Time as the dynamic aspect of the continuum

By James Arnold

Abstract

The Minkowski diagram, by which the concept of spacetime has been graphically represented and interpreted, is shown to have a pre-relativistic flaw: It depicts the relative motion of a body moving in time as-if it is moving *along with* the observer's clock, not as it is actually observed, according to its own proper time. An alternative diagram provides an accurate representation of relativistic relationships and enables heuristic insights into the nature of relativistic effects, and of time, light, and gravitation.

Introduction

The concept of four-dimensional spacetime pre-dates relativity theory, but with relativity there is a revolutionary understanding of the covariance of space and time. The prevailing understanding of the relativistic, covariant, spacetime continuum is due primarily to H. Minkowski's geometrical contribution (1908) to Einstein's Special Theory. Minkowski saw in the relativistic interpretation of the Lorentz transformations[1] the possibility for a two-dimensional representation of the peculiar interrelationships between bodies having large relative velocities. Assuming the validity of the tenets of Relativity, and assuming the correctness of the geometric representation, it was expected that the graphic would express and corroborate relativity in its mathematical form, that "physical laws might find their most perfect expression" (Minkowski, 1908, 76), and facilitate further insights into the nature of what Minkowski called the *spacetime continuum*.

Diagramming spacetime

Figure 1 is a typical Minkowski diagram, with the vertical or nearly vertical axes of two reference frames representing time and the horizontal or nearly horizontal axes representing their spatial dimensions. A defining characteristic of the diagram is a diagonal vector or vectors projecting the motion of light relative to the motion of an observer who is moving in time on the vertical axis while "at rest" in space.

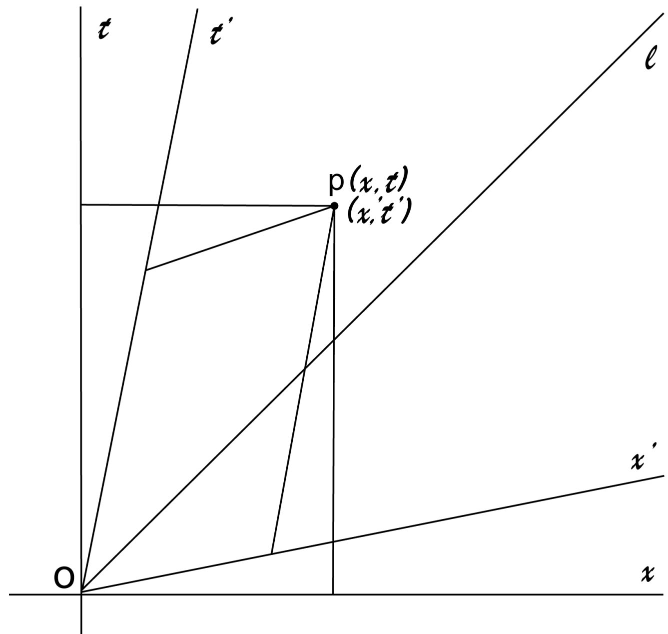

figure 1

A typical Minkowski diagram. The world-line of the observer is moving in time perpendicular to the space axis. The world-line of a ray of light is depicted as a diagonal in the observer's coordinate system, moving in equal parts of space and time. Point *p* illustrates how an event can be located differently in different coordinate systems according to their relative motion.

The world-lines in the Minkowski diagram are supposed to project the relativistic motions of bodies in spacetime. The observer's world-line moves vertically, because she is considered to be at rest in space, rather than in uniform motion – a seemingly arbitrary choice, but the clearest and simplest way of depicting relativistic relationships. The world-lines of bodies moving at great speeds relative to the observer are usually projected as moving away from the observer's initial location, again because it is the clearest way of representing relativistic relationships.

