Virtual proper time in the problems of eliminating the infrared catastrophe and of the field origin of the electron mass and self-energy in classical electrodynamics

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Abstract

With inclusion of the virtual proper time in the metric of the physical Minkowski space we pass to the four-dimensional bimetric space-time. Now a complete description of the occurring physical processes includes both physical (observable) and virtual (unobservable) objects that enter in the physical expressions. In classical electrodynamics this conversion leads to the appearance of the virtual scalar-electric field that complements the physical electromagnetic field and is a massive in the presence of sources. This allows to eliminate the infrared catastrophe and to proof the field origin of the virtual (bare) electron mass and self-energy. With inclusion of the virtual proper time in the classical quantum theory we obtain the single-particle wave Dirac equation for which the electron wave function retains the simple probabilistic interpretation. In the single-particle Dirac theory the virtual scalar-electric field shifts the physical energy levels for the hydrogen atom in an external field and this leads to two additional amendments.

Keywords: Virtual proper time – Virtual scalar-electric field - Elimination of the infrared catastrophe - Field origin of the electron mass - Single-particle wave Dirac equation.

Introduction

In Minkowski space with the metric $ds^2 = c^2dt^2 - d\mathbf{x}^2$ the proper time t is determined by the equality $ds^2 = c^2dt^2$.

For a moving particle the proper time t is measured by the clock which move with this particle and at rest relative to it [1]. Therefore, in a moving inertial reference system (further, i.r.s.) the proper time t is not measured directly by the clock of time t.

We consider the system of two expressions ds^2 given above for the invariant interval s. Thereby, we pass from the Minkowski space to the four-dimensional space-time with the double metric.

Now moving i.r.s. includes the clock of virtual proper time t that is separated from the clock of physical time t. In our context the virtual proper time t is the unobservable (immeasurable) time in a moving i.r.s. In i.r.s. at rest the clocks of time t and t are synchronized by definition .

Thus, we deal with the four-dimensional bimetric space-time in which all variables depend not only on the physical coordinates t, x, y, z, but also on the virtual proper time t. Now a complete description of the occurring physical processes include both physical (observable) and virtual (unobservable) objects entering into the physical expressions through the operations that are similar to the algebraic operations with real and imaginary numbers. So, the multiplicative operation with two virtual objects is a physical object. Thus, under the action of a suitable virtual operator on the virtual (bare) electron mass we get the physical mass and on the physical electron charge we get the virtual (bare) charge.

Introduction of virtual objects supplementing physical objects ensures the completeness of description of the occurring physical processes and allows to consider the problems that do not have solutions only in the set of physical objects. This is similar to how some algebraic real equations do not have solutions only in the set of real numbers.

For example, the classical problem of the nature of the electron mass and self-energy has a solution only for the virtual (bare) electron mass taking into account the physical electromagnetic and virtual scalar-electric fields.

In this article we will try to find the justification of the foregoing hypothesis on the inclusion of the virtual proper time in space-time metric through the study of some known problems in classical electrodynamics and quantum theory.

We use the following abbreviations:

the i.r.s. - the inertial reference system ,
the t-clock - the clock of time t ,
SEM - scalar-electromagnetic,
SE - scalar-electric.

We assume that the indices

i, j, k take on the values 1, 2, 3;
α, β, γ take on the values 0, 1, 2, 3;
μ, ν, λ take on the values 0, 1, 2, 3, 5.

I. 4-dimensional bimetric pseudo-euclidean space-time $\mathrm{V}_{4|5}$

1. 4-dimensional bimetric pseudo-euclidean space of 5-vectors $\mathbf{V}_{4|5}$

1) Space V_4

V_4 - 4-dimensional pseudo-euclidean linear space consisting of 4-vectors $x^\alpha = (x^0, x^i)$ with the metric $(ds^2)_{\mathrm{V}4} = (dx^0)^2 - (dx^i)^2$.

2) Space V_1

V_1 - 1-dimensional linear space consisting of 1-vectors (scalars) x^5 with the metric $(ds^2)_{\mathrm{V}1} = (dx^5)^2$.

3) Space V_5

V_5 - 5-dimensional pseudo-euclidean linear space consisting of 5-vectors

$x^\mu = (x^\alpha, x^5) = (x^0, x^i, x^5)$ with the metric $(ds^2)_{V5} = (dx^0)^2 - (dx^i)^2 + (dx^5)^2$.

4) Space $V_{4|5}$

A) $V_{4|5}$ - 4-dimensional linear space consisting of 5-vectors $x^\mu = (x^\alpha, x^5) \in V_5$ such that $x^\alpha x_\alpha = (x^5)^2$ and that later we will call 4|5–vectors .

B) $V_{4|5}$ - *pseudo-euclidean space with the double metric (bimetric)* which is the system

$(ds^2)_{V4|5} = (dx^0)^2 - (dx^i)^2 = (ds^2)_{V4}$,

$(ds^2)_{V4|5} = (dx^5)^2 = (ds^2)_{V1}$, or

$(ds^2)_{V4|5} = \frac{1}{2}\left[(dx^0)^2 - (dx^i)^2 + (dx^5)^2\right] = \frac{1}{2}(ds^2)_{V5}$,

$(ds^2)_{V4|5} = (dx^5)^2 = (ds^2)_{V1}$.

It is the latter form of the double metric we call *the canonical form of the metric in* $V_{4|5}$ since V_5 includes the space $V_{4|5}$.

Definition

For the 4|5–vector $x^\mu = (x^\alpha, x^5)$ the 4–vector x^α is called *the base part* and is denoted $x^\alpha = x^\mu{}_{\text{base}}$, the 1–vector (scalar) x^5 is called *the own part* and is denoted $x^5 = x^\mu{}_{\text{own}}$. Thus, the 4|5–vector $x^\mu = (x^\alpha, x^5) = (x^\mu{}_{\text{base}}, x^\mu{}_{\text{own}})$.

2. 4-dimensional bimetric pseudo-euclidean space-time $V_{4|5}$

1) The double metric in $V_{4|5}$

Let the 4-vector $x^{\alpha} = \left(x^{0},\, x^{i}\right) = \left(ct,\, \mathbf{x}\right) \in \mathrm{V}_4$, where V_4 - 4-dimensional basic space-time (Minkowski space) with the metric $ds^2 = dx^{\alpha}dx_{\alpha} = c^2dt^2 - d\mathbf{x}^2$. At each point $A\,(t\,,\mathbf{x})$ in V_4 we deal only with physically measurable (observable) coordinates t and $\mathbf{x}$.

Let the 1-vector (scalar) $x^5 = ct \in \mathrm{V}_1$, where t is the proper time. That is, the metric in V_1: $ds^2 = c^2dt^{\,2}$. Then the 5-vector $x^{\mu} = \left(x^{\alpha},\, x^{5}\right) = \left(ct,\, \mathbf{x},\, ct\right) \in \mathrm{V}_5$, where $\mathrm{V}_5 = \mathrm{V}_4 \oplus \mathrm{V}_1$ - 5-dimensional space-time with the metric

$$\left(ds^2\right)_{\mathrm{V}5} = 2\, ds^2 = dx^{\mu}dx_{\mu} = c^2dt^2 - d\mathbf{x}^2 + c^2dt^{\,2}.$$

Definition

4-dimensional bimetric pseudo-euclidean space-time $\mathrm{V}_{4|5}$ is the linear space consisting of 4|5-vectors x^{μ}, for which

a) the double metric in the projective

$$ds^2 = dx^{\alpha}dx_{\alpha} = c^2dt^2 - d\mathbf{x}^2,$$

$$ds^2 = dx^5dx_5 = c^2dt^{\,2}\ ;$$

b) the double metric in the canonical form

$$2\,ds^2 = dx^{\mu}dx_{\mu} = c^2dt^2 - d\mathbf{x}^2 + c^2dt^{\,2}\ ,$$

$$ds^2 = dx^5dx_5 = c^2dt^{\,2}\ .$$

2) Inertial reference system in the space-time $\mathrm{V}_{4|5}$

Postulate

In each moving i.r.s. there is the clock of virtual proper time t (the t -clock) that is separated from the clock of physical time t. The rate and direction of time coincide for the t - and t-clocks in each i.r.s. where the t-clock at rest.

Corollaries

α) Since the space-time $V_{4|5}$ is four-dimensional, then the virtual proper time t is not observable (immeasurable) in a moving i.r.s..

β) $s = |x^5| = \pm ct$. Here and elsewhere the sign $\pm$ corresponds to the forward \ backward direction of the virtual proper time t .

γ) If $\Delta t \neq 0$, then the interval Δs is always timelike, that is, $\Delta s^2 > 0$.

δ) The event in $V_{4|5}$ is defined by the point $A(t, \mathbf{x}, t)$. Thus, in the moving i.r.s. Σ not all components of $A(t, \mathbf{x}, t)$ corresponding to the event are physical (observable).

