partial_1 + i\partial_2)\theta_D^0 - i\partial_3\theta_D^1 + (\partial_0 + m)\theta_D^3 = 0\\
&(\partial_0 + m)\theta_D^4 + i\partial_3\theta_D^6 + i(\partial_1 - i\partial_2)\theta_D^7 = 0\\
&(\partial_0 + m)\theta_D^5 + i(\partial_1 + i\partial_2)\theta_D^6 - i\partial_3\theta_D^7 = 0\\
&-i\partial_3\theta_D^4 - i(\partial_1 - i\partial_2)\theta_D^5 + (-\partial_0 + m)\theta_D^6 = 0\\
&-i(\partial_1 + i\partial_2)\theta_D^4 + i\partial_3\theta_D^5 + (-\partial_0 + m)\theta_D^7 = 0
\end{aligned}$$

or:

$$\begin{aligned}
(-\partial_0 + m)\theta_D^0 \quad & - i\partial_3\theta_D^2 - i(\partial_1 - i\partial_2)\theta_D^3 = 0\\
(\partial_0 + m)\theta_D^4 \quad & + i\partial_3\theta_D^6 + i(\partial_1 - i\partial_2)\theta_D^7 = 0\\
(-\partial_0 + m)\theta_D^1 - i(\partial_1 + i\partial_2)\theta_D^2 \quad & + i\partial_3\theta_D^3 = 0\\
(\partial_0 + m)\theta_D^5 + i(\partial_1 + i\partial_2)\theta_D^6 \quad & - i\partial_3\theta_D^7 = 0\\
i\partial_3\theta_D^0 + i(\partial_1 - i\partial_2)\theta_D^1 \quad & + (\partial_0 + m)\theta_D^2 = 0\\
-i\partial_3\theta_D^4 - i(\partial_1 - i\partial_2)\theta_D^5 \quad & + (-\partial_0 + m)\theta_D^6 = 0\\
i(\partial_1 + i\partial_2)\theta_D^0 \quad - i\partial_3\theta_D^1 \quad & + (\partial_0 + m)\theta_D^3 = 0\\
-i(\partial_1 + i\partial_2)\theta_D^4 \quad + i\partial_3\theta_D^5 \quad & + (-\partial_0 + m)\theta_D^7 = 0
\end{aligned}$$

As [1] shows. the component pairs may be organized such that this organization exhibits the mass-generalization of Maxwell's equations, but organizing them while comparing them analogously to the Dirac equations yields:

$$\begin{aligned}
&(-\partial_0 + m)\theta_D^0 \quad - i\partial_3\theta_D^2 \quad - i(\partial_1 - i\partial_2)\theta_D^3 = 0\\
&(\partial_0 + m)\theta_D^4 \quad + i\partial_3\theta_D^6 \quad + i(\partial_1 - i\partial_2)\theta_D^7 = 0\\
&(\partial_0 + m_0)Z^0 + (\partial_3 - m_3)Z^5 - (\partial_2 - m_2)Z^6 + (\partial_1 + m_1)Z^3 = J^0\\
&(\partial_0 - m_0)Z^4 + (\partial_3 + m_3)Z^1 - (\partial_2 + m_2)Z^2 + (\partial_1 - m_1)Z^7 = J^4\\
&(-\partial_0 + m)\theta_D^1 - i(\partial_1 + i\partial_2)\theta_D^2 \quad + i\partial_3\theta_D^3 = 0\\
&(\partial_0 + m)\theta_D^5 + i(\partial_1 + i\partial_2)\theta_D^6 \quad - i\partial_3\theta_D^7 = 0\\
&(\partial_0 + m_0)Z^1 + (\partial_1 - m_1)Z^6 + (\partial_2 + m_2)Z^3 - (\partial_3 - m_3)Z^4 = J^1\\
&(\partial_0 - m_0)Z^5 + (\partial_1 + m_1)Z^2 + (\partial_2 - m_2)Z^7 - (\partial_3 + m_3)Z^0 = J^5\\
&i\partial_3\theta_D^0 + i(\partial_1 - i\partial_2)\theta_D^1 + (\partial_0 + m)\theta_D^2 = 0\\
&-i\partial_3\theta_D^4 - i(\partial_1 - i\partial_2)\theta_D^5 + (-\partial_0 + m)\theta_D^6 = 0\\
&(\partial_3 + m_3)Z^3 - (\partial_1 - m_1)Z^5 + (\partial_0 + m_0)Z^2 + (\partial_2 - m_2)Z^4 = J^2\\
&(\partial_3 - m_3)Z^7 - (\partial_1 + m_1)Z^1 + (\partial_0 - m_0)Z^6 + (\partial_2 + m_2)Z^0 = J^6\\
&i(\partial_1 + i\partial_2)\theta_D^0 \quad - i\partial_3\theta_D^1 \quad + (\partial_0 + m)\theta_D^3 = 0\\
&-i(\partial_1 + i\partial_2)\theta_D^4 \quad + i\partial_3\theta_D^5 \quad + (-\partial_0 + m)\theta_D^7 = 0\\
&(\partial_1 - m_1)Z^0 + (\partial_3 - m_3)Z^2 + (\partial_2 - m_2)Z^1 - (\partial_0 - m_0)Z^3 = J^3\\
&(\partial_1 + m_1)Z^4 + (\partial_3 + m_3)Z^6 + (\partial_2 + m_2)Z^5 - (\partial_0 + m_0)Z^7 = J^7
\end{aligned}$$