The primary interests in the diagram have generally focused on relative location of events in spacetime, as with point *p* in *figure 1*, and on the "light-cones" formed by the relative motion of light-rays. But with attention to world-lines, there can be seen a remarkable, pre-relativistic flaw in the Minkowskian projection: The world-line of a body moving relative to the observer is shown to be moving in space as observed, but moving in time as-if it is moving in synchronicity with the observer's clock, not dilated (slowed) according to theory. In other words, it isn't projected as it should be, *compared to* the observer's clock, but rather *along with* the observer's clock.

There is no more important precept in relativity theory than that in describing the relative motion of a body we must specify the frame of reference from which the observation is being made, and we must distinguish the observed body's metrics of space and time from the observer's. If for example it is said that a body is observed to travel 8 light-seconds (*ls*) in 10 seconds (*sec*), unless we are to return to the absolutes of classical physics we must specify and distinguish according to whose measures of distance and time. To fully describe the observed motion relativistically is to report that the body travels 8 *ls* in space relative to an observer's uniform or stationary spatial reference, and a number of *sec* in time (t') that is relative to the observer's corresponding temporal reference of 10 *sec* (t), at a relative velocity of v (expressed as .8, proportional to c) [2]. The relationship can be expressed by $t' = t\sqrt{(1-v^2)}$ per a Lorentz transformation, or alternately by $t' = \sqrt{(t^2-x^2)}$, with x as the relative distance traveled in space, calibrated in light-seconds, which in the above example yields either $10(\sqrt{(1-.8^2)})$ or $\sqrt{(10^2-8^2)} = 6$. Strictly speaking, therefore, the body under consideration travels a relative 8 *ls* and 6 *sec* (its clock is observed to tick 6 seconds), and we measure its travel from a reference frame at 0 *ls* and 10 *sec*.

The significance of the distinction between the pre-relativistic and relativistic accounts is most striking in the description of light: When it is said in a Newtonian perspective that light travels (approximately) 300,000 km (1 *ls*) in 1 *sec*, we make a relativistic correction and say, as in the above example, it actually travels 10 *ls* relative to the observer's uniform motion or state of rest in space and *0 sec* ($t' = 10(\sqrt{(1-0^2)}$ or $\sqrt{(10^2-10^2)})$) relative to the observer's duration of 10 *sec* in time.

To neglect the relativistic correction is to invite a serious error in one's understanding of spacetime, and yet it is an oversight built-in to the Minkowski diagram. The diagonal light-vectors (the "light-cones") in the diagram makes this conclusion unavoidable: To project the world-line of light as moving 10 *ls* in space and 10 *sec* in time (a spacetime diagonal, one side of a "light cone" in the Minkowski diagram) is to describe the relative motion of light in space, but in terms of the observer's own reference in time, treated as an independent, absolute measure. This is a critical misrepresentation of a most fundamental precept of relativity theory -- that time is relative and referential.

The Minkowski diagram has prevailed so long, the point cannot be overstressed: It is the clock of the moving body that indicates how it is moving in time, not the clock of the observer. Hence the two-dimensional representation of the world-line of the observed body must express its own clock as measured by the observer, as well as its relative distance traveled.

An alternative diagram (*figure 2*) conforming to Special Relativity and the Lorentz transformations, and treating both space and time as relative, provides a heuristic representation by means of which, as Minkowski originally envisioned, "physical laws might find their most perfect expression."

