A Rigorous Derivation of the Lorentz Transformations

It's difficult to find a derivation of the Lorentz transformations beyond a single dimension in rectangular Cartesian coordinates. Here, a rigorous general derivation is established.

An infinitesimal line element (differential) is expressed as:

$$\mathbf{r} \equiv \mathbf{r}(x^0, x^1, x^2, x^3) = \sum_{i=0}^{3} \boldsymbol{\alpha}_i r^i$$

$\therefore$

$$d\mathbf{r} \equiv \sum_{i=0}^{3} \frac{\partial \mathbf{r}}{\partial x^i} dx^i \Rightarrow \boldsymbol{\alpha}_i \equiv \frac{\partial \mathbf{r}}{\partial x^i}$$

then:

$$ds^2 \equiv d\mathbf{r} \cdot d\mathbf{r} = \left(\sum_{i=0}^{3} \boldsymbol{\alpha}_i dx^i \right) \cdot \left(\sum_{i=0}^{3} \boldsymbol{\alpha}_i dx^i \right)$$

$$= \sum_{i=0}^{3} \sum_{j=0}^{3} (\boldsymbol{\alpha}_i \cdot \boldsymbol{\alpha}_j) dx^i dx^j$$

so with: $g_{ij} \equiv \boldsymbol{\alpha}_i \cdot \boldsymbol{\alpha}_j$

$$ds^2 \equiv \sum_{i=0}^{3} g_{ij} dx^i dx^j$$

This is an invariant, unchanged in any coordinate system/reference frame - the g_{ij} specified for each coordinate system/reference frame.

In a specific coordinate system/reference frame type where the time ans space portions of this invariant may be seperated as follows:

$$ds^2 = c^2 dt dt - \sum_{i=1}^{3} g_{ij} dx^i dx^j$$

Defining velocity in a coordinate system/reference frame as follows:

$$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} \quad , \quad \vec{\mathbf{v}}' = \frac{d\vec{\mathbf{r}}'}{dt'}$$

$\vec{\mathbf{v}} = \mathbf{0}$, if and only if: $\vec{\mathbf{v}}' = \vec{\mathbf{u}}'$ (the initial velocity, measured in **K**')

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$$x^i = x^i(x^{1'}, x^{2'}, \ldots, x^{n'}) \quad , \quad \forall i \in \{ x \in N \mid 1 \le x \le n \}$$

$$x^{k'} = x^{k'}(x^1, x^2, \ldots, x^n) \quad , \quad \forall k \in \{ x \in N \mid 1 \le x \le n \}$$

so, (hereafter, using the Einstein summation convention)

$$c^2 dt^2 - g_{ij} dx^i dx^j = c^2 dt'^2 - g'_{kl} dx^{k'} dx^{l'}$$

$\therefore$

$$dx^i = \frac{\partial x^i}{\partial x^{k'}} dx^{k'} + \frac{\partial x^i}{\partial t'} dt' \quad , \quad dt = \frac{\partial t}{\partial x^{k'}} dx^{k'} + \frac{\partial t}{\partial t'} dt'$$

so:

$$c^2 \left(\frac{\partial t}{\partial x^{k'}} dx^{k'} + \frac{\partial t}{\partial t'} dt' \right)^2 - g_{ij} \left(\frac{\partial x^i}{\partial x^{k'}} dx^{k'} + \frac{\partial x^i}{\partial t'} dt' \right) \left(\frac{\partial x^j}{\partial x^{l'}} dx^{l'} + \frac{\partial x^j}{\partial t'} dt' \right) =$$

$$= c^2 dt'^2 - g'_{kl} dx^{k'} dx^{l'}$$

$\therefore$

$$\left(c^2 \frac{\partial t}{\partial x^{k'}} \frac{\partial t}{\partial x^{l'}} - g_{ij} \frac{\partial x^i}{\partial x^{k'}} \frac{\partial x^j}{\partial x^{l'}} + g'_{kl} \right) dx^{k'} dx^{l'} +$$

$$+ \left(2c^2 \frac{\partial t}{\partial x^{k'}} \frac{\partial t}{\partial t'} - 2g_{ij} \frac{\partial x^i}{\partial x^{k'}} \frac{\partial x^j}{\partial t'} \right) dx^{k'} dt' +$$

$$+ \left(c^2 \frac{\partial t}{\partial t'} \frac{\partial t}{\partial t'} - g_{ij} \frac{\partial x^i}{\partial t'} \frac{\partial x^j}{\partial t'} - c^2 \right) dt' dt' = 0$$

so:

$$c^2 \frac{\partial t}{\partial x^{k'}} \frac{\partial t}{\partial x^{l'}} - g_{ij} \frac{\partial x^i}{\partial x^{k'}} \frac{\partial x^j}{\partial x^{l'}} + g'_{kl} = 0$$

$$c^2 \frac{\partial t}{\partial x^{k'}} \frac{\partial t}{\partial t'} - g_{ij} \frac{\partial x^i}{\partial x^{k'}} \frac{\partial x^j}{\partial t'} = 0$$

$$c^2 \frac{\partial t}{\partial t'} \frac{\partial t}{\partial t'} - g_{ij} \frac{\partial x^i}{\partial t'} \frac{\partial x^j}{\partial t'} - c^2 = 0$$

Velocity components may be expressed, as follows:

$$\frac{dx^i}{dt} = v^i = \frac{\frac{\partial x^i}{\partial x^{k'}} \frac{dx^{k'}}{dt'} + \frac{\partial x^i}{\partial t'}}{\frac{\partial t}{\partial x^{k'}} \frac{dx^{k'}}{dt'} + \frac{\partial t}{\partial t'}} = \frac{\frac{\partial x^i}{\partial x^{k'}} v^{k'} + \frac{\partial x^i}{\partial t'}}{\frac{\partial t}{\partial x^{k'}} v^{k'} + \frac{\partial t}{\partial t'}}$$

$$\frac{dx^{k'}}{dt'} = v^{k'} = \frac{\frac{\partial x^{k'}}{\partial x^j}\frac{dx^j}{dt} + \frac{\partial x^i}{\partial t}}{\frac{\partial t'}{\partial x^j}\frac{dx^j}{dt} + \frac{\partial t'}{\partial t}} = \frac{\frac{\partial x^{k'}}{\partial x^j}v^j + \frac{\partial x^{k'}}{\partial t}}{\frac{\partial t'}{\partial x^j}v^j + \frac{\partial t'}{\partial t}}$$

so:

$$u^i = \frac{\frac{\partial x^i}{\partial t'}}{\frac{\partial t}{\partial t'}} \ ,\ 0 = \frac{\frac{\partial x^{k'}}{\partial x^j}u^j + \frac{\partial x^{k'}}{\partial t}}{\frac{\partial t'}{\partial x^j}u^j + \frac{\partial t'}{\partial t}} \ ,\ u^{k'} = \frac{\frac{\partial x^{k'}}{\partial t}}{\frac{\partial t'}{\partial t}} \ ,\ 0 = \frac{\frac{\partial x^j}{\partial x^{k'}}u^{k'} + \frac{\partial x^j}{\partial t'}}{\frac{\partial t}{\partial x^{k'}}u^{k'} + \frac{\partial t}{\partial t'}}$$