So:

$$\begin{aligned}
&(\partial_0 - m)\theta_D^0 + i(\partial_1 - i\partial_2)\theta_D^3 \quad + i\partial_3\theta_D^2 = 0\\
&(\partial_0 + m)\theta_D^4 + i(\partial_1 - i\partial_2)\theta_D^7 \quad + i\partial_3\theta_D^6 = 0\\
&(\partial_0 + m_0)Z^0 + (\partial_1 + m_1)Z^3 - (\partial_2 - m_2)Z^6 + (\partial_3 - m_3)Z^5 = J^0\\
&(\partial_0 - m_0)Z^4 + (\partial_1 - m_1)Z^7 - (\partial_2 + m_2)Z^2 + (\partial_3 + m_3)Z^1 = J^4\\
&(\partial_0 - m)\theta_D^1 + i(\partial_1 + i\partial_2)\theta_D^2 \quad - i\partial_3\theta_D^3 = 0\\
&(\partial_0 + m)\theta_D^5 + i(\partial_1 + i\partial_2)\theta_D^6 \quad - i\partial_3\theta_D^7 = 0\\
&(\partial_0 - m_0)Z^5 + (\partial_1 + m_1)Z^2 + (\partial_2 - m_2)Z^7 - (\partial_3 + m_3)Z^0 = J^5\\
&(\partial_0 + m_0)Z^1 + (\partial_1 - m_1)Z^6 + (\partial_2 + m_2)Z^3 - (\partial_3 - m_3)Z^4 = J^1\\
&(\partial_0 + m)\theta_D^2 + i(\partial_1 - i\partial_2)\theta_D^1 \quad + i\partial_3\theta_D^0 = 0\\
&(\partial_0 - m)\theta_D^6 + i(\partial_1 - i\partial_2)\theta_D^5 \quad + i\partial_3\theta_D^4 = 0\\
&(\partial_0 + m_0)Z^2 - (\partial_1 - m_1)Z^5 + (\partial_2 - m_2)Z^4 + (\partial_3 + m_3)Z^3 = J^2\\
&(\partial_0 - m_0)Z^6 - (\partial_1 + m_1)Z^1 + (\partial_2 + m_2)Z^0 + (\partial_3 - m_3)Z^7 = J^6\\
&(\partial_0 + m)\theta_D^3 + i(\partial_1 + i\partial_2)\theta_D^0 \quad - i\partial_3\theta_D^1 = 0\\
&(\partial_0 - m)\theta_D^7 + i(\partial_1 + i\partial_2)\theta_D^4 \quad - i\partial_3\theta_D^5 = 0\\
&(\partial_0 + m_0)Z^7 - (\partial_1 + m_1)Z^4 - (\partial_2 + m_2)Z^5 - (\partial_3 + m_3)Z^6 = J^7\\
&(\partial_0 - m_0)Z^3 - (\partial_1 - m_1)Z^0 - (\partial_2 - m_2)Z^1 - (\partial_3 - m_3)Z^2 = J^3
\end{aligned}$$