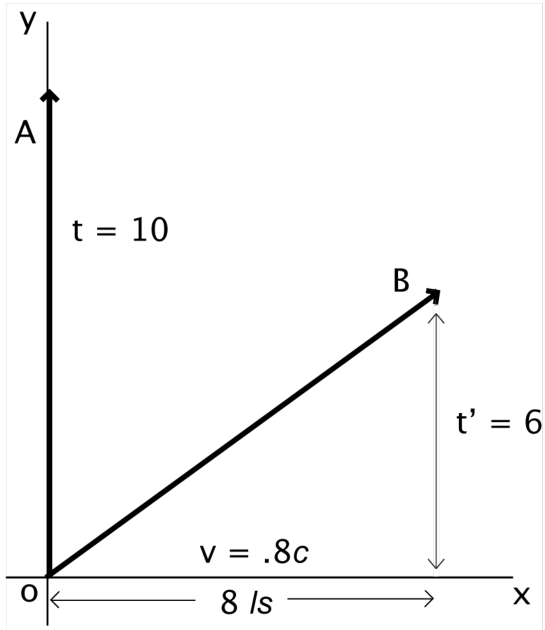

figure 2

An alternative spacetime diagram. The world-line of the observed body **B** is projected as moving in time according to its own clock as measured by the observer **A**.

The *x*-axis in *figure 2* represents space calibrated in light-seconds, while its perpendicular, the *y*-axis, represents time calibrated in seconds -- both according to observer ***A***, who is considered to be at rest and moving in time along the *y*-axis. Vector ***B*** represents a body in motion relative to ***A***.

A travels 10 *sec* in time in the scope of the diagram while "at rest" (i.e., moving perpendicular) to space in time. Body ***B***, which as a matter of convenience is located initially with ***A*** at the origin ***o***, moves from the vicinity of ***A*** at a velocity, according to ***A***, which takes it 8 *ls* in 10 *sec*. The final spacetime coordinates of ***B*** according to ***A*** (8,6) can be derived from the Lorentz transformations, or geometrically by measurement of the lengths in the diagram. By locating ***B*** at 6 seconds in time it is represented that the clock of ***B*** has moved 6 *sec* in the coordinate system of ***A***. (Note that at a velocity of .8*c* the world-line of ***B*** has already transgressed the diagonal of the Minkowskian light-cone.)

The Utility of a relativistic diagram

The so-called “invariant interval” -- a magnitude ascribed to changes in **B’**s location between two events from any coordinate system -- usually given by $s = \sqrt{(x^2 - t^2)}$ (again, with x proportional to c and calibrated with t), yields a negative square root for relative velocities less than c -- an imaginary number. But the relationship can be just as well transformed to

$$s = \sqrt{(t^2 - x^2)} \tag{1}$$

and expressed in the example by

$$s = \sqrt{(10^2 - 8^2)} = 6 \tag{2}$$

Thereby s, the *interval* is revealed in the relativistic spacetime diagram not as an abstract imaginary, graphically inexpressible, but as a physical quantity, the *proper time*, the observed speed of the clock of body ***B***.

A significant implication profiled by *figure 2* is that there are actually *two* invariants involved in a relativistic relationship: 1) the conventionally recognized interval, reinterpreted here as the proper time of ***B*** between two events along its world-line, which is invariant when measured from any reference frame, and 2) the equality of spacetime intervals of the world-lines of ***A*** and ***B***. In the Minkowski diagram the world-line of an observer is not recognized as being equivalent in length to the world-line of a body being observed; the latter is treated as a function of the observer’s time and the observer’s measure of distance traveled by the observed, so a ray of light would supposedly terminate at coordinates (10,10) after 10 seconds according to the observer, giving the light a world-line 14.14 in length. But in the relationship shown in *figure 2* between an observer and a body in relative motion (now, for the sake of comprehension, substituting s (proper time) for t', letting t and t' represent the lengths of the world-lines of ***A*** and ***B***, and setting $x = v$ so that $t' = \sqrt{(s^2 + x^2)}$), the spacetime interval of the observer (t) can be shown as necessarily equivalent to any observed world-line:

From the equation (1) for the invariant interval, which was reformulated as $s = \sqrt{(t^2 - x^2)}$ we can derive

$$t = \sqrt{(s^2 + x^2)} \tag{3}$$

which equates *t* with the hypotenuse of the triangle formed of *s* and *x*, and therefore also equal to *t'*, which *is* the hypotenuse. The world-line of ***B*** is therefore necessarily equal in length with the world-line of ***A***. We can extrapolate and declare that *all* world-lines (assuming inertial reference frames[3]) must be equal in length with all others for any given period, regardless of coordinate system.

