Two conjectures on squares of primes involving the sum of consecutive primes

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Abstract. In this paper I make a conjecture which states that there exist an infinity of squares of primes of the form 6*k - 1 that can be written as a sum of two consecutive primes plus one and also a conjecture that states that the sequence of the partial sums of odd primes contains an infinity of terms which are squares of primes of the form 6*k + 1.

Conjecture 1:

There exist an infinity of squares of primes of the form 6*k - 1 that can be written as a sum of two consecutive primes plus one.

First ten terms from this sequence:

: 5^2 = 11 + 13 + 1;
: 11^2 = 59 + 61 + 1;
: 17^2 = 139 + 149 + 1;
: 29^2 = 419 + 421 + 1;
: 53^2 = 1399 + 1409 + 1;
: 101^2 = 5099 + 5101 + 1;
: 137^2 = 9377 + 9391 + 1;
: 179^2 = 16007 + 16033 + 1;
: 251^2 = 31489 + 31511 + 1;
: 281^2 = 39461 + 39499 + 1.

Note other interesting related results:
: 41^2 = 839 + 841 + 1, where 839 is prime and 841 = 29^2 square of prime;
: 47^2 = 1103 + 1105 + 1, where 1103 is prime and 1105 is absolute Fermat pseudoprime.

Note that I haven't found in OEIS any sequence to contain the consecutive terms 5, 11, 17, 29, 53, 101..., so I presume that the conjecture above has not been enunciated before.

Note also the amount of squares of the primes of the form 6*k – 1 that can be written this way (10 from the first 31 such primes).

Conjecture 2:

The sequence of the partial sums of odd primes (see the sequence A071148 in OLEIS) contains an infinity of terms which are squares of primes of the form 6*k + 1.

First three terms from this sequence:

: 31^2 = 3 + 5 +...+ 89;
: 37^2 = 3 + 5 +...+ 107;
: 43^2 = 3 + 5 +...+ 131.