Continuing the comparison with the Maxwell-Cassano equations in the special case:
$m_0 \to -m, m_1 = m_2 = m_3 = 0$:

$$(\partial_0 - m)\theta_D^0 + i(\partial_1 - i\partial_2)\theta_D^3 \quad + i\partial_3\theta_D^2 = 0$$

$$
\begin{aligned}
&(\partial_0 + m)\theta_D^4 + i(\partial_1 - i\partial_2)\theta_D^7 + i\partial_3\theta_D^6 = 0\\
&(\partial_0 - m)Z^0 + \partial_1 Z^3 - \partial_2 Z^6 + \partial_3 Z^5 = J^0\\
&(\partial_0 + m)Z^4 + \partial_1 Z^7 - \partial_2 Z^2 + \partial_3 Z^1 = J^4\\
&(\partial_0 - m)\theta_D^1 + i(\partial_1 + i\partial_2)\theta_D^2 - i\partial_3\theta_D^3 = 0\\
&(\partial_0 + m)\theta_D^5 + i(\partial_1 + i\partial_2)\theta_D^6 - i\partial_3\theta_D^7 = 0\\
&(\partial_0 - m)Z^1 + \partial_1 Z^6 + \partial_2 Z^3 - \partial_3 Z^4 = J^1\\
&(\partial_0 + m)Z^5 + \partial_1 Z^2 + \partial_2 Z^7 - \partial_3 Z^0 = J^5\\
&(\partial_0 + m)\theta_D^2 + i(\partial_1 - i\partial_2)\theta_D^1 + i\partial_3\theta_D^0 = 0\\
&(\partial_0 - m)\theta_D^6 + i(\partial_1 - i\partial_2)\theta_D^5 + i\partial_3\theta_D^4 = 0\\
&(\partial_0 - m)Z^2 - \partial_1 Z^5 + \partial_2 Z^4 + \partial_3 Z^3 = J^2\\
&(\partial_0 + m)Z^6 - \partial_1 Z^1 + \partial_2 Z^0 + \partial_3 Z^7 = J^6\\
&(\partial_0 + m)\theta_D^3 + i(\partial_1 + i\partial_2)\theta_D^0 - i\partial_3\theta_D^1 = 0\\
&(\partial_0 - m)\theta_D^7 + i(\partial_1 + i\partial_2)\theta_D^4 - i\partial_3\theta_D^5 = 0\\
&(\partial_0 - m)Z^7 - \partial_1 Z^4 - \partial_2 Z^5 - \partial_3 Z^6 = J^7\\
&(\partial_0 + m)Z^3 - \partial_1 Z^0 - \partial_2 Z^1 - \partial_3 Z^2 = J^3
\end{aligned}
$$

or:

$$
\begin{aligned}
&(\partial_0 - m)\theta_D^0 + i(\partial_1 - i\partial_2)\theta_D^3 + i\partial_3\theta_D^2 = 0\\
&(\partial_0 + m)\theta_D^4 + i(\partial_1 - i\partial_2)\theta_D^7 + i\partial_3\theta_D^6 = 0\\
&(\partial_0 - m)\theta_D^1 + i(\partial_1 + i\partial_2)\theta_D^2 - i\partial_3\theta_D^3 = 0\\
&(\partial_0 + m)\theta_D^5 + i(\partial_1 + i\partial_2)\theta_D^6 - i\partial_3\theta_D^7 = 0\\
&(\partial_0 + m)\theta_D^2 + i(\partial_1 - i\partial_2)\theta_D^1 + i\partial_3\theta_D^0 = 0\\
&(\partial_0 - m)\theta_D^6 + i(\partial_1 - i\partial_2)\theta_D^5 + i\partial_3\theta_D^4 = 0\\
&(\partial_0 + m)\theta_D^3 + i(\partial_1 + i\partial_2)\theta_D^0 - i\partial_3\theta_D^1 = 0\\
&(\partial_0 - m)\theta_D^7 + i(\partial_1 + i\partial_2)\theta_D^4 - i\partial_3\theta_D^5 = 0\\
&(\partial_0 - m)Z^0 + \partial_1 Z^3 - \partial_2 Z^6 + \partial_3 Z^5 = J^0\\
&(\partial_0 + m)Z^4 + \partial_1 Z^7 - \partial_2 Z^2 + \partial_3 Z^1 = J^4\\
&(\partial_0 - m)Z^1 + \partial_1 Z^6 + \partial_2 Z^3 - \partial_3 Z^4 = J^1\\
&(\partial_0 + m)Z^5 + \partial_1 Z^2 + \partial_2 Z^7 - \partial_3 Z^0 = J^5\\
&(\partial_0 + m)Z^6 - \partial_1 Z^1 + \partial_2 Z^0 + \partial_3 Z^7 = J^6\\
&(\partial_0 - m)Z^2 - \partial_1 Z^5 + \partial_2 Z^4 + \partial_3 Z^3 = J^2\\
&(\partial_0 + m)Z^3 - \partial_1 Z^0 - \partial_2 Z^1 - \partial_3 Z^2 = J^3\\
&(\partial_0 - m)Z^7 - \partial_1 Z^4 - \partial_2 Z^5 - \partial_3 Z^6 = J^7
\end{aligned}
$$