It is important to note that both the Lorentz Transformations and the equation for the invariant interval indicate a Euclidean relationship between space and time, and between bodies in relative motion. For although the relationship between clocks in relative motion given by $t' = \sqrt{(t^2 - x^2)}$ is indeed parabolic, as is generally stressed in connection with the Minkowski diagram, the fact that a hypotenuse relates to the sides of a Euclidean triangle by a parabolic function presupposes the right-angle. And as *figure 2* shows, the temporal component of any body's relative motion in spacetime is at a right-angle to the observer's space axis, and parallel with the observer's own motion in time.

The spacetime diagram works to represent the relationship determined by the Lorentz transformation only if a body moving uniformly in time is actually moving perpendicular to space. Given that another body in relative motion is also moving perpendicular to space along the time-axis of its own coordinate system, its space axis must be different than that of the body taken to be at rest. Accordingly, *figure 3* shows two reference frames at once, with ***A*** and ***B*** each moving in time perpendicular to space according to their own coordinate system. It depicts, as the Minkowski diagram cannot, the curious phenomenon wherein each observer measures the other's clock as moving more slowly than her own.

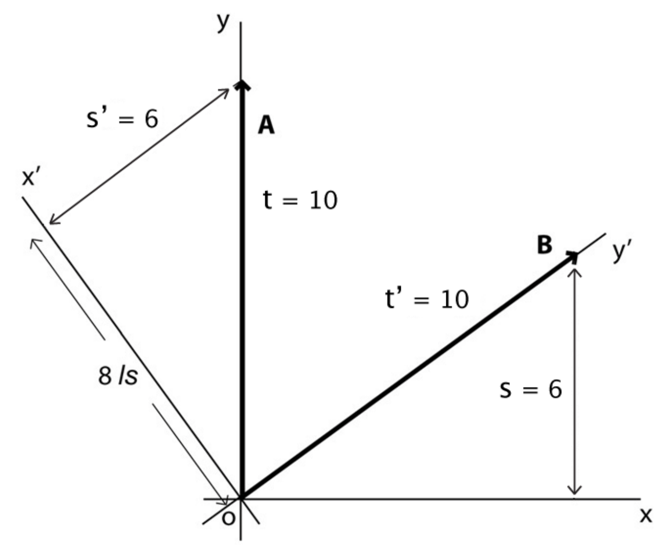

figure 3

Two coordinate systems are shown to mirror their mutual relativistic effects. By rotating the diagram either reference frame can be represented as at rest in space, and the other projected as being in relative motion.

Figure 3 is a fully accurate depiction of the relativistic relationship. It expresses the duality that students of relativity often have difficulty comprehending: Each body has its own orientation in spacetime, each moves in time perpendicular to space, and each mirrors the relativistic effects of the other.

Both the Lorentz Transformations and the (modified) equation for the invariant interval indicate a perpendicular relationship between space and time for any body (except light) at-rest or moving uniformly. The motion in time of a body **A** is perpendicular to its orientation in space, and the relative motion of a body **B**, although proceeding in a relative orientation that is partly temporal, partly spatial according to **A**, is moving uniformly in time perpendicular to space in its own coordinate system.

The relative motion of light as represented in these terms is especially noteworthy. Whereas the speed of light is commonly expressed as ~300,000 km per second, to fully describe its observed motion relativistically is to report that it travels 1 *ls* in space relative to an observer's spatial reference, and *zero* seconds in time relative to the observer's temporal reference of 1 *sec*, as is given both by the Lorentz transformations and the equation for the invariant interval. A world-line representing a ray of light in *figure 4* therefore has a spacetime interval of 10 but a proper time of zero, and lies directly along the x-axis of observer ***A***. (The interval in this case is $s = \sqrt{(10^2 - 10^2)}$.)