Remark

We assume that the results of multiplicative operations with the physical and virtual objects are similar to the ones with real and imaginary numbers. Therefore, $x^5 = ct$ is virtual, but s^2 is physical.

3) Transformation group in $V_{4|5}$

In the space-time $V_{4|5}$ isomorphic to Minkowski space V_4 as a continuous transformation group of components x^α of the 4|5–vector $x^\mu = \left(x^\alpha, x^5\right)$ we examine the Poincare group or, in a special case, the 6-parametric Lorentz group. The virtual component $x^5 = ct$ is Lorentz invariant.

Remarks

α) The last statement about transformation of components x^α remains valid also for any 4|5–vector $a^\mu = \left(a^\alpha, a^5\right)$ such that $a^\alpha a_\alpha = a_5{}^2$. The base part $a^\mu{}_{\text{base}} = a^\alpha$, and the own part $a^\mu{}_{\text{own}} = a^5$.

β) For any 4-vector a^{α} there is the couple 4|5-vectors $a^{\mu}{}_{\pm} = \left(a^{\alpha}, \pm\, a^{5}\right)$ such that $a^{\alpha}a_{\alpha} = \left(\pm\, a_{5}\right)^{2}$ and conversely. On physical reasons the signs of a^{0} and a^{5} must be coincident. Thus, there is a one-to-one correspondence between a^{α} and $a^{\mu} = \left(a^{\alpha}, a^{5}\right)$.

3. Invariant systems for 4|5–vectors in the space-time $\mathbf{V}_{4|5}$

Respect to transformations of the Lorentz group we have the invariant expressions written below in the form of systems:

1) the 4|5–vector $x^{\mu} = \left(x^{\alpha}, x^{5}\right) = \left(ct, \mathbf{x}, ct\right)$

$x^{\alpha}x_{\alpha} = \mathrm{inv}$,

$x^{5}x_{5} = \mathrm{inv}$, i.e. $x^{\mu}x_{\mu} = 2\,x_{5}^{\;2}$;

2) the 4|5–vector of velocity u^{μ} (the 4|5-velocity)

$$u^{\mu} = \frac{dx^{\mu}}{ds} = \left(u^{\alpha}, u^{5}\right) = \pm\left(\frac{dt}{dt}, \mathbf{u}, 1\right) = \pm\left(\frac{1}{\varepsilon}, \frac{\mathbf{v}}{c\,\varepsilon}, 1\right).$$ Here, u^{α} is virtual,

$$ds = \left|dx^{5}\right| = \pm\, c\,dt = \pm\, c\,\varepsilon\,dt,$$ virtual $\varepsilon = \sqrt{1 - (\mathbf{v}/c)^{2}}$, physical $\mathbf{v} = \dfrac{d\mathbf{x}}{dt}$.

Then $u^{\alpha}u_{\alpha} = 1$, $u^{5}u_{5} = 1$, i.e. $u^{\mu}u_{\mu} = 2$.

Corollary

If virtual $a^{5} \neq 0$, then $a^{\mu} = a^{5}u^{\mu}$ is the 4|5–vector $a^{\mu} = \left(a^{\alpha}, a^{5}\right)$, such that $a^{\alpha}a_{\alpha} = a_{5}^{\;2}$ or $a^{\mu}a_{\mu} = 2\,a_{5}^{\;2}$.

3) the momentum 4|5–vector (the 4|5–momentum) of a massive point particle

$$\pm\, p^{\mu} = mcu^{\mu} = \pm\left(p^{\alpha}, p^{5}\right) = \pm\left(\frac{mc}{\varepsilon}, \frac{m\mathbf{v}}{\varepsilon}, mc\right),$$

$$p^{\alpha}p_{\alpha} = m^{2}c^{2}, \quad p^{5}p_{5} = m^{2}c^{2}, \quad \text{i.e.} \quad p^{\mu}p_{\mu} = 2m^{2}c^{2} .$$

Here, m is the virtual mass of a moving particle. By value m coincides with the physical mass m_0 of a particle at rest. Hence, p^α is physical, but p^5 is virtual.

4) the energy-momentum $\pm\, p^\mu c = mc^2 u^\mu = \pm\,(E, \mathbf{p}c, \mathrm{E})$.

Here, $E = \frac{1}{\varepsilon} mc^2$, $\mathbf{p} = \frac{m\mathbf{v}}{\varepsilon}$, $\mathrm{E} = mc^2$ are respectively: *the physical energy, 3-momentum, the virtual self-energy of a massive moving particle. By value the virtual self-energy* $\mathrm{E} = mc^2$ *coincides with the physical self-energy* $E_0 = m_0 c^2$ *of a massive particle at rest.*

$E^2 - \mathbf{p}^2 \mathrm{c}^2 = m^2 c^4$,

$\mathrm{E}^2 = m^2 c^4$, i.e. $E^2 - \mathbf{p}^2 \mathrm{c}^2 + \mathrm{E}^2 = 2m^2 c^4$.

Remark

The 5-acceleration $w^\mu = \dfrac{du^\mu}{ds} = \left(w^\alpha, w^5\right)$, $w^5 = 0$, and the 5-force

$f^\mu = \dfrac{dp^\mu}{ds} = \left(f^\alpha, f^5\right)$, $f^5 = 0$, are not 4|5-vectors, since, in general case,

$w^\alpha w_\alpha \neq 0$ and $f^\alpha f_\alpha \neq 0$.

4. The mass current 4|5-vector and the energy-momentum 4|5-tensor of a particle in the space-time $\mathbf{V}_{4|5}$

The mass current 4|5-vector of a moving particle

$j^\mu_m = \rho_m c u^\mu = mc \int \delta^{4|5}\left(x^\lambda - x^\lambda(\vartheta)\right) \varepsilon(\vartheta)\, u^\mu(\vartheta)\, d\vartheta$, where

$\delta^{4|5}\left(x^\lambda - x^\lambda(\vartheta)\right) = \delta^4\left(x^\alpha - x^\alpha(\vartheta)\right)\, \delta\left(x^5 - x^5(\vartheta)\right)$, $\int \delta^{4|5}\left(x^\lambda - x^\lambda(\vartheta)\right) dx^\alpha = 1$,

$\rho_m = m\varepsilon\, \delta\left(\mathbf{x} - \mathbf{x}(t)\right) \delta\left(t - t\,(t)\right)$ is *the physical mass density,*

$\int \delta\left(\mathbf{x} - \mathbf{x}(t)\right) \delta\left(t - t\,(t)\right) d\mathbf{x} = 1$.

Let *the energy-momentum 4|5–tensor of a massive particle* $T_m^{\mu\nu} = j_m^{\mu} c u^{\nu}$.

Trace of $T_m^{\mu\nu}$: $T_{\mu}^{\mu} = 2T_{\alpha}^{\alpha} = 2\rho_m c^2$. $T_m^{\mu 5} = \left(T_m^{\alpha 5}, T_m^{55}\right)$, where $T_m^{55} = -L_m = \rho_m c^2$.

$T_m^{\mu 5} = j_m^{\mu} c u^5 = j_m^{\mu} c$, for which the conservation equation $\partial_{\mu} T_m^{\mu 5} = 0$ and $\partial_5 T_m^{\mu 5} = 0$.

The value $T_m^{\mu 0} = j_m^{\mu} c u^0 = T_m^{\mu 5} u^0$ or $T_m^{\mu 0} = \dfrac{c}{\varepsilon} j_m^{\mu} = \not{\rho}_m c^2 u^{\mu}$ is not the 4|5-vector and is usually called *the momentum density of a particle. The moment 4|5-vector of a particle* $P^{\mu 0} = \dfrac{1}{c} \int T_m^{\mu 0} d^3 x = m c u^{\mu} = p^{\mu}$ is called *the 4|5-momentum.*

For the symmetric energy-momentum 4|5–tensor of a massive particle $T_m^{\mu\nu} = (T_{\text{base}}^{\mu\nu}, T_{\text{own}}^{\mu\nu})$ *the base part* $T_{\text{base}}^{\mu\nu} = T_m^{\alpha\beta} = j_m^{\alpha} c u^{\beta}$ is the symmetric 4–tensor, *the own part* $T_{\text{own}}^{\mu\nu} = \left(T_m^{\mu 5}, T_m^{5\mu}\right)$ are two equal 4|5-vectors.

By analogy with the 4|5-vector x^{μ} for the 4|5-vector $T^{\mu 5} = \left(T_m^{\alpha 5}, T_m^{55}\right)$ there are the invariant equalities: $T_{\alpha 5} T^{\alpha 5} = T_{55}{}^2$, $T_{\mu 5} T^{\mu 5} = 2T_{55}{}^2$.