So, extending the Dirac equation beyond the source/sink free case (so looking beyond just eigenvalues and eigenvectors); and writing in matrix form, and comparing:

$$
\begin{pmatrix}
(\partial_0 - m) & 0 & 0 & 0 & i\partial_3 & 0 & i(\partial_1 - i\partial_2) & 0\\
0 & (\partial_0 + m) & 0 & 0 & 0 & i\partial_3 & 0 & i(\partial_1 - i\partial_2)\\
0 & 0 & (\partial_0 - m) & 0 & i(\partial_1 + i\partial_2) & 0 & -i\partial_3 & 0\\
0 & 0 & 0 & (\partial_0 + m) & 0 & i(\partial_1 + i\partial_2) & 0 & -i\partial_3\\
i\partial_3 & 0 & i(\partial_1 - i\partial_2) & 0 & (\partial_0 + m) & 0 & 0 & 0\\
0 & i\partial_3 & 0 & i(\partial_1 - i\partial_2) & 0 & (\partial_0 - m) & 0 & 0\\
i(\partial_1 + i\partial_2) & 0 & -i\partial_3 & 0 & 0 & 0 & (\partial_0 + m) & 0\\
0 & i(\partial_1 + i\partial_2) & 0 & -i\partial_3 & 0 & 0 & 0 & (\partial_0 - m)
\end{pmatrix}
\begin{pmatrix}
\theta_D^0\\ \theta_D^4\\ \theta_D^1\\ \theta_D^5\\ \theta_D^2\\ \theta_D^6\\ \theta_D^3\\ \theta_D^7
\end{pmatrix}
=
\begin{pmatrix}
\Phi^0\\ \Phi^4\\ \Phi^1\\ \Phi^5\\ \Phi^2\\ \Phi^6\\ \Phi^3\\ \Phi^7
\end{pmatrix}
$$

$$
\begin{aligned}
&(\partial_0 - m)Z^2 - \partial_1 Z^5 + \partial_2 Z^4 + \partial_3 Z^3 = J^2\\
&(\partial_0 + m)Z^6 - \partial_1 Z^1 + \partial_2 Z^0 + \partial_3 Z^7 = J^6\\
&(\partial_0 + m)Z^3 - \partial_1 Z^0 - \partial_2 Z^1 - \partial_3 Z^2 = J^3\\
&(\partial_0 - m)Z^7 - \partial_1 Z^4 - \partial_2 Z^5 - \partial_3 Z^6 = J^7
\end{aligned}
$$

$$
\begin{pmatrix}
(\partial_0 - m) & 0 & 0 & \partial_3 & 0 & -\partial_2 & \partial_1 & 0\\
0 & (\partial_0 + m) & \partial_3 & 0 & -\partial_2 & 0 & 0 & \partial_1\\
0 & -\partial_3 & (\partial_0 - m) & 0 & 0 & \partial_1 & \partial_2 & 0\\
-\partial_3 & 0 & 0 & (\partial_0 + m) & \partial_1 & 0 & 0 & \partial_2\\
0 & \partial_2 & 0 & -\partial_1 & (\partial_0 - m) & 0 & \partial_3 & 0\\
\partial_2 & 0 & -\partial_1 & 0 & 0 & (\partial_0 + m) & 0 & \partial_3\\
-\partial_1 & 0 & -\partial_2 & 0 & -\partial_3 & 0 & (\partial_0 + m) & 0\\
0 & -\partial_1 & 0 & -\partial_2 & 0 & -\partial_3 & 0 & (\partial_0 - m)
\end{pmatrix}
\begin{pmatrix}
Z^0\\ Z^4\\ Z^1\\ Z^5\\ Z^2\\ Z^6\\ Z^3\\ Z^7
\end{pmatrix}
=
\begin{pmatrix}
J^0\\ J^4\\ J^1\\ J^5\\ J^2\\ J^6\\ J^3\\ J^7
\end{pmatrix}
$$

