Seven conjectures on the triplets of primes p, q, r where q = p + 4 and r = p + 6

Marius Coman
email: mariuscoman13@gmail.com

Abstract. In this paper I make seven conjectures on the triplets of primes [p, q, r], where q = p + 4 and r = p + 6, conjectures involving primes, squares of primes, c-primes, m-primes, c-composites and m-composites (the last four notions are defined in previous papers, see for instance the paper "Conjecture that states that any Carmichael number is a cm-composite".

Conjecture 1:

There exist an infinity of triplets of primes [p, q, r], where q = p + 4 and r = p + 6.

The ordered sequence of these triplets is:
[7, 11, 13], [13, 17, 19], [37, 41, 43], [97, 101, 103], [103, 107, 109], [193, 197, 199], [223, 227, 229], [307, 311, 313], [457, 461, 463], [613, 617, 619], [823, 827, 829], [853, 857, 859], [877, 881, 883], [1087, 1091, 1093], [1297, 1301, 1303], [1423, 1427, 1429], [1447, 1451, 1453], [1483, 1487, 1489], [1663, 1667, 1669], [1693, 1697, 1699], [1783, 1787, 1789], [1873, 1877, 1879], [1993, 1997, 1999], [2083, 2087, 2089], [2137, 2141, 2143], [2377, 2381, 2383] ...

Conjecture 2:

There exist an infinity of triplets of primes [p, q, r], where q = p + 4 and r = p + 6, such that s = p + q + r is a prime.

The ordered sequence of the quadruplets [p, q, r, s] is:
[7, 11, 13, 31], [457, 461, 463, 1381], [1087, 1091, 1093, 3271], [1663, 1667, 1669, 4999], [2137, 2141, 2143, 6421] ...

Conjecture 3:

There exist an infinity of triplets of primes [p, q, r], where q = p + 4 and r = p + 6, such that p + q + r is a square of a prime s.

The ordered sequence of the quadruplets [p, q, r, s] is:
[13, 17, 19, 7], [37, 41, 43, 11], [613, 617, 619, 43] ...

Conjecture 4:

There exist an infinity of triplets of primes [p, q, r], where q = p + 4 and r = p + 6, such that s = p + q + r is a c-prime, without being a prime or a square of a prime.

The first such quadruplets [p, q, r, s] are:

: [97, 101, 103, 301], because 301 = 7*43 and 43 – 7 + 1 = 37, prime;
: [103, 107, 109, 319], because 319 = 11*29 and 29 – 11 + 1 = 19, prime;
: [193, 197, 199, 589], because 589 = 19*31 and 31 – 19 + 1 = 13, prime;
: [223, 227, 229, 679], because 679 = 7*97 and 97 - 7 + 1 = 91 = 7*13 and 13 – 7 + 1 = 7, prime;
: [823, 827, 829, 2479], because 2479 = 37*67 and 67 – 37 + 1 = 31, prime;
: [853, 857, 859, 2569], because 2569 = 7*367 and 367 – 7 + 1 = 361, square of prime;
: [877, 881, 883, 2641], because 2641 = 19*139 and 139 – 19 + 1 = 121, square of prime;
: [1297, 1301, 1303, 3901], because 3901 = 47*83 and 83 – 47 + 1 = 37, prime;
: [1423, 1427, 1429, 4279], because 4279 = 11*389 and 389 – 11 + 1 = 379, prime;
: [1447, 1451, 1453, 4351], because 4351 = 19*229 and 229 – 19 + 1 = 211, prime;
: [1693, 1697, 1699, 5089], because 5089 = 7*727 and 727 – 7 + 1 = 721 = 7*103 and 103 – 7 + 1 = 97, prime;
: [1783, 1787, 1789, 5359], because 5359 = 23*233 and 233 – 23 + 1 = 211, prime;
: [1867, 1871, 1873, 5611], because 5611 = 31*181 and 181 – 31 + 1 = 151, prime;
: [1873, 1877, 1879, 5629], because 5629 = 13*433 and 433 – 13 + 1 = 421, prime;
: [1993, 1997, 1999, 5989], because 5989 = 53*113 and 113 – 53 + 1 = 61, prime;
: [2083, 2087, 2089, 6259], because 6259 = 11*569 and 569 – 11 + 1 = 559 = 13*43 and 43 – 13 + 1 = 31, prime;
: [2377, 2381, 2383, 7141], because 7141 = 37*193 and 193 – 37 + 1 = 157, prime.

Conjecture 5:

There exist an infinity of triplets of primes [p, q, r], where q = p + 4 and r = p + 6, such that s = p + q + r is a m-prime, without being a prime or a square of a prime.

The first such quadruplets [p, q, r, s] are:

: [97, 101, 103, 301], because 301 = 7*43 and 43 + 7 - 1 = 37, square of prime;
: [103, 107, 109, 319], because 319 = 11*29 and 29 + 11 - 1 = 39 = 3*13 and 3 + 13 – 1 = 15 = 3*5 and 3 + 5 – 1 = 7, prime;
: [193, 197, 199, 589], because 589 = 19*31 and 31 + 19 - 1 = 49, square of prime;
: [223, 227, 229, 679], because 679 = 7*97 and 97 + 7 - 1 = 103, prime;
: [823, 827, 829, 2479], because 2479 = 37*67 and 67 + 37 - 1 = 103, prime;
: [853, 857, 859, 2569], because 2569 = 7*367 and 367 + 7 - 1 = 373, prime;
: [877, 881, 883, 2641], because 2641 = 19*139 and 139 + 19 + 1 = 157, prime;
: [1447, 1451, 1453, 4351], because 4351 = 19*229 and 229 + 19 + 1 = 247, prime;
: [1693, 1697, 1699, 5089], because 5089 = 7*727 and 727 + 7 - 1 = 733, prime;
: [1867, 1871, 1873, 5611], because 5611 = 31*181 and 181 + 31 - 1 = 151, prime.
: [2083, 2087, 2089, 6259], because 6259 = 11*569 and 569 + 11 - 1 = 573 = 3*193 and 193 – 3 + 1 = 191, prime;
: [2377, 2381, 2383, 7141], because 7141 = 37*193 and 193 + 37 - 1 = 229, prime.

Conjecture 6:

There exist an infinity of triplets of primes [p, q, r], where q = p + 4 and r = p + 6, such that s = p + q + r is a c-composite.

The first such quadruplets [p, q, r, s] are:
: [307, 311, 313, 931], because 931 = 7*7*19 and 7*7 – 19 + 1 = 31, prime;
: [1483, 1487, 1489, 4459], because 4459 = 7*7*7*13 and 7*13 – 7*7 + 1 = 43, prime.

Conjecture 7:

There exist an infinity of triplets of primes [p, q, r], where q = p + 4 and r = p + 6, such that s = p + q + r is a c-composite.

The first such quadruplets [p, q, r, s] are:
: [307, 311, 313, 931], because 931 = 7*7*19 and 7*7 + 19 - 1 = 67, prime;
: [1483, 1487, 1489, 4459], because 4459 = 7*7*7*13 and 7*13 + 7*7 - 1 = 139, prime.

Observations:

: It can be seen that any from the first 26 triplets [p, q, r] falls at least in one of the cases involved by the Conjectures 2-7;
: For all the first 26 triplets [p, q, r] the number s = p + q + r is a prime or a product of two prime factors;
: Both of the triplets from above that are c-composites are also m-composites so they are cm-composites;
: Most of the triplets from above that are c-primes are also m-primes so they are cm-primes.