Respectively, for the 4|5-tensor $T_m^{\mu\nu}$ there are the invariant equalities:

$$T_{\alpha\beta} T^{\alpha\beta} = T_{\gamma 5} T^{\gamma 5}, \qquad \alpha < \beta ,$$

$$T_{\alpha\nu} T^{\alpha\nu} = 2T_{\gamma 5} T^{\gamma 5}, \qquad \alpha < \nu .$$

5. The charge current 4|5-vector of a massive particle

The charge current 4|5-vector of a massive particle with the physical charge e and the virtual mass m $j_e^{\mu} = \rho_e c u^{\mu}$ may be written as $j_e^{\mu} = \dfrac{e}{m} j_m^{\mu} = \dfrac{e}{mc} T_m^{\mu 5}$, where *the virtual charge density* $\rho_e = \dfrac{e}{m} \rho_m$.

If we assume the positive direction of time t and t, then the charge current 4|5-vector of a particle $j^{\mu} = \rho c u^{\mu} = \left(j^{\alpha}, j^5\right) = (\not{\rho} c, \mathbf{j}, \rho c)$, where ρ is the virtual charge density, $\not{\rho} = \dfrac{\rho}{\varepsilon}$ is the physical charge density, $\mathbf{j} = \rho c \mathbf{u} = \not{\rho}\mathbf{v}$. Here,

$j_\alpha j^\alpha = j_5^{\,2}$ or $\not{\rho}^2 c^2 - \mathbf{j}^2 = \rho^2 c^2$.

The equation of current continuity $\partial_\mu j^\mu = \partial_\alpha j^\alpha = 0$. That is, $\partial_5 j^5 = \dfrac{\partial \rho}{\partial t} = 0$.

Then the physical charge $Q = \int \not{\rho}\, dV = \int \rho\, d\mathrm{V}$, $dV = \varepsilon\, d\mathrm{V} = d^3 x$.

II. The scalar-electromagnetic field in the space-time $\mathrm{V}_{4|5}$

1. 5-potential of the SEM-field

Let the scalar-electromagnetic potential $A^\mu(x^\nu) = (A^\alpha, A^5) = (\varphi, \mathbf{A}, \phi)$ is the 5-vector $x^\lambda \in \mathrm{V}_5$, $\mathrm{V}_5 = \mathrm{V}_4 \oplus \mathrm{V}_1$, but not 4|5-vector $x^\nu \in \mathrm{V}_{4|5}$. That is, the scalar potential ϕ is virtual and respect to the transformations of the Lorentz group (of boosts and spatial rotations) takes place the inequality $A^\alpha A_\alpha \neq A_5^{\,2}$ or $\varphi^2 - \mathbf{A}^2 \neq \phi^2$.

In the case of a massive SEM-field with sources the 4-potential $A^\alpha(x^\nu)$ depends explicitly on the virtual proper time t, i.e. $\partial_5 A^\alpha \neq 0$. Thus, the massive SEM-field with the 5-potential $A^\mu(x^\nu)$ is considered in $\mathrm{V}_{4|5}$, where the 4|5-vector $x^\nu = (x^\alpha, x^5)$, $x^\alpha x_\alpha = x_5^{\,2}$. The theory of a massive SEM-field is not gauge invariant.

In the case of a massless SEM-field without sources the 5-potential A^μ does not depend explicitly on the virtual proper time t, that is, $\partial_5 A^\alpha = 0$. Thus, the massless SEM-field is considered actually in V_4 and is given by the 5-potential $A^\mu(x^\alpha)$, $x^\alpha \in \mathrm{V}_4$.

The theory of a massless SEM-field is invariant respect to gauge transformations of the potential A_μ: $A_\mu \to A_\mu{}' = A_\mu - \partial_\mu f$, where f: $\partial_\mu \partial^\mu f = 0$.

In what follows we will use mainly the Heaviside-Lorentz system of units, where $e^2 = 4\pi\alpha$, $\mathbf{h} = c = 1$.

2. 5-tensor of the SEM-field strengths

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = (\mathbf{E},\mathbf{H},\mathrm{C},-\mathbf{Э}) = \begin{pmatrix} 0 & E_x & E_y & E_z & C \\ -E_x & 0 & -H_z & H_y & -Э_x \\ -E_y & H_z & 0 & -H_x & -Э_y \\ -E_z & -H_y & H_x & 0 & -Э_z \\ -C & Э_x & Э_y & Э_z & 0 \end{pmatrix}, \text{ where}$$

the physical electric field $\mathbf{E} = -\operatorname{grad}\varphi - \dfrac{\partial\mathbf{A}}{\partial t}$, the physical magnetic field $\mathbf{H} = \operatorname{rot}\mathbf{A}$,

the virtual scalar field $\mathrm{C} = \dfrac{\partial\phi}{\partial t} - \dfrac{\partial\varphi}{\partial t}$, the virtual electric field $\mathbf{Э} = -\operatorname{grad}\phi - \dfrac{\partial\mathbf{A}}{\partial t}$.

For the antisymmetric 5-tensor $F_{\mu\nu} = \left(F_{\mu\nu}^{\text{base}}, F_{\mu\nu}^{\text{own}}\right)$ *the base part* $F_{\mu\nu}^{\text{base}} = F_{\alpha\beta} = (\mathbf{E},\mathbf{H})$ is the antisymmetric 4-tensor of the physical EM-field , *the own part* $F_{\mu\nu}^{\text{own}} = \left(F_{\mu 5}, F_{5\mu}\right)$, where $F_{\mu 5} = (\mathrm{C},-\mathbf{Э}, 0) = -F_{5\mu}$, are two opposite 5-vectors of the virtual SE-field which is not observable directly in a moving i.r.s.

By analogy with the 4|5-vector A^μ , for the 4|5-tensor $F_{\mu\nu}$ we have the inequalities:

$$F_{\alpha\beta}F^{\alpha\beta} \neq F_{\gamma 5}F^{\gamma 5}, \qquad \alpha < \beta , \qquad \text{i.e.} \quad \mathbf{H}^2 - \mathbf{E}^2 \neq \mathrm{C}^2 - \mathbf{Э}^2 ,$$

$$F_{\alpha\nu}F^{\alpha\nu} \neq 2F_{\gamma 5}F^{\gamma 5}, \qquad \alpha < \nu .$$

In the general case, $F_{\gamma 5}F^{\gamma 5} \neq F_{55}{}^2$, i.e. $\mathrm{C}^2 - \mathbf{Э}^2 \neq 0$. The equality takes place in the special case for a plane SEM-wave.

3. Transformation of the virtual SE-field strengths

The physical EM- and virtual SE- fields transform independently under the Lorentz group. As a result of boosts the virtual SE-field transforms as the 4-vector $F^{\alpha 5} = (\mathrm{C},\mathbf{Э})$:

$$\mathrm{C}' = \frac{1}{\varepsilon}\,(\mathrm{C} + \mathbf{V}\,\mathbf{Э})\ ,\quad \mathbf{Э}' = \mathbf{Э} + \frac{\mathbf{V}}{\varepsilon}\left(\frac{\mathbf{V}\,\mathbf{Э}}{\varepsilon+1} + \mathrm{C}\right),\quad \text{where}\quad \varepsilon = \sqrt{1 - V^2}\,,\quad V = |\,\mathbf{V}\,|\,.$$

4. The first union of the SEM-field equations

$\partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} + \partial_\lambda F_{\mu\nu} = 0\,,$ i.e.

the physical equations: $\mathrm{rot}\,\mathbf{E} = -\dfrac{\partial\,\mathbf{H}}{\partial t}\ ,\quad \mathrm{div}\,\mathbf{H} = 0\,,$

the virtual equations: $\mathrm{rot}\,\mathbf{Э} = -\dfrac{\partial\,\mathbf{H}}{\partial t}\ ,\quad \mathrm{grad}\,\mathrm{C} = \dfrac{\partial\,\mathbf{E}}{\partial t} - \dfrac{\partial\,\mathbf{Э}}{\partial t}\ .$

Remark

Physical equations consist only of physical terms. Virtual equations consist only of virtual terms.

5. The charge current 5-vector

$j^{\,\mu} = \left(j^{\,\alpha}, j^{\,5}\right) = \left(\not{\phi},\, \mathbf{j},\, \rho\right),$ where ρ is virtual, but $j^{\,\mu} \neq \rho\, u^{\,\mu}$. Thus, $j^{\,\mu}$ is not the 4|5-vector. From this, $j_\alpha\, j^{\,\alpha} \neq j_5^{\,2}$ or $\not{\phi}^{\,2} - \mathbf{j}^{\,2} \neq \rho^{\,2}$.