The matrix is equivalent (have the same solution set) under the elementary row operation of interchanging rows, so interchanging rows 4 & 5:

$$
\begin{pmatrix}
(\partial_0-m) & 0 & 0 & \partial_3 & 0 & -\partial_2 & \partial_1 & 0 \\
0 & (\partial_0+m) & \partial_3 & 0 & -\partial_2 & 0 & 0 & \partial_1 \\
0 & -\partial_3 & (\partial_0-m) & 0 & 0 & \partial_1 & \partial_2 & 0 \\
-\partial_3 & 0 & 0 & (\partial_0+m) & \partial_1 & 0 & 0 & \partial_2 \\
\partial_2 & 0 & -\partial_1 & 0 & 0 & (\partial_0+m) & 0 & \partial_3 \\
0 & \partial_2 & 0 & -\partial_1 & (\partial_0-m) & 0 & \partial_3 & 0 \\
-\partial_1 & 0 & -\partial_2 & 0 & -\partial_3 & 0 & (\partial_0+m) & 0 \\
0 & -\partial_1 & 0 & -\partial_2 & 0 & -\partial_3 & 0 & (\partial_0-m)
\end{pmatrix}
\begin{pmatrix} Z^0 \\ Z^4 \\ Z^1 \\ Z^5 \\ Z^2 \\ Z^6 \\ Z^3 \\ Z^7 \end{pmatrix}
=
\begin{pmatrix} J^0 \\ J^4 \\ J^1 \\ J^5 \\ J^2 \\ J^6 \\ J^3 \\ J^7 \end{pmatrix}
$$

To retain equality under the elementary row operation of interchanging columns 4 & 5 , the rows of vectors $\mathbf{Z}$ & $\mathbf{J}$ are interchanged:

$$
\begin{pmatrix}
(\partial_0-m) & 0 & 0 & \partial_3 & -\partial_2 & 0 & \partial_1 & 0 \\
0 & (\partial_0+m) & \partial_3 & 0 & 0 & -\partial_2 & 0 & \partial_1 \\
0 & -\partial_3 & (\partial_0-m) & 0 & \partial_1 & 0 & \partial_2 & 0 \\
-\partial_3 & 0 & 0 & (\partial_0+m) & 0 & \partial_1 & 0 & \partial_2 \\
\partial_2 & 0 & -\partial_1 & 0 & (\partial_0+m) & 0 & 0 & \partial_3 \\
0 & \partial_2 & 0 & -\partial_1 & 0 & (\partial_0-m) & \partial_3 & 0 \\
-\partial_1 & 0 & -\partial_2 & 0 & 0 & -\partial_3 & (\partial_0+m) & 0 \\
0 & -\partial_1 & 0 & -\partial_2 & -\partial_3 & 0 & 0 & (\partial_0-m)
\end{pmatrix}
\begin{pmatrix} Z^0 \\ Z^4 \\ Z^1 \\ Z^5 \\ Z^6 \\ Z^2 \\ Z^3 \\ Z^7 \end{pmatrix}
=
\begin{pmatrix} J^0 \\ J^4 \\ J^1 \\ J^5 \\ J^6 \\ J^2 \\ J^3 \\ J^7 \end{pmatrix}
$$

Now, consider each matrix as a sum:

$$
\begin{pmatrix}
(\partial_0-m) & 0 & 0 & 0 & i\partial_3 & 0 & i(\partial_1-i\partial_2) & 0 \\
0 & (\partial_0+m) & 0 & 0 & 0 & i\partial_3 & 0 & i(\partial_1-i\partial_2) \\
0 & 0 & (\partial_0-m) & 0 & i(\partial_1+i\partial_2) & 0 & -i\partial_3 & 0 \\
0 & 0 & 0 & (\partial_0+m) & 0 & i(\partial_1+i\partial_2) & 0 & -i\partial_3 \\
i\partial_3 & 0 & i(\partial_1-i\partial_2) & 0 & (\partial_0+m) & 0 & 0 & 0 \\
0 & i\partial_3 & 0 & i(\partial_1-i\partial_2) & 0 & (\partial_0-m) & 0 & 0 \\
i(\partial_1+i\partial_2) & 0 & -i\partial_3 & 0 & 0 & 0 & (\partial_0+m) & 0 \\
0 & i(\partial_1+i\partial_2) & 0 & -i\partial_3 & 0 & 0 & 0 & (\partial_0-m)
\end{pmatrix}
=
$$

$$
=
\begin{pmatrix}
(\partial_0-m) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & (\partial_0+m) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & (\partial_0-m) & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & (\partial_0+m) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & (\partial_0+m) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & (\partial_0-m) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & (\partial_0+m) & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & (\partial_0-m)
\end{pmatrix}
+
$$