∴

$$\frac{\partial x^i}{\partial t'} = u^i\frac{\partial t}{\partial t'} \quad , \quad \frac{\partial x^j}{\partial x^{k'}}u^{k'} = -\frac{\partial x^j}{\partial t'}$$

$$\frac{\partial x^{k'}}{\partial t} = u^{k'}\frac{\partial t'}{\partial t} \quad , \quad \frac{\partial x^{k'}}{\partial x^j}u^j = -\frac{\partial x^{k'}}{\partial t}$$

so:

$$c^2\frac{\partial t}{\partial t'}\frac{\partial t}{\partial t'} - g_{ij}u^iu^j\frac{\partial t}{\partial t'}\frac{\partial t}{\partial t'} - c^2 = 0$$

$$\left(c^2 - |\vec{\mathbf{u}}|^2\right)\frac{\partial t}{\partial t'}\frac{\partial t}{\partial t'} - c^2 = 0 \quad , \quad \left(|\vec{\mathbf{u}}|^2 \equiv g_{ij}u^iu^j\right)$$

∴

$$\boxed{\frac{\partial t}{\partial t'} = \frac{1}{\sqrt{1 - \frac{|\vec{\mathbf{u}}|^2}{c^2}}} \equiv \gamma}$$

so:

$$\frac{\partial x^i}{\partial t'} = u^i\gamma$$

and:

$$c^2\frac{\partial t}{\partial x^{k'}}\gamma - g_{ij}\frac{\partial x^i}{\partial x^{k'}}u^j\gamma = 0 \quad , so \quad \frac{\partial t}{\partial x^{k'}} = g_{ij}\frac{\partial x^i}{\partial x^{k'}}\frac{u^j}{c^2}$$

∴

$$\frac{\partial t}{\partial x^{k'}}u^{k'} = g_{ij}\frac{\partial x^i}{\partial x^{k'}}u^{k'}\frac{u^j}{c^2} = -g_{ij}\frac{\partial x^i}{\partial t'}\frac{u^j}{c^2} = -g_{ij}u^i\gamma\frac{u^j}{c^2} = -\gamma\frac{|\vec{\mathbf{u}}|^2}{c^2}$$

and:

$$c^2\frac{\partial t}{\partial x^{k'}}\gamma - g_{ij}\frac{\partial x^i}{\partial x^{k'}}u^j\gamma = 0$$

∴

$$g_{ij}\frac{\partial x^i}{\partial x^{k'}}u^j = c^2\frac{\partial t}{\partial x^{k'}}$$

but:

$$c^2\frac{\partial t}{\partial x^{k'}}\frac{\partial t}{\partial x^{l'}}u^{l'} - g_{ij}\frac{\partial x^i}{\partial x^{k'}}\frac{\partial x^j}{\partial x^{l'}}u^{l'} + g'_{kl}u^{l'} = 0$$

∴

$$c^2\frac{\partial t}{\partial x^{k'}}\left(-\gamma\frac{|\vec{\mathbf{u}}|^2}{c^2}\right) - g_{ij}\frac{\partial x^i}{\partial x^{k'}}\left(-\frac{\partial x^j}{\partial t'}\right) + g'_{kl}u^{l'} = 0$$

∴

$$c^2\frac{\partial t}{\partial x^{k'}}\left(-\gamma\frac{|\vec{\mathbf{u}}|^2}{c^2}\right) + \left(g_{ij}\frac{\partial x^i}{\partial x^{k'}}u^j\right)\gamma + g'_{kl}u^{l'} = 0$$

∴

$$c^2\frac{\partial t}{\partial x^{k'}}\left(-\gamma\frac{|\vec{\mathbf{u}}|^2}{c^2}\right) + \left(c^2\frac{\partial t}{\partial x^{k'}}\right)\gamma + g'_{kl}u^{l'} = 0$$

∴

$$\frac{\partial t}{\partial x^{k'}}\left\{-\gamma|\vec{\mathbf{u}}|^2 + \gamma c^2\right\} + g'_{kl}u^{l'} = 0$$

∴

$$\frac{\partial t}{\partial x^{k'}}\gamma c^2\left(1 - \frac{|\vec{\mathbf{u}}|^2}{c^2}\right) = -g'_{kl}u^{l'}$$

so:

$$\frac{\partial t}{\partial x^{k'}} = -g'_{kl}\frac{u^{l'}}{c^2}\gamma$$

and so:

$$c^2\left(-g'_{kl}\frac{u^{l'}}{c^2}\gamma\right)(\gamma) - g_{ij}\frac{\partial x^i}{\partial x^{k'}}(u^j\gamma) = 0$$

∴

$$g_{ij}\frac{\partial x^i}{\partial x^{k'}}u^j = -g'_{kl}u^{l'}\gamma$$

$$c^2\left(-g'_{kr}\frac{u^{r'}}{c^2}\gamma\right)\left(-g'_{sl}\frac{u^{s'}}{c^2}\gamma\right) - g_{ij}\frac{\partial x^i}{\partial x^{k'}}\frac{\partial x^j}{\partial x^{l'}} + g'_{kl} = 0$$

$$g_{ij}\frac{\partial x^i}{\partial x^{k'}}\frac{\partial x^j}{\partial x^{l'}} = g'_{kl} + g'_{kr}g'_{sl}\frac{u^{r'}}{c}\frac{u^{s'}}{c}\gamma^2$$

$$g_{ij}\frac{\partial x^i}{\partial x^{k'}}\frac{\partial x^j}{\partial x^{l'}}u^{l'} = g'_{kl}u^{l'} + g'_{kr}g'_{sl}\frac{u^{r'}}{c}\frac{u^{s'}}{c}\gamma^2u^{l'}$$

$$= -g_{ij}\frac{\partial x^i}{\partial x^{k'}}u^j\gamma = g'_{kl}u^{l'}\gamma^2$$

$$u^{l'}\left\{g'_{kl}\gamma^2 - g'_{kl} - g'_{kr}g'_{sl}\frac{u^{r'}}{c}\frac{u^{s'}}{c}\gamma^2\right\} = 0$$

this system of N equations in N variables ($u^{l'}$) is
NOT Linearly Independant.
for N=3:

$$\forall k = 1,2,3\ :$$

$$u^{1'}\{g'_{k1}\gamma^2 - g'_{k1} - g'_{kr}g'_{1s}\frac{u^{r'}}{c}\frac{u^{s'}}{c}\gamma^2\} +$$
$$+\, u^{2'}\{g'_{k2}\gamma^2 - g'_{k2} - g'_{kr}g'_{2s}\frac{u^{r'}}{c}\frac{u^{s'}}{c}\gamma^2\} +$$
$$+\, u^{3'}\{g'_{k3}\gamma^2 - g'_{k3} - g'_{kr}g'_{3s}\frac{u^{r'}}{c}\frac{u^{s'}}{c}\gamma^2\} = 0$$

$$k = 1\ :$$
$$u^{1'}\{\gamma^2 - 1 - \frac{u^{1'}}{c}\frac{u^{1'}}{c}\gamma^2\} + u^{2'}\{-\frac{u^{1'}}{c}\frac{u^{2'}}{c}\gamma^2\} + u^{3'}\{-\frac{u^{1'}}{c}\frac{u^{3'}}{c}\gamma^2\} = 0$$

$$k = 2\ :$$
$$u^{1'}\{-\frac{u^{2'}}{c}\frac{u^{1'}}{c}\gamma^2\} + u^{2'}\{\gamma^2 - 1 - \frac{u^{2'}}{c}\frac{u^{2'}}{c}\gamma^2\} + u^{3'}\{-\frac{u^{2'}}{c}\frac{u^{3'}}{c}\gamma^2\} = 0$$

$$k = 3\ :$$
$$u^{1'}\{-\frac{u^{3'}}{c}\frac{u^{1'}}{c}\gamma^2\} + u^{2'}\{-\frac{u^{3'}}{c}\frac{u^{2'}}{c}\gamma^2\} + u^{3'}\{\gamma^2 - 1 - \frac{u^{1'}}{c}\frac{u^{3'}}{c}\gamma^2\} = 0$$