6. The Lagrangian of a massive SEM-field with sources

It is the system of two Lagrangians : *the full Lagrangian* $\mathrm{L} = \mathrm{L}_{\mathrm{f}} + \mathrm{L}_{\mathrm{int}}$,

where $\mathrm{L}_{\mathrm{f}} = \mathrm{L}_{\mathrm{SEM}} = -\dfrac{1}{4}\, F_{\mu\nu} F^{\,\mu\nu} + м^2 A_\mu A^\mu =$

$= \dfrac{1}{2}\left(\mathbf{E}^2 - \mathbf{H}^2 + \mathbf{Э}^2 - \mathrm{C}^2\right) + м^2\left(\varphi^2 - \mathbf{A}^2 + \phi^2\right)\ ,\quad \mathrm{L}_{\mathrm{int}} = -A_\mu\, j^{\,\mu}\,,$

$м$ - *the virtual mass of quantum SE-field,* and *the own part of Lagrangians* L and L_{f} :

$$\mathrm{L}_{\mathrm{own}} = \mathrm{L}^{\mathrm{f}}_{\mathrm{own}} = \mathrm{L}_{\mathrm{SE}} = -\frac{1}{2}\, F_{\alpha 5} F^{\,\alpha 5} + \frac{1}{2}\, м^2 A_\alpha A^\alpha = \frac{1}{2}\left(\mathbf{Э}^2 - \mathrm{C}^2\right) + \frac{1}{2}\, м^2\left(\varphi^2 - \mathbf{A}^2\right).$$

The interaction term does not appear in own part of the Lagrangian L. Hence

$\mathrm{L}_{\mathrm{SEM}} = \mathrm{L}_{\mathrm{SE}} + \mathrm{L}_{\mathrm{EM}} + м^2 A_5^{\,2}$ and $\mathrm{L}_{\mathrm{SE}} \neq \mathrm{L}_{\mathrm{EM}}$. *The base part of the Lagrangian* L_{f}

$$\mathrm{L}^{\mathrm{f}}_{\mathrm{base}} = \mathrm{L}_{\mathrm{EM}} = -\frac{1}{4}\, F_{\alpha\beta} F^{\,\alpha\beta} + \frac{1}{2}\, м^2 A_\alpha A^\alpha = \frac{1}{2}\left(\mathbf{E}^2 - \mathbf{H}^2\right) + \frac{1}{2}\, м^2\left(\varphi^2 - \mathbf{A}^2\right)\ .$$

7. The second union of equations of a massive SEM-field with sources. Elimination of the infrared divergences

Proca equations for a massive SEM-field with sources are obtained from the Lagrangians L and $\mathrm{L}_{\mathrm{own}}$ as the system

$$\partial_\nu F^{\nu\mu} + 2м^2 A^\mu = j^\mu, \qquad \partial_5 F^{5\alpha} + м^2 A^\alpha = 0 ,$$ or

the physical equations: $\operatorname{div}\mathbf{E} + 2м^2\varphi = \not{\rho} + \dfrac{\partial \mathrm{C}}{\partial t}$, $\quad \operatorname{rot}\mathbf{H} + 2м^2\mathbf{A} = \mathbf{j} + \dfrac{\partial \mathbf{E}}{\partial t} + \dfrac{\partial \mathbf{Э}}{\partial t}$,

$\dfrac{\partial \mathrm{C}}{\partial t} = м^2\varphi$, $\dfrac{\partial \mathbf{Э}}{\partial t} = м^2\mathbf{A}$. From this, $\operatorname{div}\mathbf{E} = \not{\rho} - \dfrac{\partial \mathrm{C}}{\partial t}$, $\operatorname{rot}\mathbf{H} = \mathbf{j} + \dfrac{\partial \mathbf{E}}{\partial t} - \dfrac{\partial \mathbf{Э}}{\partial t}$,

or $\operatorname{div}\mathbf{E} + м^2\varphi = \not{\rho}$, $\operatorname{rot}\mathbf{H} + м^2\mathbf{A} = \mathbf{j} + \dfrac{\partial \mathbf{E}}{\partial t}$.

The virtual equation: $\operatorname{div}\mathbf{Э} + 2м^2\phi = \rho - \dfrac{\partial \mathrm{C}}{\partial t}$.

Corollary

As follows from the equations of a massive SEM-field, the virtual SE-field varies in time t and, therefore, is massive in the presence of field sources . The small virtual mass $м$ of a quantum SE-field protects from the infrared catastrophe in QED [2]. The physical EM-field (the observable base part of SEM-field) is massless and long-range. The EM-field is responsible for the space-time propagation. The massive virtual SE-field (the unobservable own part of SEM-field) may be related to the Coulomb interaction.

8. Wave equations for a massive SEM-field with sources

Using the Stückelberg Lagrangians with the interaction term

$$\mathrm{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2}\left(\partial_\mu A^\mu\right)^2 + м^2 A_\mu A^\mu - A_\mu\, j^\mu ,$$

$$\mathrm{L}_{\mathrm{own}} = -\frac{1}{2} F_{\alpha 5} F^{\alpha 5} - \frac{1}{2}\left(\partial_5 A^5\right)^2 + \frac{1}{2} м^2 A_\alpha A^\alpha ,$$

we obtain the system of SEM-field equations

$$\partial_\nu F^{\nu\mu} + 2\textit{м}^2 A^\mu = j^\mu ,$$

$$\partial_5 F^{5\alpha} + \textit{м}^2 A^\alpha = 0 \ , \quad \text{with the condition} \quad \partial_\mu A^\mu = 0 \, .$$

That is equivalent to the system of wave equations for the 5-potential A^μ

$$\left(\mathrm{l} - 2\textit{м}^2 \right) A^\mu = - j^\mu , \quad \text{where} \quad \mathrm{l} = - \partial_\nu \partial^\nu ,$$

$$\left(\partial_5{}^2 + \textit{м}^2 \right) A^\alpha = 0 \ , \qquad \text{with the condition} \quad \partial_5 A^5 = 0 \, .$$

Then the system of wave equations for the SEM-field strengths

$$\left(\mathrm{l} - 2\textit{м}^2 \right) F^{\mu\nu} = - J^{\mu\nu} , \qquad \text{where} \quad J^{\mu\nu} = \partial^\mu j^\nu - \partial^\nu j^\mu ,$$

$$\left(\partial_5{}^2 + \textit{м}^2 \right) F^{\alpha\beta} = 0 \ , \qquad \text{or}$$

$$\left(\mathbf{W} - \textit{м}^2 \right) F^{\mu\nu} = - J^{\mu\nu} , \qquad \text{where} \quad \mathbf{W} = - \partial_\gamma \partial^\gamma ,$$

$$\left(\partial_5{}^2 + \textit{м}^2 \right) F^{\alpha\beta} = 0 \ .$$

From here we find the system of virtual wave equations for the strengths **Э** and C

$$\left(\mathbf{W} - \textit{м}^2 \right) \mathbf{Э} = \operatorname{grad} \rho + \frac{\partial \mathbf{j}}{\partial t} \ ,$$

$$\left(\partial_5{}^2 + \textit{м}^2 \right) \mathbf{Э} = 0 \ ,$$

$$\left(\mathbf{W} - \textit{м}^2 \right) \mathrm{C} = \frac{\partial \not{\rho}}{\partial t} - \frac{\partial \rho}{\partial t} \ ,$$

$$\left(\partial_5{}^2 + \textit{м}^2 \right) \mathrm{C} = 0 \ .$$

9. The equation of current continuity. Conserved charges

From the SEM-field equations it follows the equation of current continuity $\partial_\mu j^\mu = 0$ together with the condition $\partial_5 j^5 = 0$. Therefore, in $\mathrm{V}_{4|5}$ the physical charge $Q_0 = \int j^0 \left(x^\lambda \right) d^3 x = \int \not{\rho} \, d^3 x$, the virtual charge $Q_5 = \int j^5 \left(x^\lambda \right) d^3 x = \int \rho \, d^3 x$, where $| Q_0 | > | Q_5 |$, since $\not{\rho} = \dfrac{\rho}{\varepsilon}$, are conserved in time: $\dfrac{d}{dt} Q_0 = 0$, $\dfrac{d}{dt} Q_5 = 0$.

10. The canonical energy-momentum tensor of a massive SEM-field

From the full Lagrangian L_{f} of a massive SEM-field we can obtain the energy-momentum tensor $T^{\mu\nu} = (T^{\mu\nu}_{\text{base}}\,,\ T^{\mu\nu}_{\text{own}})$, or in the matrix form

$$T^{\mu\nu} = \begin{pmatrix} T^{\alpha\beta} & T^{\alpha 5} \\ T^{5\alpha} & T^{55} \end{pmatrix} = \begin{pmatrix} \mathrm{w} & \mathbf{g} & \mathrm{v} \\ \mathbf{S} & -\hat{\sigma} & \mathbf{h} \\ \mathrm{v} & \mathbf{R} & \mathrm{u} \end{pmatrix}.$$

All equalities below are given with an accuracy to terms that disappear upon integration over d^3x in $\mathrm{V}_{4|5}$.