$$
+
\begin{pmatrix}
0 & 0 & 0 & 0 & i\partial_3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & i\partial_3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -i\partial_3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -i\partial_3 \\
i\partial_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & i\partial_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -i\partial_3 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -i\partial_3 & 0 & 0 & 0 & 0
\end{pmatrix}
+
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & i\partial_1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & i\partial_1 \\
0 & 0 & 0 & 0 & i\partial_1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & i\partial_1 & 0 & 0 \\
0 & 0 & i\partial_1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & i\partial_1 & 0 & 0 & 0 & 0 \\
i\partial_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & i\partial_1 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
+
$$

$$
+
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & \partial_2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \partial_2 \\
0 & 0 & 0 & 0 & -\partial_2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\partial_2 & 0 & 0 \\
0 & 0 & \partial_2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \partial_2 & 0 & 0 & 0 & 0 \\
-\partial_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\partial_2 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

and:

$$
\begin{pmatrix}
(\partial_0 - m) & 0 & 0 & \partial_3 & -\partial_2 & 0 & \partial_1 & 0 \\
0 & (\partial_0 + m) & \partial_3 & 0 & 0 & -\partial_2 & 0 & \partial_1 \\
0 & -\partial_3 & (\partial_0 - m) & 0 & \partial_1 & 0 & \partial_2 & 0 \\
-\partial_3 & 0 & 0 & (\partial_0 + m) & 0 & \partial_1 & 0 & \partial_2 \\
\partial_2 & 0 & -\partial_1 & 0 & (\partial_0 + m) & 0 & 0 & \partial_3 \\
0 & \partial_2 & 0 & -\partial_1 & 0 & (\partial_0 - m) & \partial_3 & 0 \\
-\partial_1 & 0 & -\partial_2 & 0 & 0 & -\partial_3 & (\partial_0 + m) & 0 \\
0 & -\partial_1 & 0 & -\partial_2 & -\partial_3 & 0 & 0 & (\partial_0 - m)
\end{pmatrix} =
$$

$$
= \begin{pmatrix}
(\partial_0 - m) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & (\partial_0 + m) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & (\partial_0 - m) & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & (\partial_0 + m) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & (\partial_0 + m) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & (\partial_0 - m) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & (\partial_0 + m) & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & (\partial_0 - m)
\end{pmatrix} +
$$

$$
+ \begin{pmatrix}
0 & 0 & 0 & 0 & -\partial_2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\partial_2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \partial_2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \partial_2 \\
\partial_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \partial_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\partial_2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\partial_2 & 0 & 0 & 0 & 0
\end{pmatrix} + \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & \partial_1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \partial_1 \\
0 & 0 & 0 & 0 & \partial_1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \partial_1 & 0 & 0 \\
0 & 0 & -\partial_1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\partial_1 & 0 & 0 & 0 & 0 \\
-\partial_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\partial_1 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} +
$$

$$
+ \begin{pmatrix}
0 & 0 & 0 & \partial_3 & 0 & 0 & 0 & 0 \\
0 & 0 & \partial_3 & 0 & 0 & 0 & 0 & 0 \\
0 & -\partial_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\partial_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \partial_3 \\
0 & 0 & 0 & 0 & 0 & 0 & \partial_3 & 0 \\
0 & 0 & 0 & 0 & 0 & -\partial_3 & 0 & 0 \\
0 & 0 & 0 & 0 & -\partial_3 & 0 & 0 & 0
\end{pmatrix}
$$

In this form the transformations are easy to see:

$$
\begin{pmatrix}
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0
\end{pmatrix} i\overline{\partial_3} \Leftrightarrow \begin{pmatrix}
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0
\end{pmatrix} \partial_2
$$

$$
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} i\overline{\partial_1} \Leftrightarrow \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} \partial_1
$$

$$\begin{pmatrix} 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&-1&0&0&0\\ 0&0&0&0&0&-1&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ -1&0&0&0&0&0&0&0\\ 0&-1&0&0&0&0&0&0 \end{pmatrix}\overline{\partial_2} \Leftrightarrow \begin{pmatrix} 0&0&0&1&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&-1&0&0&0&0&0&0\\ -1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&-1&0&0\\ 0&0&0&0&-1&0&0&0 \end{pmatrix}\partial_3$$

By the invertible matrix theorem each matrix is invertible. Thus, these transformations are oneto, bijective.
From the first matrices on each side of the sum, the rest of the transformations are even more easily seen. The full set of transformations follow.