Note: $1 = \gamma^2\left(1 - \frac{|\vec{\mathbf{u}}|^2}{c^2}\right)$

$\therefore$

$$\gamma^2\frac{|\vec{\mathbf{u}}|^2}{c^2} = \gamma^2 - 1 = (\gamma+1)(\gamma-1)$$

so:

$$\left(\frac{\gamma^2}{\gamma+1}\right)\frac{|\vec{\mathbf{u}}|^2}{c^2} = \gamma - 1 \quad ,so:\quad \left(\frac{\gamma^2}{\gamma+1}\right)\frac{|\vec{\mathbf{u}}|^2}{c^2} + 2 = \gamma + 1$$

let $\Delta \equiv \begin{vmatrix} \left(-\frac{u^{1'}}{c}\frac{u^{1'}}{c}\gamma^2 - 1 - \gamma^2\right) & \left(-\frac{u^{2'}}{c}\frac{u^{1'}}{c}\gamma^2\right) & \left(-\frac{u^{3'}}{c}\frac{u^{1'}}{c}\gamma^2\right) \\ \left(-\frac{u^{1'}}{c}\frac{u^{2'}}{c}\gamma^2\right) & \left(-\frac{u^{2'}}{c}\frac{u^{2'}}{c}\gamma^2 - 1 - \gamma^2\right) & \left(-\frac{u^{3'}}{c}\frac{u^{2'}}{c}\gamma^2\right) \\ \left(-\frac{u^{1'}}{c}\frac{u^{3'}}{c}\gamma^2\right) & \left(-\frac{u^{2'}}{c}\frac{u^{3'}}{c}\gamma^2\right) & \left(-\frac{u^{3'}}{c}\frac{u^{3'}}{c}\gamma^2 - 1 - \gamma^2\right) \end{vmatrix}$

$$= (-\frac{u^{1'}}{c}\frac{u^{1'}}{c}\gamma^2 - 1 - \gamma^2)\{(-\frac{u^{2'}}{c}\frac{u^{2'}}{c}\gamma^2 - 1 - \gamma^2)(-\frac{u^{3'}}{c}\frac{u^{3'}}{c}\gamma^2 - 1 - \gamma^2) - (-\frac{u^{2'}}{c}\frac{u^{3'}}{c}\gamma^2)(-\frac{u^{3'}}{c}\frac{u^{2'}}{c}\gamma^2)\} +$$
$$-\,(-\frac{u^{2'}}{c}\frac{u^{1'}}{c}\gamma^2)\{(-\frac{u^{1'}}{c}\frac{u^{2'}}{c}\gamma^2)(-\frac{u^{3'}}{c}\frac{u^{3'}}{c}\gamma^2 - 1 - \gamma^2) - (-\frac{u^{1'}}{c}\frac{u^{3'}}{c}\gamma^2)(-\frac{u^{3'}}{c}\frac{u^{2'}}{c}\gamma^2)\} +$$
$$+\,(-\frac{u^{3'}}{c}\frac{u^{1'}}{c}\gamma^2)\{(-\frac{u^{1'}}{c}\frac{u^{2'}}{c}\gamma^2)(-\frac{u^{2'}}{c}\frac{u^{3'}}{c}\gamma^2) - (-\frac{u^{1'}}{c}\frac{u^{3'}}{c}\gamma^2)(-\frac{u^{2'}}{c}\frac{u^{2'}}{c}\gamma^2 - 1 - \gamma^2)\}$$

$$= \gamma^6[-(\frac{u^{1'}}{c}\frac{u^{1'}}{c} + \gamma^{-2} - 1)\{(\frac{u^{2'}}{c}\frac{u^{2'}}{c} + \gamma^{-2} - 1)(\frac{u^{3'}}{c}\frac{u^{3'}}{c} + \gamma^{-2} - 1) - (\frac{u^{2'}}{c}\frac{u^{3'}}{c})(\frac{u^{3'}}{c}\frac{u^{2'}}{c})\} +$$
$$+\,(\frac{u^{2'}}{c}\frac{u^{1'}}{c})\{(\frac{u^{1'}}{c}\frac{u^{2'}}{c})(\frac{u^{3'}}{c}\frac{u^{3'}}{c} + \gamma^{-2} - 1) - (\frac{u^{1'}}{c}\frac{u^{3'}}{c})(\frac{u^{3'}}{c}\frac{u^{2'}}{c})\} +$$
$$-\,(\frac{u^{3'}}{c}\frac{u^{1'}}{c})\{(\frac{u^{1'}}{c}\frac{u^{2'}}{c})(\frac{u^{2'}}{c}\frac{u^{3'}}{c}) - (\frac{u^{1'}}{c}\frac{u^{3'}}{c})(\frac{u^{2'}}{c}\frac{u^{2'}}{c} + \gamma^{-2} - 1)\}$$

$$= \gamma^6[-(\frac{u^{1'}}{c}\frac{u^{1'}}{c} + \gamma^{-2} - 1)\{(\frac{u^{2'}}{c}\frac{u^{2'}}{c})(\frac{u^{3'}}{c}\frac{u^{3'}}{c}) + (\gamma^{-2} - 1)[(\frac{u^{2'}}{c}\frac{u^{2'}}{c}) + (\frac{u^{3'}}{c}\frac{u^{3'}}{c})] +$$
$$+\,(\gamma^{-2} - 1)^2 - (\frac{u^{2'}}{c}\frac{u^{2'}}{c})(\frac{u^{3'}}{c}\frac{u^{3'}}{c})\} +$$
$$+\,(\frac{u^{2'}}{c}\frac{u^{1'}}{c})\{(\frac{u^{1'}}{c}\frac{u^{2'}}{c})(\frac{u^{3'}}{c}\frac{u^{3'}}{c}) + (\gamma^{-2} - 1)(\frac{u^{1'}}{c}\frac{u^{2'}}{c}) - (\frac{u^{1'}}{c}\frac{u^{3'}}{c})(\frac{u^{3'}}{c}\frac{u^{2'}}{c})\} +$$
$$-\,(\frac{u^{3'}}{c}\frac{u^{1'}}{c})\{(\frac{u^{1'}}{c}\frac{u^{2'}}{c})(\frac{u^{2'}}{c}\frac{u^{3'}}{c}) - (\frac{u^{1'}}{c}\frac{u^{3'}}{c})(\frac{u^{2'}}{c}\frac{u^{2'}}{c}) - (\frac{u^{1'}}{c}\frac{u^{3'}}{c})(\gamma^{-2} - 1)\}]$$