The base part of the energy-momentum tensor $T^{\mu\nu}_{\text{base}} = T^{\alpha\beta}$ has the physical components: *the energy density* $(c = 1)$

$$\mathrm{w} = T^{00} = \frac{1}{2}\left(\mathbf{E}^2 + \mathbf{H}^2 - \mathbf{Э}^2 - \mathrm{C}^2\right) - м^2\left(\varphi^2 - \mathbf{A}^2 + \phi^2\right) = T^{00}_{\text{EM}} - T^{00}_{\text{SE}} - м^2 A_\mu A^\mu \ ,$$

g - *the momentum density 3-vector,* $c\,\mathbf{g} = \left\{T^{0i}\right\} = [\mathbf{E}\,\mathbf{H}] - \mathrm{C}\mathbf{Э} = \left\{T^{0i}\right\}_{\text{EM}} - \left\{T^{0i}\right\}_{\text{SE}}$,

S - *the energy flux density 3-vector (the Poynting vector)* , $\frac{1}{c}\,\mathbf{S} = \left\{T^{i0}\right\} = [\mathbf{E}\,\mathbf{H}] - \mathrm{C}\mathbf{Э}$,

the stress 3-tensor $-\hat{\sigma} = \left\{T^{ij}\right\}$.

Trace of the base part of the energy-momentum tensor

$$T^{\alpha}_{\alpha} = T^{0}_{0} + T^{i}_{i} = -\left(\mathbf{Э}^2 - \mathrm{C}^2\right) = -\,2\,\mathrm{L}_{\text{SE}} \neq 0\,.$$

Trace of the full energy-momentum tensor

$$T^{\mu}_{\mu} = T^{\alpha}_{\alpha} + T^{5}_{5} = -\frac{1}{2}\left(\mathbf{E}^2 - \mathbf{H}^2 + \mathbf{Э}^2 - \mathrm{C}^2\right) - м^2\left(\varphi^2 - \mathbf{A}^2 + \phi^2\right) = -\,\mathrm{L}_{\text{SEM}} \neq 0\,.$$

The own part of the energy-momentum tensor $T^{\mu\nu}_{\text{own}} = \left(T^{\mu 5}, T^{5\mu}\right)$ has the components:

$$\mathrm{u} = T^{55} = -\frac{1}{2}\left(\mathbf{E}^2 - \mathbf{H}^2 - \mathbf{Э}^2 + \mathrm{C}^2\right) - м^2\left(\varphi^2 - \mathbf{A}^2 + \phi^2\right) = \mathrm{L}_{\text{SE}} - \mathrm{L}_{\text{EM}} - м^2 A_\mu A^\mu \neq 0,$$

virtual: $\mathrm{v} = T^{05} = \mathbf{Э}\mathbf{E}$, $c\,\mathbf{h} = \left\{T^{i5}\right\} = [\mathbf{Э}\,\mathbf{H}] + \mathrm{C}\mathbf{E}$, $\frac{1}{c}\,\mathbf{R} = \left\{T^{5i}\right\} = [\mathbf{Э}\,\mathbf{H}] + \mathrm{C}\mathbf{E}$.

Corollary

The virtual SE-field brings in the negative contribution to the physical total energy and momentum of SEM-field. Thus, for hydrogen atom in an external field the virtual SE-field shifts the observable energy levels and this leads to two additional amendments (see III.8).

11. The origin of an electron virtual mass

The consideration of only the physical EM-field cannot explain the origin of mass, self-energy and momentum of an electron. The stability of an electron cannot be achieved only through physical electromagnetic forces [3,4]. It should also take into account the massive virtual SE-field.

In the space-time $V_{4|5}$ *the virtual mass* m *of an electron has the origin of a massive SEM-field* and is explained by the presence of the virtual self-SEM-field of an electron. The latter corresponds to the nonzero base part of the energy-momentum 5–vector of a massive SEM-field, i.e. , $T^{\mu 5}_{\text{base}} = T^{\alpha 5} \neq 0$.

Since the momentum density of the virtual self-SEM-field of an electron

$\mathbf{h} = \frac{1}{c}\left\{T^{i5}\right\} = \frac{1}{c}\left([\mathbf{Э}\mathbf{H}] + \mathrm{C}\mathbf{E}\right)$, where $c = 1$, then the 3-momentum

$$\boldsymbol{И} = \int \mathbf{h}\; d^3x \;=\; \frac{1}{c}\int\left([\mathbf{Э}\mathbf{H}] + \mathrm{C}\mathbf{E}\right) d^3x\ .$$

In i.r.s., where the electron at rest, $\mathrm{C}' = 0$, $\mathbf{H}' = \mathbf{0}$. From the transformation of the SEM-field strengths (see II.3) it follows that $\mathrm{C} = \frac{1}{c}\,\mathbf{V}\mathbf{Э}$, $\mathbf{H} = \frac{1}{c}\,[\mathbf{V}\mathbf{E}]$.

Then, $\boldsymbol{И} = \frac{1}{c^2}\,\mathbf{V}\int \mathbf{Э}\mathbf{E}\; d^3x \;=\; m\mathbf{V}$, where the virtual mass of an electron

$m \;=\; \frac{1}{c^2}\int \mathbf{Э}\mathbf{E}\; d^3x$, $T^{05} = \mathbf{Э}\mathbf{E}$. Thus, the virtual *self-energy* $mc^2 \;=\; \int \mathbf{Э}\mathbf{E}\; d^3x$.

Also, we have proved the equality $\frac{1}{c}\int T^{\alpha 5}\, d^{3}x = \int j_{m}^{\alpha}\, d^{3}x$, where j_{m}^{α} - the virtual mass current 4-vector (see I.4).

12. Definition of the current 5-vector

For the base part of the energy-momentum 5–vector of a SEM-field $T_{\text{base}}^{\mu 5} = T^{\alpha 5}$ we have the following: $T^{\alpha 5} \neq 0 \;\Rightarrow\; м \neq 0$, and conversely , $T^{\alpha 5} = 0 \;\Leftarrow\; м = 0$, where $м$ - the virtual mass of a quantum SE-field.

We accept the following statement: *the mass current 5-vector* j_{m}^{μ} *has a SEM-origin if the SEM-field is massive.* This statement about the SEM-origin of the mass current is expressed through the equality $j_{m}^{\mu} = T^{\mu 5}$, which takes place if the mass current 4-vector $j_{m}^{\alpha} = T^{\alpha 5} \neq 0$. Then *the charge current 5-vector* $j_{e}^{\mu} = j^{\mu} = \frac{e}{m} T^{\mu 5}$.

Corollaries

α) The source (the charge current) creates only the massive SEM-field since

$j^{\alpha} \neq 0 \;\Rightarrow\; м \neq 0$ or $\not{\rho} \neq 0 \,\vee\, \mathbf{j} \neq 0 \;\Rightarrow\; м \neq 0$, where $\not{\rho} = \frac{\rho}{\varepsilon}$.

β) The massless SEM-field is free of the charge current, that is, $м = 0 \;\Rightarrow\; j^{\alpha} = 0$ or $м = 0 \;\Rightarrow\; \not{\rho} = 0 \,\wedge\, \mathbf{j} = 0$.

Remark

In the space-time V_4 the physical EM-field is massless and long-range. But *in the space-time* $\mathrm{V}_{4|5}$ *the virtual SE-field is massive in the presence of field sources* and , therefore, is not long-range (see II.7, Corollary). In the absence of field sources the virtual SE-field may be massless and long-range.

13. The equation of motion of the charged particle in an external massive SEM-field

$f_{\mathrm{i}}^{\,\mu} = j_\nu F^{\mu\nu} - \text{м}^2 \partial^{\mu}\left(A_\nu A^\nu\right)$, where j^{ν} - the charge current 5-vector, $f_{\mathrm{i}}^{\,\mu}$ - the 5-force acting on the particle with the physical charge e and the virtual mass m .

This particle moves in an external massive SEM-field in the forward direction of time t and time t . Other hand, $f_{\mathrm{i}}^{\,\mu} = \dfrac{d}{ds} T_m^{\mu 5} = \rho_m \dfrac{du^{\mu}}{ds}$, where ρ_m - the physical mass density of a particle (see I.4) and $T_m^{\mu 5}$ - the momentum density 4|5-vector of a particle.