$(\partial_0 \pm m) \Leftrightarrow (\overline{\partial_0} \pm m)$
$\begin{pmatrix} 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&-1&0&0&0&0&0\\ 0&0&0&-1&0&0&0&0\\ -1&0&0&0&0&0&0&0\\ 0&-1&0&0&0&0&0&0 \end{pmatrix}\partial_1 \Leftrightarrow \begin{pmatrix} 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0 \end{pmatrix}i\overline{\partial_1}$
$\begin{pmatrix} 0&0&0&0&-1&0&0&0\\ 0&0&0&0&0&-1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1\\ 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&-1&0&0&0&0&0\\ 0&0&0&-1&0&0&0&0 \end{pmatrix}\partial_2 \Leftrightarrow \begin{pmatrix} 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&-1&0\\ 0&0&0&0&0&0&0&-1\\ 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&-1&0&0&0&0&0\\ 0&0&0&-1&0&0&0&0 \end{pmatrix}i\overline{\partial_3}$
$\begin{pmatrix} 0&0&0&1&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&-1&0&0&0&0&0&0\\ -1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&-1&0&0\\ 0&0&0&0&-1&0&0&0 \end{pmatrix}\partial_3 \Leftrightarrow \begin{pmatrix} 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&-1&0&0&0\\ 0&0&0&0&0&-1&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ -1&0&0&0&0&0&0&0\\ 0&-1&0&0&0&0&0&0 \end{pmatrix}\overline{\partial_2}$
[Dirac (barred)]

Let:

$$\boldsymbol{\sigma}_2^0 = \begin{pmatrix} 1&0\\ 0&1 \end{pmatrix} \quad , \quad \boldsymbol{\sigma}_2^1 = \begin{pmatrix} 0&1\\ 1&0 \end{pmatrix} \quad , \quad \mathbf{0}_2 = \begin{pmatrix} 0&0\\ 0&0 \end{pmatrix}$$

then, more compactly:

$(\partial_0 \pm m) \Leftrightarrow (\overline{\partial_0} \pm m) \qquad \Rightarrow x^0 = \bar{x}^0$
$\begin{pmatrix} \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 & \boldsymbol{\sigma}_2^0 \\ \mathbf{0}_2 & \mathbf{0}_2 & \boldsymbol{\sigma}_2^0 & 0 \\ \mathbf{0}_2 & -\boldsymbol{\sigma}_2^0 & \mathbf{0}_2 & \mathbf{0}_2 \\ -\boldsymbol{\sigma}_2^0 & \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 \end{pmatrix} \partial_1 \Leftrightarrow \begin{pmatrix} \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 & \boldsymbol{\sigma}_2^0 \\ \mathbf{0}_2 & \mathbf{0}_2 & \boldsymbol{\sigma}_2^0 & 0 \\ \mathbf{0}_2 & \boldsymbol{\sigma}_2^0 & \mathbf{0}_2 & \mathbf{0}_2 \\ \boldsymbol{\sigma}_2^0 & \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 \end{pmatrix} i\overline{\partial_1} \Rightarrow x^1 = -i \begin{pmatrix} \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 & \boldsymbol{\sigma}_2^0 \\ \mathbf{0}_2 & \mathbf{0}_2 & \boldsymbol{\sigma}_2^0 & 0 \\ \mathbf{0}_2 & \boldsymbol{\sigma}_2^0 & \mathbf{0}_2 & \mathbf{0}_2 \\ \boldsymbol{\sigma}_2^0 & \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 \end{pmatrix}^{-1} \begin{pmatrix} \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 & \boldsymbol{\sigma}_2^0 \\ \mathbf{0}_2 & \mathbf{0}_2 & \boldsymbol{\sigma}_2^0 & 0 \\ \mathbf{0}_2 & -\boldsymbol{\sigma}_2^0 & \mathbf{0}_2 & \mathbf{0}_2 \\ -\boldsymbol{\sigma}_2^0 & \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 \end{pmatrix} \bar{x}^1$
$\begin{pmatrix} \mathbf{0}_2 & \mathbf{0}_2 & -\boldsymbol{\sigma}_2^0 & \mathbf{0}_2 \\ \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 & \boldsymbol{\sigma}_2^0 \\ \boldsymbol{\sigma}_2^0 & \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 \\ \mathbf{0}_2 & -\boldsymbol{\sigma}_2^0 & \mathbf{0}_2 & \mathbf{0}_2 \end{pmatrix} \partial_2 \Leftrightarrow \begin{pmatrix} \mathbf{0}_2 & \mathbf{0}_2 & \boldsymbol{\sigma}_2^0 & \mathbf{0}_2 \\ \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 & -\boldsymbol{\sigma}_2^0 \\ \boldsymbol{\sigma}_2^0 & \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 \\ \mathbf{0}_2 & -\boldsymbol{\sigma}_2^0 & \mathbf{0}_2 & \mathbf{0}_2 \end{pmatrix} i\overline{\partial_3} \Rightarrow x^2 = -i \begin{pmatrix} \mathbf{0}_2 & \mathbf{0}_2 & \boldsymbol{\sigma}_2^0 & \mathbf{0}_2 \\ \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 & -\boldsymbol{\sigma}_2^0 \\ \boldsymbol{\sigma}_2^0 & \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 \\ \mathbf{0}_2 & -\boldsymbol{\sigma}_2^0 & \mathbf{0}_2 & \mathbf{0}_2 \end{pmatrix}^{-1} \begin{pmatrix} \mathbf{0}_2 & \mathbf{0}_2 & -\boldsymbol{\sigma}_2^0 & \mathbf{0}_2 \\ \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 & \boldsymbol{\sigma}_2^0 \\ \boldsymbol{\sigma}_2^0 & \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 \\ \mathbf{0}_2 & -\boldsymbol{\sigma}_2^0 & \mathbf{0}_2 & \mathbf{0}_2 \end{pmatrix} x^3$
$\begin{pmatrix} \mathbf{0}_2 & \boldsymbol{\sigma}_2^1 & \mathbf{0}_2 & \mathbf{0}_2 \\ -\boldsymbol{\sigma}_2^1 & \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 \\ \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 & \boldsymbol{\sigma}_2^1 \\ \mathbf{0}_2 & \mathbf{0}_2 & -\boldsymbol{\sigma}_2^1 & \mathbf{0}_2 \end{pmatrix} \partial_3 \Leftrightarrow \begin{pmatrix} \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 & \boldsymbol{\sigma}_2^0 \\ \mathbf{0}_2 & \mathbf{0}_2 & -\boldsymbol{\sigma}_2^0 & \mathbf{0}_2 \\ \mathbf{0}_2 & \boldsymbol{\sigma}_2^0 & \mathbf{0}_2 & \mathbf{0}_2 \\ -\boldsymbol{\sigma}_2^0 & \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 \end{pmatrix} \overline{\partial_2} \Rightarrow x^2 = \begin{pmatrix} \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 & \boldsymbol{\sigma}_2^0 \\ \mathbf{0}_2 & \mathbf{0}_2 & -\boldsymbol{\sigma}_2^0 & \mathbf{0}_2 \\ \mathbf{0}_2 & \boldsymbol{\sigma}_2^0 & \mathbf{0}_2 & \mathbf{0}_2 \\ -\boldsymbol{\sigma}_2^0 & \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 \end{pmatrix}^{-1} \begin{pmatrix} \mathbf{0}_2 & \boldsymbol{\sigma}_2^1 & \mathbf{0}_2 & \mathbf{0}_2 \\ -\boldsymbol{\sigma}_2^1 & \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 \\ \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 & \boldsymbol{\sigma}_2^1 \\ \mathbf{0}_2 & \mathbf{0}_2 & -\boldsymbol{\sigma}_2^1 & \mathbf{0}_2 \end{pmatrix} \bar{x}^3$
[Dirac (barred)]

$\begin{pmatrix} \theta_D^0 \\ \theta_D^4 \\ \theta_D^1 \\ \theta_D^5 \\ \theta_D^2 \\ \theta_D^6 \\ \theta_D^3 \\ \theta_D^7 \end{pmatrix} = \begin{pmatrix} Z^0 \\ Z^4 \\ Z^1 \\ Z^5 \\ Z^6 \\ Z^2 \\ Z^3 \\ Z^7 \end{pmatrix}$	$\begin{pmatrix} \Phi^0 \\ \Phi^4 \\ \Phi^1 \\ \Phi^5 \\ \Phi^2 \\ \Phi^6 \\ \Phi^3 \\ \Phi^7 \end{pmatrix} = \begin{pmatrix} J^0 \\ J^4 \\ J^1 \\ J^5 \\ J^6 \\ J^2 \\ J^3 \\ J^7 \end{pmatrix}$

This proves that the mass-generalized Maxwell's equations (Maxwell-Cassano equations) is a more general analysis of fundamental-elementary particle phenomena.
It further proves that the Lagrangian is far simpler than that consisting of the Glashow-Salam-Weinberg + fermion + Higgs + Yukawa kludge.

Also, it explains the group structure and architecture of the fermions [2].

It also proves that those with wealth to seek the truth choose not to do so, but with all deceivableness and unrighteousness in them they have not the love of the truth, but rather embrace strong delusion, that they profess a lie.

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and further readings

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