$$= \gamma^6[-(-\frac{u^{2'}}{c}\frac{u^{2'}}{c} - \frac{u^{3'}}{c}\frac{u^{3'}}{c})\{(\frac{u^{2'}}{c}\frac{u^{2'}}{c})(\frac{u^{3'}}{c}\frac{u^{3'}}{c}) - (\frac{|u|^2}{c^2})(\frac{u^{2'}}{c}\frac{u^{2'}}{c}) - (\frac{|u|^2}{c^2})(\frac{u^{3'}}{c}\frac{u^{3'}}{c}) +$$
$$+\,(\frac{|u|^2}{c^2})(\frac{|u|^2}{c^2}) - (\frac{u^{2'}}{c}\frac{u^{3'}}{c})(\frac{u^{3'}}{c}\frac{u^{2'}}{c})\} +$$
$$+\,(\frac{u^{2'}}{c}\frac{u^{1'}}{c})\{(\frac{u^{1'}}{c}\frac{u^{2'}}{c})(\frac{u^{3'}}{c}\frac{u^{3'}}{c}) - (\frac{|u|^2}{c^2})(\frac{u^{1'}}{c}\frac{u^{2'}}{c}) - (\frac{u^{1'}}{c}\frac{u^{3'}}{c})(\frac{u^{3'}}{c}\frac{u^{2'}}{c})\} +$$
$$+\,(\frac{u^{3'}}{c}\frac{u^{1'}}{c})\{(\frac{u^{1'}}{c}\frac{u^{2'}}{c})(\frac{u^{2'}}{c}\frac{u^{3'}}{c}) - (\frac{u^{1'}}{c}\frac{u^{3'}}{c})(\frac{u^{2'}}{c}\frac{u^{2'}}{c}) + (\frac{|u|^2}{c^2})(\frac{u^{1'}}{c}\frac{u^{3'}}{c})\}]$$

$$= \gamma^6[-(-(\frac{u^{2'}}{c})^2 - (\frac{u^{3'}}{c})^2)\{(\frac{u^{2'}}{c})^2(\frac{u^{3'}}{c})^2 - (\frac{u^{2'}}{c})^2[(\frac{u^{1'}}{c})^2 + (\frac{u^{2'}}{c})^2 + (\frac{u^{3'}}{c})^2] +$$
$$-\,(\frac{u^{3'}}{c})^2[(\frac{u^{1'}}{c})^2 + (\frac{u^{2'}}{c})^2 + (\frac{u^{3'}}{c})^2] +$$
$$+\,[(\frac{u^{1'}}{c})^2 + (\frac{u^{2'}}{c})^2 + (\frac{u^{3'}}{c})^2][(\frac{u^{1'}}{c})^2 + (\frac{u^{2'}}{c})^2 + (\frac{u^{3'}}{c})^2] - (\frac{u^{2'}}{c})^2(\frac{u^{3'}}{c})^2\} +$$
$$+\,\{(\frac{u^{1'}}{c})^2(\frac{u^{2'}}{c})^2(\frac{u^{3'}}{c})^2 - (\frac{u^{1'}}{c})^2(\frac{u^{2'}}{c})^2[(\frac{u^{1'}}{c})^2 + (\frac{u^{2'}}{c})^2 + (\frac{u^{3'}}{c})^2] +$$
$$-\,(\frac{u^{1'}}{c})^2(\frac{u^{2'}}{c})^2(\frac{u^{3'}}{c})^2\} +$$
$$-\,\{(\frac{u^{1'}}{c})^2(\frac{u^{2'}}{c})^2(\frac{u^{3'}}{c})^2 - (\frac{u^{1'}}{c})^2(\frac{u^{2'}}{c})^2(\frac{u^{3'}}{c})^2 + (\frac{u^{1'}}{c})^2(\frac{u^{3'}}{c})^2[(\frac{u^{1'}}{c})^2 + (\frac{u^{2'}}{c})^2 + (\frac{u^{3'}}{c})^2]\}]$$

$$= \gamma^6[\{(\frac{u^{2'}}{c})^2 + (\frac{u^{3'}}{c})^2\}\{-(\frac{u^{2'}}{c})^2(\frac{u^{1'}}{c})^2 - (\frac{u^{2'}}{c})^2(\frac{u^{2'}}{c})^2 +$$
$$-\,(\frac{u^{3'}}{c})^2(\frac{u^{1'}}{c})^2 - (\frac{u^{3'}}{c})^2(\frac{u^{2'}}{c})^2 - (\frac{u^{3'}}{c})^2(\frac{u^{3'}}{c})^2 +$$
$$+\,(\frac{u^{1'}}{c})^2(\frac{u^{1'}}{c})^2 - (\frac{u^{1'}}{c})^2(\frac{u^{2'}}{c})^2 - (\frac{u^{1'}}{c})^2(\frac{u^{3'}}{c})^2 +$$
$$+\,(\frac{u^{2'}}{c})^2(\frac{u^{1'}}{c})^2 - (\frac{u^{2'}}{c})^2(\frac{u^{2'}}{c})^2 - (\frac{u^{2'}}{c})^2(\frac{u^{3'}}{c})^2 +$$

$$+ (\frac{u^{3'}}{c})^2(\frac{u^{1'}}{c})^2 - (\frac{u^{3'}}{c})^2(\frac{u^{2'}}{c})^2 - (\frac{u^{3'}}{c})^2(\frac{u^{3'}}{c})^2 - (\frac{u^{2'}}{c})^2(\frac{u^{3'}}{c})^2\} +$$
$$+ \{-(\frac{u^{1'}}{c})^2(\frac{u^{2'}}{c})^2(\frac{u^{1'}}{c})^2 - (\frac{u^{1'}}{c})^2(\frac{u^{2'}}{c})^2(\frac{u^{2'}}{c})^2 - (\frac{u^{1'}}{c})^2(\frac{u^{2'}}{c})^2(\frac{u^{3'}}{c})^2\} +$$
$$+ \{-(\frac{u^{1'}}{c})^2(\frac{u^{3'}}{c})^2(\frac{u^{1'}}{c})^2 - (\frac{u^{1'}}{c})^2(\frac{u^{3'}}{c})^2(\frac{u^{2'}}{c})^2 - (\frac{u^{1'}}{c})^2(\frac{u^{3'}}{c})^2(\frac{u^{3'}}{c})^2\}]$$
$$= \gamma^6[(\frac{u^{2'}}{c})^2(\frac{u^{1'}}{c})^2(\frac{u^{1'}}{c})^2 + (\frac{u^{2'}}{c})^2(\frac{u^{1'}}{c})^2(\frac{u^{2'}}{c})^2 + (\frac{u^{2'}}{c})^2(\frac{u^{1'}}{c})^2(\frac{u^{3'}}{c})^2 +$$
$$+ (\frac{u^{3'}}{c})^2(\frac{u^{1'}}{c})^2(\frac{u^{1'}}{c})^2 + (\frac{u^{3'}}{c})^2(\frac{u^{1'}}{c})^2(\frac{u^{2'}}{c})^2 + (\frac{u^{3'}}{c})^2(\frac{u^{1'}}{c})^2(\frac{u^{3'}}{c})^2 +$$
$$- (\frac{u^{1'}}{c})^2(\frac{u^{2'}}{c})^2(\frac{u^{1'}}{c})^2 - (\frac{u^{1'}}{c})^2(\frac{u^{2'}}{c})^2(\frac{u^{2'}}{c})^2 - (\frac{u^{1'}}{c})^2(\frac{u^{2'}}{c})^2(\frac{u^{3'}}{c})^2 +$$
$$- (\frac{u^{1'}}{c})^2(\frac{u^{3'}}{c})^2(\frac{u^{1'}}{c})^2 - (\frac{u^{1'}}{c})^2(\frac{u^{3'}}{c})^2(\frac{u^{2'}}{c})^2 - (\frac{u^{1'}}{c})^2(\frac{u^{3'}}{c})^2(\frac{u^{3'}}{c})^2]$$
$$= 0 \ \checkmark$$