Hence, the physical equations:

$$f_{\mathrm{i}}^{\,0} = \mathbf{j}\,\mathbf{E} + \rho\text{C} - \text{м}^2 \partial^{0}\left(A_\nu A^\nu\right) , \quad \mathbf{f}_{\mathrm{i}} = \not{\rho}\,\mathbf{E} - \rho\text{Э} + [\mathbf{j}\,\mathbf{H}] - \text{м}^2\, \nabla\left(A_\nu A^\nu\right) ,$$

the virtual equation: $f_{\mathrm{i}}^{\,5} = \mathbf{j}\,\text{Э} - \not{\rho}\text{C} - \text{м}^2 \partial^{5}\left(A_\nu A^\nu\right)$. However , $f_{\mathrm{i}}^{\,5} = \rho_m \dfrac{du^5}{ds} = 0$.

Therefore, $0 = j_\nu F^{5\nu} - \text{м}^2 \partial^{5}\left(A_\nu A^\nu\right)$. We can see that $\partial_5\left(A_\nu A^\nu\right) = 0$.

Then, $j_\nu F^{5\nu} = \mathbf{j}\,\text{Э} - \not{\rho}\text{C} = 0$, i.e. $\not{\rho}\text{C} = \mathbf{j}\,\text{Э}$ or $\text{C} = \mathbf{v}\text{Э}$.

On the other hand, the 5-force acting on a moving charge from an external massive SEM-field with the energy-momentum tensor $T^{\mu\nu}$, is equal $f_{\mathrm{f}}^{\,\mu} = \partial_\nu T^{\mu\nu}$.

From the equality $f_{\mathrm{f}}^{\,\mu} = f_{\mathrm{i}}^{\,\mu}$ follows that $f_{\mathrm{f}}^{\,5} = f_{\mathrm{i}}^{\,5} = 0$. That is, $\partial_\nu T^{5\nu} = 0$.

Therefore, in $\text{V}_{4|5}$ the values $P^{50} = \int T^{50}\left(x^{\lambda}\right) d^3x$ and $P^{55} = \int T^{55}\left(x^{\lambda}\right) d^3x$ are conserved in time: $\dfrac{d}{dt} P^{50} = 0$, $\dfrac{d}{dt} P^{55} = 0$.

14. Equations of the virtual SE-waves. The plane SEM-wave

We can obtain the equations of SEM-waves from the equations for a massive SEM-field, when $j^{\mu} = \text{м} = 0$. In particular, the equations of the virtual SE-waves are

$$\operatorname{div} \text{Э} = -\frac{\partial \text{C}}{\partial t} , \quad \operatorname{rot} \text{Э} = \mathbf{0} , \quad \operatorname{grad} \text{C} = -\frac{\partial \text{Э}}{\partial t} .$$

Here below the dot above denotes the differentiation with respect to time t .

Let the propagation direction of a plane SEM-wave $\mathbf{n} // Ox$. We can find i.r.s.

in which $A^0 = \varphi = \text{const} \neq 0$. Then $\mathbf{E} = -\,\dot{\mathbf{A}}$, $\mathbf{H} = \text{rot}\,\mathbf{A}$.

Thus, $\mathbf{E} = [\mathbf{H}\,\mathbf{n}]$, $\mathbf{H} = [\mathbf{n}\,\mathbf{E}]$, $|\,\mathbf{E}\,| = |\,\mathbf{H}\,|$, $\mathbf{n}\,\mathbf{E} = 0$, $\mathbf{n}\,\mathbf{H} = 0$, $\mathbf{E}\,\mathbf{H} = 0$.

That is, *the physical EM-waves are transverse.*

Further, $\mathbf{Э} = -\,\text{grad}\,\phi = \mathbf{n}\,\dot{\phi}$, $\mathrm{C} = \dot{\phi}$.

Thus, $\mathbf{Э} = \mathbf{n}\,\mathrm{C}$, $\mathrm{C} = \mathbf{n}\,\mathbf{Э}$, $\mathbf{Э} // \mathbf{n}$, $|\,\mathrm{C}\,| = |\,\mathbf{Э}\,|$, $\mathbf{Э}\mathbf{E} = 0$, $\mathbf{Э}\mathbf{H} = 0$.

That is, *the virtual SE-waves are longitudinal*.

Since $\mathbf{H}^2 = \mathbf{E}^2$, $\mathrm{C}^2 = \mathbf{Э}^2$, then $T_{55} = 0$ and the energy density of a plane SEM-wave

$T_{00} = \mathbf{E}^2 - \mathbf{Э}^2$. The Poynting vector $\mathbf{S} = [\mathbf{E}\,\mathbf{H}] - \mathrm{C}\,\mathbf{Э} = \mathbf{S}_{\text{EM}} - \mathbf{S}_{\text{SE}} = \mathbf{n}\left(\mathbf{E}^2 - \mathbf{Э}^2\right) = \mathbf{n}\,T_{00}$.

That is, $T_{00} = \mathbf{n}\,\mathbf{S}$.

III. The single-particle wave Dirac equation in the space-time $\mathrm{V}_{4|5}$

1. The Lagrangian for the single-particle wave Dirac equation

Let $\psi_\eta(x)$ - the wave function of a particle with the virtual mass $\eta\, m$ and spin $\frac{1}{2}$.

The value $\eta = \pm\, 1$ corresponds to the forward / backward direction of time t and t ,

to the sign of a particle mass $\pm\, m$ and, therefore, to the sign of an energy-momentum.

For the case of a plane wave $\psi_\eta(x) = u_\eta(p)\, \exp\left(-\, i\,\eta\, p^\mu x_\mu\right)$, where

$p^\mu = \left(E,\, \mathbf{p},\, E_0\right)$, $x^\mu = \left(\mathit{t},\, \mathbf{x},\, t\right) \in \mathrm{V}_{4|5}$.

The full Lagrangian of a particle in the absence of an external SEM-field has the form

$\mathsf{L}_\eta = \frac{i}{2}\left(\bar{\psi}_\eta \gamma^\mu \partial_\mu \psi_\eta - \partial_\mu \bar{\psi}_\eta \gamma^\mu \psi_\eta\right) - m\, \bar{\psi}_\eta \left(1 + \eta\gamma^5\right)\psi_\eta$, the own part of Lagrangian

$\mathsf{L}_\eta^{\text{own}} = \frac{i}{2}\left(\bar{\psi}_\eta \gamma^5 \partial_5 \psi_\eta - \partial_5 \bar{\psi}_\eta \gamma^5 \psi_\eta\right) - \eta\, m\, \bar{\psi}_\eta \gamma^5 \psi_\eta$. Here, γ^μ - the Dirac matrices in the standard representation, satisfying the condition $\gamma^{\mu}\gamma^{\nu} + \gamma^{\nu}\gamma^{\mu} = 2g^{\mu\nu}$, where $g^{\mu\nu}$ - the metric coefficients in V_5 , $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$. Hence, $(\gamma^0)^2 = 1$, $(\gamma^i)^2 = -1$, $(\gamma^5)^2 = 1$.

Remark

To harmonize the following we take that the matrices γ^{α} are virtual, γ^5 is physical .

2. The single-particle wave Dirac equation in the absence of an external SEM-field

From the system of Lagrangians L_{η} and L_{η}^{own} we obtain the system of two equations

$$\left[\gamma^{\mu}\hat{p}_{\mu} - \left(1 + \eta\gamma^5\right)m\right]\psi_{\eta}(x) = 0, \tag{1}$$

$$\gamma^5\left(\hat{p}_5 - \eta m\right)\psi_{\eta}(x) = 0, \tag{2}$$

where $\hat{p}_{\mu} = i\partial_{\mu}$, or in the difference of the equations (1) and (2)

$$\left(\gamma^{\alpha}\hat{p}_{\alpha} - m\right)\psi_{\eta}(x) = 0, \tag{3}$$

$$\gamma^5\left(\hat{p}_5 - \eta m\right)\psi_{\eta}(x) = 0. \tag{2}$$

The equation (3) is the classical wave Dirac equation in the space-time V_4. Solutions of the equation (3) are the particle states $\psi_{\eta}(x)$ with two values $\eta = \pm 1$. That is, the classical wave Dirac equation is the two-particle equation [5].

However, $\{(1),(2)\}$ and $\{(3),(2)\}$ are the single-particle systems. That is, their solutions are the particle states $\psi_{\eta}(x)$ that correspond to the definite value η in the operator part. The system $\{(1),(2)\}$ we will call *the single-particle wave Dirac equation for a particle with the mass* ηm *and spin* $\frac{1}{2}$ *in the absence of an external SEM-field in the space-time* $V_{4|5}$.