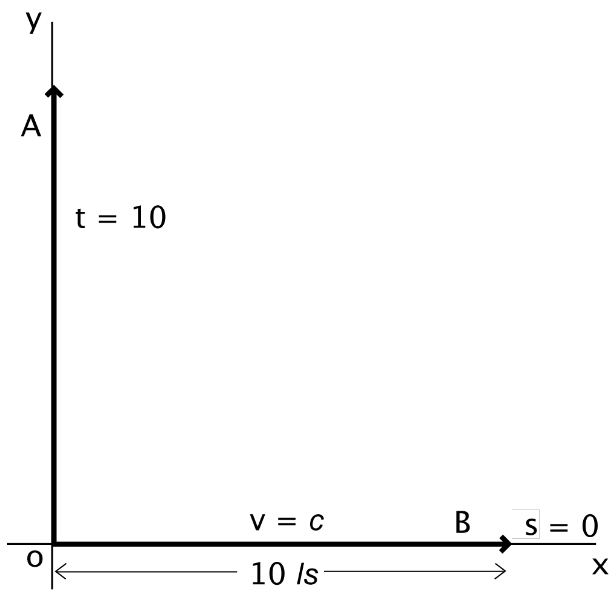

figure 4

The world-line of a ray of light is projected as moving along the observer's space-axis, conforming to its absence of clock speed according to the observer.

The world-line of light doesn't move in time relative to the observer, therefore it is a misrepresentation to place in on a diagonal, which in effect treats the observer's clock as absolute.

The relativistic representation at once provides a visual explanation for *c* as a limiting velocity (given the invariance of world-lines described above, a vector along the *x*-axis will have the maximum possible relative extension in space, equal to the observer's extension in time) and an explanation for the invariant measure of *c* (again by the invariance of world-lines, an observer will always measure light as moving as far along the space axis as she moves along the time axis). These aspects of light have remained inexplicable by adherence to the Minkowski diagram, and have prevented the realization of Minkowski's original vision -- that "physical laws might find their most perfect expression" in a two-dimensional spacetime projection.

The relationships projected in the relativistic diagram can be expressed in terms of three corollaries thus far:

1. The speed of light is a limit. If the world-lines of bodies in relative motion are taken as having the same spacetime interval but with varying spatial and temporal components according to their relative spacetime trajectories, the limiting spatial velocity is the interval of a world-line along the space axis measured in terms of the same interval along the corresponding time axis. (A vector drawn along the x-axis in *figure 4* to represent a ray of light extends as far along the *x*-axis as time elapses for the observer in the duration of the diagram. There is no vector that can extend further (i.e., move faster) in space than one that has a temporal component of zero.)

2. The speed of light is invariant. Due to the equivalence of the observer's and the observed world-lines, each observer will measure light as traveling the same distance in space as time elapses in that observer's reference frame, and though the measure of the spatial distance traveled by a beam of light between events will vary between reference frames, the rate will always be agreed upon.

3. The speed of light and the speed of time are equivalent. Given the equivalence of world-lines, given the perpendicular relationship between space and time expressed by the Lorentz transformations, and given the world-line of light as lying along the *x*-axis, distance in time must be equal to distance in space: one second in time is the same distance, but in a perpendicular direction, as ~300,000 km in space.

Time as motion in space

As the relativistic diagram shows, and as the Lorentz transformations express, each body has its own clock and moves in time with its own orientation in space; to move in time in one coordinate system is to move partly in time, partly in space according to another; the time of a body considered to be in motion relative to another moves across the space of the other.