Note:

$$c^2 \frac{\partial t}{\partial x^{k'}} u^{k'} \frac{\partial t}{\partial x^{l'}} u^{l'} - g_{ij} \frac{\partial x^i}{\partial x^{k'}} u^{k'} \frac{\partial x^j}{\partial x^{l'}} u^{l'} + g'_{kl} u^{k'} u^{l'} = 0$$

so:

$$c^2 \left(-\gamma \frac{|\vec{\mathbf{u}}|^2}{c^2}\right)\left(-\gamma \frac{|\vec{\mathbf{u}}|^2}{c^2}\right) - g_{ij}\left(-\frac{\partial x^i}{\partial t'}\right)\left(-\frac{\partial x^j}{\partial t'}\right) + g'_{kl} u^{k'} u^{l'} = 0$$

$$c^2 \left(-\gamma \frac{|\vec{\mathbf{u}}|^2}{c^2}\right)\left(-\gamma \frac{|\vec{\mathbf{u}}|^2}{c^2}\right) - g_{ij}(-u^i \gamma)(-u^j \gamma) + |\vec{\mathbf{u}}'|^2 = 0$$

$$|\vec{\mathbf{u}}|^2 \gamma^2 \frac{|\vec{\mathbf{u}}|^2}{c^2} - \gamma^2 |\vec{\mathbf{u}}|^2 + |\vec{\mathbf{u}}'|^2 = 0$$

$\therefore$

$$|\vec{\mathbf{u}}'|^2 = |\vec{\mathbf{u}}|^2 \gamma^2 \left(1 - \frac{|\vec{\mathbf{u}}|^2}{c^2}\right) = |\vec{\mathbf{u}}|^2$$

Now,

$$g_{ij} \frac{\partial x^i}{\partial x^{k'}} \frac{\partial x^j}{\partial x^{l'}} = g'_{kl} + g'_{kr} g'_{sl} \frac{u^{r'}}{c} \frac{u^{s'}}{c} \gamma^2$$ is very general; allowing any curviliear coordinate system for each reference frame

In order tp get to specific Lorentz Transformations, consider Rectangular Cartesian Coordinates for each reference frame:

$$g_{ij} = \delta_{ij} \quad , \quad g'_{kl} = \delta_{kl}$$

$\therefore$

$$\delta_{ij} \frac{\partial x^i}{\partial x^{k'}} \frac{\partial x^j}{\partial x^{l'}} = \delta_{kl} + \delta_{kr}\delta_{sl} \frac{u^{r'}}{c} \frac{u^{s'}}{c} \gamma^2 = \delta_{kl} + \frac{u^{k'}}{c} \frac{u^{l'}}{c} \gamma^2$$

$$= \delta_{kl} + \frac{u^{k'}}{c} \frac{u^{l'}}{c} \gamma^2 = \delta_{kl} + \frac{u^{k'}}{c} \frac{u^{l'}}{c} \left(\frac{\gamma^2}{\gamma+1}\right)\left\{\left(\frac{\gamma^2}{\gamma+1}\right)\frac{|\vec{\mathbf{u}}|^2}{c^2} + 2\right\}$$

$$= \delta_{kl} + \frac{u^{k'}}{c} \frac{u^{l'}}{c} \left\{\left(\frac{\gamma^2}{\gamma+1}\right)^2 \frac{|\vec{\mathbf{u}}|^2}{c^2} + 2\left(\frac{\gamma^2}{\gamma+1}\right)\right\}$$

$$= \delta_{kl} + \frac{u^{k'}}{c} \frac{u^{l'}}{c} \left(\frac{\gamma^2}{\gamma+1}\right)^2 \frac{|\vec{\mathbf{u}}|^2}{c^2} +$$
$$+ \left(\frac{\gamma^2}{\gamma+1}\right)\left\{\frac{u^{l'}}{c} \frac{u^{k'}}{c} + \frac{u^{k'}}{c} \frac{u^{l'}}{c}\right\}$$

$$= \delta_{kl} + \frac{u^{k'}}{c} \frac{u^{l'}}{c} \left(\frac{\gamma^2}{\gamma+1}\right)^2 \frac{|\vec{\mathbf{u}}|^2}{c^2} +$$
$$+ \left(\frac{\gamma^2}{\gamma+1}\right)\left\{\delta_{il} \frac{u^{i'}}{c} \frac{u^{k'}}{c} + \delta_{kj} \frac{u^{j'}}{c} \frac{u^{l'}}{c}\right\}$$

$$= \delta_{ij}\delta^i_k\delta^j_l + \delta_{ij} \frac{u^{i'}}{c} \frac{u^{j'}}{c} \frac{u^{k'}}{c} \frac{u^{l'}}{c} \left(\frac{\gamma^2}{\gamma+1}\right)^2 +$$
$$+ \left(\frac{\gamma^2}{\gamma+1}\right)\left\{\delta_{ij}\delta^j_l \frac{u^{i'}}{c} \frac{u^{k'}}{c} + \delta_{ij}\delta^i_k \frac{u^{j'}}{c} \frac{u^{l'}}{c}\right\}$$

$$= \delta_{ij}\{\delta^i_k\delta^j_l + \frac{u^{i'}}{c} \frac{u^{j'}}{c} \frac{u^{k'}}{c} \frac{u^{l'}}{c} \left(\frac{\gamma^2}{\gamma+1}\right)^2 +$$
$$+ \left(\frac{\gamma^2}{\gamma+1}\right)\{\delta^j_l \frac{u^{i'}}{c} \frac{u^{k'}}{c} + \delta^i_k \frac{u^{j'}}{c} \frac{u^{l'}}{c}\}\}$$

$$= \delta_{ij}\left\{\delta^i_k + \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{i'}}{c} \frac{u^{k'}}{c}\right\}\left\{\delta^j_l + \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{j'}}{c} \frac{u^{l'}}{c}\right\}$$

thus:

$$\frac{\partial x^i}{\partial x^{k'}} = \delta^i_k + \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{i'}}{c} \frac{u^{k'}}{c}$$

so, between reference frames using
Rectangular Cartesian Coordinate Systems,

a Lorentz Transformation is given by:

$x^i = x^{j'}\left\{\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{i'}}{c}\frac{u^{j'}}{c} + \delta^i_j\right\} - \gamma u^{i'} t'$
$t = \gamma\left(t' - \frac{u^{i'}}{c^2}x^{i'}\right)$

$\frac{\partial x}{\partial x'} = \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{1'}}{c} + 1$	$\frac{\partial x}{\partial y'} = \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{2'}}{c}$	$\frac{\partial x}{\partial z'} = \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{3'}}{c}$	$\frac{\partial x}{\partial t'} = -u^{1'}\gamma$
$\frac{\partial y}{\partial x'} = \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{1'}}{c}$	$\frac{\partial y}{\partial y'} = \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{2'}}{c} + 1$	$\frac{\partial y}{\partial z'} = \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{3'}}{c}$	$\frac{\partial y}{\partial t'} = -u^{2'}\gamma$
$\frac{\partial z}{\partial x'} = \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{1'}}{c}$	$\frac{\partial z}{\partial y'} = \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{2'}}{c}$	$\frac{\partial z}{\partial z'} = \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{3'}}{c} + 1$	$\frac{\partial z}{\partial t'} = -u^{3'}\gamma$
$\frac{\partial t}{\partial x'} = -\frac{u^{1'}}{c^2}\gamma$	$\frac{\partial t}{\partial y'} = -\frac{u^{2'}}{c^2}\gamma$	$\frac{\partial t}{\partial z'} = -\frac{u^{3'}}{c^2}\gamma$	$\frac{\partial t}{\partial t'} = \gamma$

so:

$$\frac{\partial x}{\partial x'}\frac{\partial x}{\partial x'} + \frac{\partial x}{\partial y'}\frac{\partial x}{\partial y'} + \frac{\partial x}{\partial z'}\frac{\partial x}{\partial z'} = \gamma^2\frac{u^{1'}}{c}\frac{u^{1'}}{c} + 1$$
$$\frac{\partial x}{\partial x'}\frac{\partial y}{\partial x'} + \frac{\partial x}{\partial y'}\frac{\partial y}{\partial y'} + \frac{\partial x}{\partial z'}\frac{\partial y}{\partial z'} = \gamma^2\frac{u^{1'}}{c}\frac{u^{2'}}{c}$$
$$\frac{\partial x}{\partial x'}\frac{\partial z}{\partial x'} + \frac{\partial x}{\partial y'}\frac{\partial z}{\partial y'} + \frac{\partial x}{\partial z'}\frac{\partial z}{\partial z'} = \gamma^2\frac{u^{1'}}{c}\frac{u^{3'}}{c}$$
$$\frac{\partial x}{\partial x'}\frac{\partial t}{\partial x'} + \frac{\partial x}{\partial y'}\frac{\partial t}{\partial y'} + \frac{\partial x}{\partial z'}\frac{\partial t}{\partial z'} = -\gamma^2\frac{u^{1'}}{c^2}$$
$$\frac{\partial y}{\partial x'}\frac{\partial x}{\partial x'} + \frac{\partial y}{\partial y'}\frac{\partial x}{\partial y'} + \frac{\partial y}{\partial z'}\frac{\partial x}{\partial z'} = \gamma^2\frac{u^{2'}}{c}\frac{u^{1'}}{c}$$
$$\frac{\partial y}{\partial x'}\frac{\partial y}{\partial x'} + \frac{\partial y}{\partial y'}\frac{\partial y}{\partial y'} + \frac{\partial y}{\partial z'}\frac{\partial y}{\partial z'} = \gamma^2\frac{u^{2'}}{c}\frac{u^{2'}}{c} + 1$$
$$\frac{\partial y}{\partial x'}\frac{\partial z}{\partial x'} + \frac{\partial y}{\partial y'}\frac{\partial z}{\partial y'} + \frac{\partial y}{\partial z'}\frac{\partial z}{\partial z'} = \gamma^2\frac{u^{2'}}{c}\frac{u^{3'}}{c}$$
$$\frac{\partial y}{\partial x'}\frac{\partial t}{\partial x'} + \frac{\partial y}{\partial y'}\frac{\partial t}{\partial y'} + \frac{\partial y}{\partial z'}\frac{\partial t}{\partial z'} = -\gamma^2\frac{u^{2'}}{c^2}$$
$$\frac{\partial z}{\partial x'}\frac{\partial x}{\partial x'} + \frac{\partial z}{\partial y'}\frac{\partial x}{\partial y'} + \frac{\partial z}{\partial z'}\frac{\partial x}{\partial z'} = \gamma^2\frac{u^{3'}}{c}\frac{u^{1'}}{c}$$
$$\frac{\partial z}{\partial x'}\frac{\partial y}{\partial x'} + \frac{\partial z}{\partial y'}\frac{\partial y}{\partial y'} + \frac{\partial z}{\partial z'}\frac{\partial y}{\partial z'} = \gamma^2\frac{u^{3'}}{c}\frac{u^{2'}}{c}$$
$$\frac{\partial z}{\partial x'}\frac{\partial z}{\partial x'} + \frac{\partial z}{\partial y'}\frac{\partial z}{\partial y'} + \frac{\partial z}{\partial z'}\frac{\partial z}{\partial z'} = \gamma^2\frac{u^{3'}}{c}\frac{u^{3'}}{c} + 1$$
$$\frac{\partial z}{\partial x'}\frac{\partial t}{\partial x'} + \frac{\partial z}{\partial y'}\frac{\partial t}{\partial y'} + \frac{\partial z}{\partial z'}\frac{\partial t}{\partial z'} = -\gamma^2\frac{u^{3'}}{c^2}$$
$$\frac{\partial t}{\partial x'}\frac{\partial t}{\partial x'} + \frac{\partial t}{\partial y'}\frac{\partial t}{\partial y'} + \frac{\partial t}{\partial z'}\frac{\partial t}{\partial z'} = \gamma^2\frac{1}{c^2}\frac{|\vec{\mathbf{u}}|^2}{c^2}$$

Note, also:

$$\frac{\partial x^k}{\partial t'} = -u^{k'}\gamma \quad ,so \quad u^k = -u^{k'}$$

But,

$$\hat{\boldsymbol{\alpha}}_j dx^j = d\mathbf{r} = d\mathbf{r}' = \hat{\boldsymbol{\alpha}}'_k dx^{k'}$$
$$= \hat{\boldsymbol{\alpha}}_j \frac{\partial x^j}{\partial x^{k'}} dx^{k'}$$

$\therefore$

$$\hat{\boldsymbol{\alpha}}'_k = \hat{\boldsymbol{\alpha}}_j \frac{\partial x^j}{\partial x^{k'}}$$

let,

$$\hat{\mathbf{i}} \equiv \frac{\partial \mathbf{r}}{\partial x^1} \quad , \quad \hat{\mathbf{j}} \equiv \frac{\partial \mathbf{r}}{\partial x^2} \quad , \quad \hat{\mathbf{k}} \equiv \frac{\partial \mathbf{r}}{\partial x^3} \quad , \quad \hat{\mathbf{t}} \equiv \frac{\partial \mathbf{r}}{\partial x^0},$$

so:

$$\hat{\mathbf{i}}' = \hat{\mathbf{i}}\frac{\partial x}{\partial x'} + \hat{\mathbf{j}}\frac{\partial y}{\partial x'} + \hat{\mathbf{k}}\frac{\partial z}{\partial x'} + \hat{\mathbf{t}}\frac{\partial t}{\partial x'}$$
$$\hat{\mathbf{j}}' = \hat{\mathbf{i}}\frac{\partial x}{\partial y'} + \hat{\mathbf{j}}\frac{\partial y}{\partial y'} + \hat{\mathbf{k}}\frac{\partial z}{\partial y'} + \hat{\mathbf{t}}\frac{\partial t}{\partial y'}$$
$$\hat{\mathbf{k}}' = \hat{\mathbf{i}}\frac{\partial x}{\partial z'} + \hat{\mathbf{j}}\frac{\partial y}{\partial z'} + \hat{\mathbf{k}}\frac{\partial z}{\partial z'} + \hat{\mathbf{t}}\frac{\partial t}{\partial z'}$$
$$\hat{\mathbf{t}}' = \hat{\mathbf{i}}\frac{\partial x}{\partial t'} + \hat{\mathbf{j}}\frac{\partial y}{\partial t'} + \hat{\mathbf{k}}\frac{\partial z}{\partial t'} + \hat{\mathbf{t}}\frac{\partial t}{\partial t'}$$

for the embedded 3-space:

$$\begin{matrix} \hat{\mathbf{i}}\cdot\hat{\mathbf{i}}=1 & \hat{\mathbf{i}}\cdot\hat{\mathbf{j}}=0 & \hat{\mathbf{i}}\cdot\hat{\mathbf{k}}=0 \\ \hat{\mathbf{j}}\cdot\hat{\mathbf{i}}=0 & \hat{\mathbf{j}}\cdot\hat{\mathbf{j}}=1 & \hat{\mathbf{j}}\cdot\hat{\mathbf{k}}=0 \\ \hat{\mathbf{k}}\cdot\hat{\mathbf{i}}=0 & \hat{\mathbf{k}}\cdot\hat{\mathbf{j}}=0 & \hat{\mathbf{k}}\cdot\hat{\mathbf{k}}=1 \end{matrix} \quad \Rightarrow \quad g_{ij} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

for the 4-space-time:

$$\begin{matrix} \hat{\mathbf{i}}\cdot\hat{\mathbf{i}}=1 & \hat{\mathbf{i}}\cdot\hat{\mathbf{j}}=0 & \hat{\mathbf{i}}\cdot\hat{\mathbf{k}}=0 & \hat{\mathbf{i}}\cdot\hat{\mathbf{t}}=0 \\ \hat{\mathbf{j}}\cdot\hat{\mathbf{i}}=0 & \hat{\mathbf{j}}\cdot\hat{\mathbf{j}}=1 & \hat{\mathbf{j}}\cdot\hat{\mathbf{k}}=0 & \hat{\mathbf{i}}\cdot\hat{\mathbf{t}}=0 \\ \hat{\mathbf{k}}\cdot\hat{\mathbf{i}}=0 & \hat{\mathbf{k}}\cdot\hat{\mathbf{j}}=0 & \hat{\mathbf{k}}\cdot\hat{\mathbf{k}}=1 & \hat{\mathbf{i}}\cdot\hat{\mathbf{t}}=0 \\ \hat{\mathbf{t}}\cdot\hat{\mathbf{i}}=0 & \hat{\mathbf{t}}\cdot\hat{\mathbf{j}}=0 & \hat{\mathbf{t}}\cdot\hat{\mathbf{k}}=0 & \hat{\mathbf{t}}\cdot\hat{\mathbf{t}}=-c^2 \end{matrix} \quad \Rightarrow \quad g_{ij} \equiv \begin{pmatrix} -c^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{aligned}
\hat{\mathbf{i}}' &= \hat{\mathbf{i}}\left[\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{1'}}{c}+1\right]+\hat{\mathbf{j}}\left[\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{1'}}{c}\right]+ \\
&\quad +\hat{\mathbf{k}}\left[\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right]+\hat{\mathbf{t}}\left[-\frac{u^{1'}}{c^2}\gamma\right] \\
\hat{\mathbf{j}}' &= \hat{\mathbf{i}}\left[\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{2'}}{c}\right]+\hat{\mathbf{j}}\left[\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{2'}}{c}+1\right]+ \\
&\quad +\hat{\mathbf{k}}\left[\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{2'}}{c}\right]+\hat{\mathbf{t}}\left[-\frac{u^{2'}}{c^2}\gamma\right] \\
\hat{\mathbf{k}}' &= \hat{\mathbf{i}}\left[\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{3'}}{c}\right]+\hat{\mathbf{j}}\left[\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{3'}}{c}\right]+ \\
&\quad +\hat{\mathbf{k}}\left[\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{3'}}{c}+1\right]+\hat{\mathbf{t}}\left[-\frac{u^{3'}}{c^2}\gamma\right] \\
\hat{\mathbf{t}}' &= \hat{\mathbf{i}}(-u^{1'}\gamma)+\hat{\mathbf{j}}(-u^{2'}\gamma)+\hat{\mathbf{k}}(-u^{3'}\gamma)+\hat{\mathbf{t}}(\gamma)
\end{aligned}$$

so:

$$\begin{aligned}
\hat{\mathbf{i}}' &= \hat{\mathbf{i}}+\left[\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c^2}\right]\vec{\mathbf{u}}'+\hat{\mathbf{t}}\left(-\frac{u^{1'}}{c^2}\gamma\right) \\
\hat{\mathbf{j}}' &= \hat{\mathbf{j}}+\left[\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c^2}\right]\vec{\mathbf{u}}'+\hat{\mathbf{t}}\left(-\frac{u^{2'}}{c^2}\gamma\right) \\
\hat{\mathbf{k}}' &= \hat{\mathbf{k}}+\left[\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c^2}\right]\vec{\mathbf{u}}'+\hat{\mathbf{t}}\left(-\frac{u^{3'}}{c^2}\gamma\right) \\
\hat{\mathbf{t}}' &= \gamma\hat{\mathbf{t}}-\gamma\vec{\mathbf{u}}'
\end{aligned}$$

But

$$\begin{aligned}
\hat{\boldsymbol{\alpha}}'_k dx^{k'} = d\mathbf{r}' = d\mathbf{r} &= \hat{\boldsymbol{\alpha}}_j dx^j \\
&= \hat{\boldsymbol{\alpha}}'_k \frac{\partial x^{k'}}{\partial x^j} dx^j
\end{aligned}$$

∴

$$\hat{\boldsymbol{\alpha}}_j = \hat{\boldsymbol{\alpha}}'_k \frac{\partial x^{k'}}{\partial x^j} \quad , \quad \frac{\partial x^j}{\partial x^{k'}}\frac{\partial x^{k'}}{\partial x^i} = \delta^j_i \quad \textit{and} \quad \frac{\partial x^{k'}}{\partial x^i}\frac{\partial x^i}{\partial x^{l'}} = \delta^k_l$$

$$\hat{\mathbf{i}}' \equiv \frac{\partial \mathbf{r}'}{\partial x^{1'}} \ , \ \hat{\mathbf{j}}' \equiv \frac{\partial \mathbf{r}'}{\partial x^{2'}} \ , \ \hat{\mathbf{k}}' \equiv \frac{\partial \mathbf{r}'}{\partial x^{3'}} \ , \ \hat{\mathbf{t}}' \equiv \frac{\partial \mathbf{r}'}{\partial x^{0'}},$$

so:

$$\begin{aligned}
\hat{\mathbf{i}} &= \hat{\mathbf{i}}'\frac{\partial x'}{\partial x}+\hat{\mathbf{j}}'\frac{\partial y'}{\partial x}+\hat{\mathbf{k}}'\frac{\partial z'}{\partial x}+\hat{\mathbf{t}}'\frac{\partial t'}{\partial x} \\
\hat{\mathbf{j}} &= \hat{\mathbf{i}}'\frac{\partial x'}{\partial y}+\hat{\mathbf{j}}'\frac{\partial y'}{\partial y}+\hat{\mathbf{k}}'\frac{\partial z'}{\partial y}+\hat{\mathbf{t}}'\frac{\partial t'}{\partial y} \\
\hat{\mathbf{k}} &= \hat{\mathbf{i}}'\frac{\partial x'}{\partial z}+\hat{\mathbf{j}}'\frac{\partial y'}{\partial z}+\hat{\mathbf{k}}'\frac{\partial z'}{\partial z}+\hat{\mathbf{t}}'\frac{\partial t'}{\partial z} \\
\hat{\mathbf{t}} &= \hat{\mathbf{i}}'\frac{\partial x'}{\partial t}+\hat{\mathbf{j}}'\frac{\partial y'}{\partial t}+\hat{\mathbf{k}}'\frac{\partial z'}{\partial t}+\hat{\mathbf{t}}'\frac{\partial t'}{\partial t}
\end{aligned}$$