3. Second-order equations

The squares of operators of the system $\{(1),(2)\}$ lead to the system $\{(4),(5)\}$ for which the solutions are also the single-particle states $\psi_\eta(x)$ with a definite value η

$$\hat{p}_\mu \hat{p}^\mu \psi_\eta = 2\left(1+\eta\gamma^5\right) m^2 \psi_\eta \ , \tag{4}$$

$$\hat{p}_5 \hat{p}^5 \psi_\eta = m^2 \psi_\eta \ . \tag{5}$$

Half of the sum of operators of the equation (4) for two values $\eta = +1$ and $\eta = -1$ leads to the system

$$\hat{p}_\mu \hat{p}^\mu \psi_\eta = 2m^2 \psi_\eta \ ,$$

$$\hat{p}_5 \hat{p}^5 \psi_\eta = m^2 \psi_\eta \ ,$$

from which in the difference of equations we obtain the system

$$\hat{p}_\alpha \hat{p}^\alpha \psi_\eta = m^2 \psi_\eta \ ,$$

$$\hat{p}_5 \hat{p}^5 \psi_\eta = m^2 \psi_\eta \ .$$

The latter two systems are two-particle systems. That is, their solutions are the states with two values $\eta = \pm 1$ corresponding to two signs of an energy-momentum.

4. The electron-positron vacuum. The single-particle wave Dirac equation in the presence of an external massless SEM-field. The polarization of vacuum

The classical wave Dirac equation in the space-time V_4 is the two-particle equation. The single particle equation obtained from it is considered only in the absence of an external EM-field and with “ the Dirac vacuum “ in which all states with negative energies are employed [5]. Dirac proposed the theory of holes which describes particles of both signs of charge in the presence of an external EM-field. This theory denotes the passage to many-particle quantum field theory. In this case, the solution of the wave Dirac equation has no simple probabilistic interpretation, since it must describe the processes of creation and annihilation of electron-positron pairs [6].

The single-particle wave Dirac equation in the space-time $V_{4|5}$ may be considered in the presence of an external SEM-field and allows predict the phenomenon of vacuum polarization. In this case, the wave function of a single particle retains the simple probabilistic interpretation.

Let us consider the single-particle system $\{(1),(2)\}$ for $\eta = +1$ corresponding to the positive energy of a particle. If we accept that this system describes the vacuum of electron /positron states, then in the presence of an external massless SEM-field with 5-potential $A^{\mu}(x)$ this system may be written in the form:

$$\gamma^{\mu}\left[2\hat{p}_{\mu} - (+e-e)A_{\mu}\right]\psi_{\eta} = 2\left(1+\eta\gamma^{5}\right)m\psi_{\eta} \ ,$$

$$\left[2\hat{p}_{5} - (+e-e)A_{5}\right]\psi_{\eta} = 2\eta\gamma^{5}m\psi_{\eta} \ ,$$

where $\pm e$ - the electron / positron charge. In the case of a plane wave

$$\psi_{\eta}(x) = u_{\eta}(p)\exp\left(-i\eta p^{\mu}x_{\mu}\right) =$$

$$= \psi^{\eta}_{+e}(x)\exp\left[-i(-e)A^{\mu}x_{\mu}\right] = \psi^{\eta}_{-e}(x)\exp\left[-i(+e)A^{\mu}x_{\mu}\right] , \quad \text{where}$$

$$\psi^{\eta}_{\pm e}(x) = \psi_{\eta}(x)\exp\left[-i(\pm e)A^{\mu}x_{\mu}\right] = u_{\eta}(p)\exp\left\{-i\left[\eta p^{\mu} + (\pm e)A^{\mu}\right]x_{\mu}\right\}.$$

Thus, the original system $\{(1),(2)\}$ on the one hand describes the electron\positron vacuum, and on the other hand in the presence of an external SEM-field this one is the sum of two single-particle systems. In this case, $\psi_{\eta}(x)$ is the solution of the original system if and only if $\psi^{\eta}_{\pm e}(x)$ are the solutions of the single-particle systems with the appropriate charge sign

$$\gamma^{\mu}\left[\hat{p}_{\mu}-(\pm e)A_{\mu}\right]\psi^{\eta}_{\pm e}=\left(1+\eta\gamma^{5}\right)m\psi^{\eta}_{\pm e}\ , \qquad (6)^{\pm}$$

$$\gamma^{5}\left[\hat{p}_{5}-(\pm e)A_{5}\right]\psi^{\eta}_{\pm e}=\eta\gamma^{5}m\psi^{\eta}_{\pm e}\ . \qquad (7)^{\pm}$$

Each of the two systems $\{(6)^{\pm},(7)^{\pm}\}$ is the single-particle Dirac equation in the space-time $V_{4|5}$, respectively for the electron\positron in the presence of an external SEM-field with 5-potential $A^{\mu}(x)$. The sign of the electron\positron energy is determined by the value η. Thus, in the space-time $V_{4|5}$ an external SEM-field induces the appearance of single-particle electron\positron states in vacuum (vacuum polarization).

5. CM – symmetry of the single-particle Dirac equation

The single-particle Dirac equation as the system $\{(6)^{\pm},(7)^{\pm}\}$ possesses the property of **CM**-symmetry relatively 2-operation:

1) **C** - inversion of the charge sign: $\pm e \rightarrow \mp e$,

2) **M** - inversion of the mass sign: $\eta=\pm 1 \rightarrow \overline{\eta}=\mp 1$, i.e. the sign inversion of the energy-momentum.

6. The spin 5-tensor

Let in the standard representation of the Dirac matrices: $\gamma^{5}=i\gamma^{0}\gamma^{1}\gamma^{2}\gamma^{3}$, $\left(\gamma^{5}\right)^{2}=1$, $\boldsymbol{\alpha}=\gamma^{0}\boldsymbol{\gamma}$. In addition, we define the following matrices: $\phi=i\gamma^{0}\gamma^{5}=-\gamma^{1}\gamma^{2}\gamma^{3}$, $(\phi)^{2}=1$, $\boldsymbol{\beta}=\gamma^{5}\boldsymbol{\gamma}$. Then the spin 5-tensor $\sigma^{\mu\nu}=\frac{i}{2}\left[\gamma^{\mu},\gamma^{\nu}\right]$ in the matrix form

$$\sigma^{\mu\nu}=(i\boldsymbol{\alpha},-\boldsymbol{\sigma},\phi,-i\boldsymbol{\beta})=\begin{pmatrix} 0 & i\alpha_1 & i\alpha_2 & i\alpha_3 & \phi \\ -i\alpha_1 & 0 & \sigma_3 & -\sigma_2 & -i\beta_1 \\ -i\alpha_2 & -\sigma_3 & 0 & \sigma_1 & -i\beta_2 \\ -i\alpha_3 & \sigma_2 & -\sigma_1 & 0 & -i\beta_3 \\ -\phi & i\beta_1 & i\beta_2 & i\beta_3 & 0 \end{pmatrix},\quad \text{where}$$

the matrices $\boldsymbol{\alpha},\boldsymbol{\sigma}$ are physical, $\phi,\boldsymbol{\beta}$ are virtual.

7. The angular momentum 5-tensor

$\Lambda^{\mu\nu} = i\left(x^{\mu}\partial^{\nu} - x^{\nu}\partial^{\mu}\right) + \frac{1}{2}\sigma^{\mu\nu}$ or in the matrix form

$$\Lambda^{\mu\nu} = -\left(\mathbf{K},\, \mathbf{J},\, \mathrm{O},\, -\mathbf{N}\right) = -\begin{pmatrix} 0 & K_1 & K_2 & K_3 & \mathrm{O} \\ -K_1 & 0 & -J_3 & J_2 & -N_1 \\ -K_2 & J_3 & 0 & -J_1 & -N_2 \\ -K_3 & -J_2 & J_1 & 0 & -N_3 \\ -\mathrm{O} & N_1 & N_2 & N_3 & 0 \end{pmatrix}, \quad \text{where}$$

$$\mathbf{K} = i\left(\mathbf{r}\,\frac{\partial}{\partial t} + t\,\vec{\nabla}\right) - \frac{1}{2}\,i\,\boldsymbol{\alpha}\,, \qquad \mathbf{N} = i\left(\mathbf{r}\,\frac{\partial}{\partial t} + t\,\vec{\nabla}\right) - \frac{1}{2}\,i\,\boldsymbol{\beta}\,,$$

$$\mathbf{J} = \frac{1}{i}\left[\mathbf{r}\,\vec{\nabla}\right] + \frac{1}{2}\,\boldsymbol{\sigma}\,, \qquad \mathrm{O} = \frac{1}{i}\left(t\,\frac{\partial}{\partial t} - t\,\frac{\partial}{\partial t}\right) - \frac{1}{2}\,\boldsymbol{\gamma}\,.$$

Here, the operators $\mathbf{K}$, $\mathbf{J}$ are physical, O, $\mathbf{N}$ are virtual. $T = \frac{1}{2}\boldsymbol{\gamma}$ is the 1-dimensional operator of the virtual temporal spin of an electron, and that has the eigenvalues $\pm\frac{1}{2}$.