It is difficult, perhaps impossible to conceive how our everyday experience could involve "moving", space-wise, in time -- especially when our experience is of "maintaining" or "enduring" (rather than "moving") more-or-less parallel in time with everything we commonly observe. But the difficulty is to be expected, considering that temporal motion is in a fourth dimension, and we are creatures attuned to a three-dimensional experience distinct from our sense of duration. It is even more incomprehensible that to move in time is to move 300,000 km across space -- and in just one second. We don't seem to be moving at any great speed even relative to the stars; how could the whole universe of mass be moving at *c* in time although even on a galactic scale we seem to be moving as-if in a common-sense, three-dimensional order. It is a question ripe for speculation, but the phenomenon itself has immediate, informative implications.

Time and Momentum

The notion of a spacetime continuum, and of time as moving in space, indicates a dynamic aspect of time than is not fully appreciated even in relativity theory, due in part perhaps to a residue of the pre-relativistic and common-sense regard for space and time as being independent and fundamentally different. But if spacetime is a continuum, if time moves in space, or rather, if a body moves in space by moving in time, then temporal motion must be dynamic, and possessing of momentum and kinetic energy.

The component of relative time in momentum is obscured even in the relativistic formulation $p = m_0 v / \sqrt{(1-v^2/c^2)}$.

If for the sake of isolating considerations of space and time in momentum we set m_0 to unity, and for simplicity, as before, set *v* proportional to *c*, we have

$$p_{st} = v / \sqrt{(1-v^2)} \qquad (4)$$

Projecting this on a spacetime diagram in *figure 5* with body **B** impinging on **A**, we set *t* to unity, $x = vt$, and $t' = \sqrt{(1-x^2)}$ and thereby arrive at

$$p_{st} = x/t' \qquad (5)$$

(Note: Having established that *s*, the "invariant interval" is just the proper time of an observed body, and given the invariance of the length of world-lines, we can henceforth dispense with *s*, and return to using *t'* as the relative time of the observed, as in *figure 2*.)

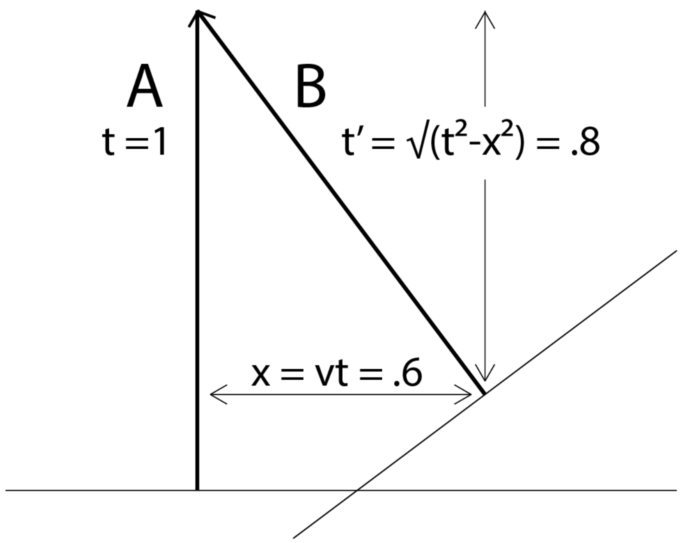

figure 5

> In an event that transpires at 1 second on **A**'s clock, body **B** impinges on **A** at a velocity of *.6c*, with momentum of $m_0x/(\sqrt{t^2-x^2})$, or m_0x/t', or m_0vt/t', which express the temporal dynamic of **B**'s motion in **A**'s coordinate system. (Note that the diagram could be rotated to treat **A** as impinging on **B** with the same momentum, given equal masses.)

In a more general expression of t', with t allowed to vary,

$$t' = \sqrt{(t^2-(vt)^2)} \tag{10}$$

$$t' = t\sqrt{(1-v^2)} \tag{11}$$

(which identifies the geometric source and significance of the Lorentz term $1-v^2$).

Now allowing m_0 to vary, and substituting t_r ("relative time") for t',

$$p = m_0vt/t_r \tag{12}$$

thus obtaining a more revealing expression than that with the standard use of the Lorentz term, profiling the dynamic role of time. We can thereby recognize momentum as a function of the relative motion of time in space. In other words, momentum is not just the product of the motion of a mass in time and space, it is directly related to the motion in time by one body in the spatial aspect of the coordinate system of another.