The nonzero components of the 5-tensor $\Lambda^{\mu\nu}$: $\Lambda^{\mu\nu}_{\text{base}} = \Lambda^{\alpha\beta} = -\left(\mathbf{K},\, \mathbf{J}\right)$ - the 4-tensor and $\Lambda^{\mu\nu}_{\text{own}} = \left(\Lambda^{\mu 5},\, \Lambda^{5\mu}\right)$. The base part of the 5-vector $\Lambda^{\mu 5}$ is the 4-vector $\Lambda^{\mu 5}_{\text{base}} = \Lambda^{\alpha 5} = \left(\mathrm{O},\, \mathbf{N}\right)$.

8. The equation of second order for an electron with the positive energy in an external massless SEM-field

The classical wave Dirac equation in the space-time V_4 predicts the energy levels of hydrogen atom in an external EM-field without additional amendments on the Lamb shift and hyperfine structure [5]. The single-particle wave Dirac equation in the space-time $\mathrm{V}_{4|5}$ predicts two additional amendments for the energy levels of hydrogen atom in an external SEM-field.

We consider the system of equations $\{(6)^{+}, (7)^{+}\}$ for an electron with the positive energy, that is, when $\eta = +1$

$$\left[\gamma^{\mu}\left(\hat{p}_{\mu}-eA_{\mu}\right)-\left(1+\gamma^{5}\right)m\right]\psi_{e}=0\ , \tag{6$^+$}$$

$$\gamma^{5}\left[\left(\hat{p}_{5}-eA_{5}\right)-m\right]\psi_{e}=0\ . \tag{7$^+$}$$

Applying the adjoint operator to each equation, we obtain the system

$$\left[\left(\hat{p}_{\mu}-eA_{\mu}\right)\left(\hat{p}^{\mu}-eA^{\mu}\right)-\frac{e}{2}\,\sigma^{\mu\nu}F_{\mu\nu}-2\left(1+\gamma^{5}\right)m^{2}\right]\psi_{e}=0\ , \tag{8}$$

$$\left[\left(\hat{p}_{5}-eA_{5}\right)\left(\hat{p}^{5}-eA^{5}\right)-m^{2}\right]\psi_{e}=0\ . \tag{9}$$

In the difference of equations (8) and (9) we obtain the system

$$\left[\left(\hat{p}_{\alpha}-eA_{\alpha}\right)\left(\hat{p}^{\alpha}-eA^{\alpha}\right)-\frac{e}{2}\,\sigma^{\mu\nu}F_{\mu\nu}-\left(1+2\gamma^{5}\right)m^{2}\right]\psi_{e}=0\ , \tag{10}$$

$$\left[\left(\hat{p}_{5}-eA_{5}\right)\left(\hat{p}^{5}-eA^{5}\right)-m^{2}\right]\psi_{e}=0\ . \tag{9}$$

The system $\{(10),(9)\}$ in the differential form (11)

$$\left\{\left(i\frac{\partial}{\partial t}-e\varphi\right)^{2}-\left(i\vec{\nabla}+e\mathbf{A}\right)^{2}-\left(1+2\gamma^{5}\right)m^{2}+e\left[\boldsymbol{\sigma}\mathbf{H}-\boldsymbol{\phi}\mathrm{C}-i\left(\boldsymbol{\alpha}\mathbf{E}+\boldsymbol{\beta}\text{Э}\right)\right]\right\}\psi_{e}=0\ ,$$

$$\left[\left(i\frac{\partial}{\partial t}-e\phi\right)^{2}-m^{2}\right]\psi_{e}=0\ . \tag{12}$$

In the case of a plane wave : $\psi_{e}(x)=u(p)\exp\left[-i\left(p^{\mu}+eA^{\mu}\right)x_{\mu}\right]=$

$$=u(p)\exp\left\{-i\left[(E+e\varphi)\,t-(\mathbf{p}+e\mathbf{A})\mathbf{x}+(E_{0}+e\phi)\,t\ \right]\right\}.$$

The second and fourth physical terms in square brackets in the equation (11) appear as a result of interaction of the electron with the external virtual SE-field. It can suppose that these terms correspond to two additional amendments for the energy levels of hydrogen atom in an external SEM-field. The second term takes into account *“the virtual temporal spin”* of an electron.

9. The canonical energy-momentum tensor. Conservation equations. Charge and mass current 5-vectors of an electron.

From the Lagrangian L_η for $\eta = +1$ we obtain the energy-momentum 5-tensor

$T^{\mu\nu} = \frac{i}{2}\left(\overline{\psi}\gamma^\mu \partial^\nu\psi - \partial^\nu\overline{\psi}\,\gamma^\mu\psi\right)$ for which $\partial_\nu T^{\mu\nu} = 0$. From here it follows that in $V_{4|5}$ the values $P^{\mu 0} = \int T^{\mu 0}\left(x^\lambda\right) d^3x$ and $P^{\mu 5} = \int T^{\mu 5}\left(x^\lambda\right) d^3x$, where $x^\lambda \in V_{4|5}$, are conserved in time: $\frac{d}{dt}P^{\mu 0} = 0$, $\frac{d}{dt}P^{\mu 5} = 0$.

From the equation (2) and conjugate equation we obtain the momentum density 5-vector $T^{\mu 5} = \frac{i}{2}\left(\overline{\psi}\gamma^\mu \partial^5\psi - \partial^5\overline{\psi}\,\gamma^\mu\psi\right) = m\,\overline{\psi}\gamma^\mu\psi = j_m^\mu$, where j_m^μ - the mass current 5-vector. Then the charge current 5-vector $j_e^\mu = \frac{e}{m}T^{\mu 5} = e\,\overline{\psi}\gamma^\mu\psi$ and $\partial_\mu T^{\mu 5} = 0$. From this it follows that in $V_{4|5}$ the values $P^{05} = \int T^{05}\left(x^\lambda\right) d^3x$ and $P^{55} = \int T^{55}\left(x^\lambda\right) d^3x$ are conserved in time: $\frac{d}{dt}P^{05} = 0$, $\frac{d}{dt}P^{55} = 0$.

From L_η^{own} we obtain $T^{55} = \frac{i}{2}\left(\overline{\psi}\gamma^5 \partial^5\psi - \partial^5\overline{\psi}\,\gamma^5\psi\right)$ or $T^{55} = m\,\overline{\psi}\gamma^5\psi = j_m^5$.

That is, T^{55} is the scalar density of the particle mass and $\partial_5 T^{55} = 0$.

10. The physical and virtual electron charge and mass

The physical electron charge $Q_0 = \int j_e^0\left(x^\lambda\right) d^3x = e\int \overline{\psi}\gamma^0\psi\, d^3x = e$,

and is conserved in time t: $\frac{d}{dt}Q_0 = 0$.

Since $j_e^5 = e\,\overline{\psi}\gamma^5\psi$ is the pseudo-scalar, then the virtual electron charge $Q_5 = e\int \left|\overline{\psi}\gamma^5\psi\right| d^3x$, and is conserved in time t : $\frac{d}{dt}Q_5 = 0$. It can be assumed that the virtual electron charge Q_5 is *the "bare" charge* e_0 *of an electron.* That is,

$Q_5 = e_0$ and $|Q_5| > |Q_0|$ [6].

The virtual electron mass $\mathrm{M}_0 = \int j_m^0(x^\lambda)\, d^3x = m \int \bar{\psi}\gamma^0\psi\, d^3x = m$ is conserved in time t: $\frac{d}{dt}\mathrm{M}_0 = 0$. Then the physical electron mass $\mathrm{M}_5 = m \int \left|\bar{\psi}\gamma^5\psi\right| d^3x$, and is conserved in time t : $\frac{d}{dt}\mathrm{M}_5 = 0$. It can be assumed that the virtual electron mass $\mathrm{M}_0 = m$ is *the "bare" mass* m_0 *of an electron.* That is, $\mathrm{M}_0 = m_0$ and $|\mathrm{M}_5| > |\mathrm{M}_0|$.

Conclusion

With inclusion of the virtual proper time in the metric of Minkowski space we passed to the four-dimensional bimetric space-time. As a result, we operated with physical (observable) and virtual (unobservable) objects entering into the physical expressions like how with real and imaginary numbers.

Introduction of virtual objects supplementing physical objects ensured the completeness of description of the occurring physical processes and led us to the following results.

1. The consistent and closed electrodynamics was built for the scalar-electromagnetic field (the SEM-field). The virtual scalar-electric field (the SE-field) is massive in the presence of field sources. The virtual mass protects from the infrared catastrophe in QED. The physical EM-field is massless. It was shown that the mass, self-energy and current for an electron have the origin of the massive SEM-field .

2. In classical quantum theory it was found a form of the single-particle wave Dirac equation for which the electron wave function retains the simple probabilistic interpretation. It was shown that in the single-particle Dirac theory the virtual scalar-electric field shifts the physical energy levels and this leads to two additional amendments for the hydrogen atom in an external field.

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