Two more corollaries of the principle of the continuum, the covariance of space and time, follow:

4. Motion in time is dynamic, and possessing of kinetic energy and momentum. Furthermore, it may be inferred that the motion of mass in time is absolute as-such (i.e., motion in time is incessant), and as motion across space, it is the actual basis of kinetic energy.

5. Uniform motion is both relative and absolute. A body that moves in time moves in space, and absolutely; it can only be considered "at rest" as a matter of convenience. Uniform motion in spacetime is, however, relative between bodies, varying in their spatial and temporal components.

Time and Gravitation

Perhaps the most significant benefit of this conception of time is that it can finally account for the energy associated with gravitation in General Relativity. To think of gravitation as a deformation of the geometry of spacetime in the presence of mass has always entailed the problematic of resolving the essentially static principle of geometry with the force-like interactions and sustained pressures between the surfaces of gravitating bodies. But if time is dynamic, and time is relentless, then the evident force between bodies interacting gravitationally can be attributed to the "force" of motion in time: If two bodies are moving freely, divergent in time due to their relative motion, and at least one of them is massive enough to produce a significant curvature of spacetime in its vicinity, the other will veer toward it, and if direct contact is made, there will be a continuous pressure as each continues to seek a vector in time perpendicular to its own orientation in space. This is a precise description of what we observe and experience, it supplies the explanation for the energy that cannot be attributed by the general-relativistic geometric description of gravitation, and it renders the search for a quantum theory of gravity unnecessary and inappropriate.

I've written elsewhere: "The source of the energy usually identified as gravitational can… be attributed to an intrinsic and ceaseless dynamic of mass-energy moving in time, independent of gravitation, and obscured by the conflation of gravitation and inertial acceleration in circumstances when they happen to coincide (as at a gravitational surface) but revealed by a recognition of their fundamental distinction.” (Arnold, 2013)

6. “Gravitational energy” is *temporal* energy. If gravitation is the geometric warping of spacetime in the presence of mass, as geometry it bears no similarity to force, and cannot account for the persistence of weight. The kinetic energy of motion in time across space is entirely adequate to account for motion that is warped by gravitation, and for the continuous pressure (weight) induced at a gravitational surface.

Conclusion

The unique characteristic of time as the dynamic aspect of the spacetime continuum has been largely unexplored, I believe in large part due to Minkowski’s original graphic misrepresentation.

By means of a relativistic correction to the Minkowski diagram a number of clarifications and corollaries have been illustrated and propounded: The invariant length of world-lines; uniform motion in time as perpendicular to space and relative motion in spacetime as a different orientation of time to space; the misidentification of the “invariant interval”; the reason *c* is absolute and invariable; how two reference frames can each regard the other’s clock as moving more slowly; that motion in time is absolute and dynamic, and not just the *condition* but the *source* of relative motion in space; and that the energy mis-identified with gravitation is due to continuous motion in time.

End Notes

1. The Lorentz Transformations are $t' = \sqrt{((t-v)/(1-v^2))}$ and $x' = \sqrt{((x-vt)/(1-v^2))}$, with t as time, x as distance, and v as velocity proportional to c.

2. As a matter of convenience t is generally multiplied by c so that space and time can be expressed in distances of the same scale. I prefer instead to calibrate them by giving time in seconds (*sec*) and space in light-seconds (*ls*).

3. Inertial acceleration and local gravitational influences are incidental, and need not be considered here.

References

Arnold J: Gravitation, force, and time, *Eur. Sci. Journal* **9**, 24, (2013)

Minkowski H: Space and time, in *The Principle of Relativity*, H.A. Lorentz, A. Einstein, H. Minkowski, and H. Weyl, trans: W. Perrett and G.B. Jeffery, 1923 (1908)