Applying Cramer’s Rule, let:

$$\Delta \equiv \begin{vmatrix}
\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{1'}}{c}+1 & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{1'}}{c} & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{1'}}{c} & -\frac{u^{1'}}{c^2}\gamma \\
\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{2'}}{c} & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{2'}}{c}+1 & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{2'}}{c} & -\frac{u^{2'}}{c^2}\gamma \\
\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{3'}}{c} & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{3'}}{c} & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{3'}}{c}+1 & -\frac{u^{3'}}{c^2}\gamma \\
-u^{1'}\gamma & -u^{2'}\gamma & -u^{3'}\gamma & \gamma
\end{vmatrix}$$

$$\Delta_{11} \equiv \begin{vmatrix} \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{2'}}{c}+1 & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{2'}}{c} & -\frac{u^{2'}}{c^2}\gamma \\ \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{3'}}{c} & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{3'}}{c}+1 & -\frac{u^{3'}}{c^2}\gamma \\ -u^{2'}\gamma & -u^{3'}\gamma & \gamma \end{vmatrix}$$

$$\Delta_{12} \equiv \begin{vmatrix} \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{1'}}{c} & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{1'}}{c} & -\frac{u^{1'}}{c^2}\gamma \\ \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{3'}}{c} & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{3'}}{c}+1 & -\frac{u^{3'}}{c^2}\gamma \\ -u^{2'}\gamma & -u^{3'}\gamma & \gamma \end{vmatrix}$$

$$\Delta_{13} \equiv \begin{vmatrix} \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{1'}}{c} & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{1'}}{c} & -\frac{u^{1'}}{c^2}\gamma \\ \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{2'}}{c}+1 & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{2'}}{c} & -\frac{u^{2'}}{c^2}\gamma \\ -u^{2'}\gamma & -u^{3'}\gamma & \gamma \end{vmatrix}$$

$$\Delta_{14} \equiv \begin{vmatrix} \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{1'}}{c} & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{1'}}{c} & -\frac{u^{1'}}{c^2}\gamma \\ \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{2'}}{c}+1 & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{2'}}{c} & -\frac{u^{2'}}{c^2}\gamma \\ \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{3'}}{c} & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{3'}}{c}+1 & -\frac{u^{3'}}{c^2}\gamma \end{vmatrix}$$

$$\Delta_{21} \equiv \begin{vmatrix} \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{2'}}{c} & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{2'}}{c} & -\frac{u^{2'}}{c^2}\gamma \\ \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{3'}}{c} & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{3'}}{c}+1 & -\frac{u^{3'}}{c^2}\gamma \\ -u^{1'}\gamma & -u^{3'}\gamma & \gamma \end{vmatrix}$$

$$\Delta_{22} \equiv \begin{vmatrix} \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{1'}}{c}+1 & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{1'}}{c} & -\frac{u^{1'}}{c^2}\gamma \\ \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{3'}}{c} & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{3'}}{c}+1 & -\frac{u^{3'}}{c^2}\gamma \\ -u^{1'}\gamma & -u^{3'}\gamma & \gamma \end{vmatrix}$$

$$\Delta_{23} \equiv \begin{vmatrix} \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{1'}}{c}+1 & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{1'}}{c} & -\frac{u^{1'}}{c^2}\gamma \\ \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{2'}}{c} & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{2'}}{c} & -\frac{u^{2'}}{c^2}\gamma \\ -u^{1'}\gamma & -u^{3'}\gamma & \gamma \end{vmatrix}$$

$$\Delta_{24} \equiv \begin{vmatrix} \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{1'}}{c}+1 & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{1'}}{c} & -\frac{u^{1'}}{c^2}\gamma \\ \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{2'}}{c} & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{2'}}{c} & -\frac{u^{2'}}{c^2}\gamma \\ \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{3'}}{c} & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{3'}}{c}+1 & -\frac{u^{3'}}{c^2}\gamma \end{vmatrix}$$

$$\Delta_{31} \equiv \begin{vmatrix} \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{2'}}{c} & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{2'}}{c}+1 & -\frac{u^{2'}}{c^2}\gamma \\ \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{3'}}{c} & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{3'}}{c} & -\frac{u^{3'}}{c^2}\gamma \\ -u^{1'}\gamma & -u^{2'}\gamma & \gamma \end{vmatrix}$$

$$\Delta_{32} \equiv \begin{vmatrix} \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{1'}}{c}+1 & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{1'}}{c} & -\frac{u^{1'}}{c^2}\gamma \\ \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{3'}}{c} & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{3'}}{c} & -\frac{u^{3'}}{c^2}\gamma \\ -u^{1'}\gamma & -u^{2'}\gamma & \gamma \end{vmatrix}$$

$$\Delta_{33} \equiv \begin{vmatrix} \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{1'}}{c}+1 & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{1'}}{c} & -\frac{u^{1'}}{c^2}\gamma \\ \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{2'}}{c} & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{2'}}{c}+1 & -\frac{u^{2'}}{c^2}\gamma \\ -u^{1'}\gamma & -u^{2'}\gamma & \gamma \end{vmatrix}$$

$$\Delta_{34} \equiv \begin{vmatrix} \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1\prime}}{c}\frac{u^{1\prime}}{c}+1 & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2\prime}}{c}\frac{u^{1\prime}}{c} & -\frac{u^{1\prime}}{c^2}\gamma \\ \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1\prime}}{c}\frac{u^{2\prime}}{c} & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2\prime}}{c}\frac{u^{2\prime}}{c}+1 & -\frac{u^{2\prime}}{c^2}\gamma \\ \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1\prime}}{c}\frac{u^{3\prime}}{c} & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2\prime}}{c}\frac{u^{3\prime}}{c} & -\frac{u^{3\prime}}{c^2}\gamma \end{vmatrix}$$

$$\Delta_{41} \equiv \begin{vmatrix} \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1\prime}}{c}\frac{u^{2\prime}}{c} & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2\prime}}{c}\frac{u^{2\prime}}{c}+1 & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3\prime}}{c}\frac{u^{2\prime}}{c} \\ \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1\prime}}{c}\frac{u^{3\prime}}{c} & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2\prime}}{c}\frac{u^{3\prime}}{c} & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3\prime}}{c}\frac{u^{3\prime}}{c}+1 \\ -u^{1\prime}\gamma & -u^{2\prime}\gamma & -u^{3\prime}\gamma \end{vmatrix}$$

$$\Delta_{42} \equiv \begin{vmatrix} \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1\prime}}{c}\frac{u^{1\prime}}{c}+1 & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2\prime}}{c}\frac{u^{1\prime}}{c} & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3\prime}}{c}\frac{u^{1\prime}}{c} \\ \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1\prime}}{c}\frac{u^{3\prime}}{c} & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2\prime}}{c}\frac{u^{3\prime}}{c} & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3\prime}}{c}\frac{u^{3\prime}}{c}+1 \\ -u^{1\prime}\gamma & -u^{2\prime}\gamma & -u^{3\prime}\gamma \end{vmatrix}$$

$$\Delta_{43} \equiv \begin{vmatrix} \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1\prime}}{c}\frac{u^{1\prime}}{c}+1 & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2\prime}}{c}\frac{u^{1\prime}}{c} & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3\prime}}{c}\frac{u^{1\prime}}{c} \\ \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1\prime}}{c}\frac{u^{2\prime}}{c} & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2\prime}}{c}\frac{u^{2\prime}}{c}+1 & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3\prime}}{c}\frac{u^{2\prime}}{c} \\ -u^{1\prime}\gamma & -u^{2\prime}\gamma & -u^{3\prime}\gamma \end{vmatrix}$$

$$\Delta_{44} \equiv \begin{vmatrix} \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1\prime}}{c}\frac{u^{1\prime}}{c}+1 & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2\prime}}{c}\frac{u^{1\prime}}{c} & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3\prime}}{c}\frac{u^{1\prime}}{c} \\ \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1\prime}}{c}\frac{u^{2\prime}}{c} & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2\prime}}{c}\frac{u^{2\prime}}{c}+1 & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3\prime}}{c}\frac{u^{2\prime}}{c} \\ \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1\prime}}{c}\frac{u^{3\prime}}{c} & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2\prime}}{c}\frac{u^{3\prime}}{c} & \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3\prime}}{c}\frac{u^{3\prime}}{c}+1 \end{vmatrix}$$

$\therefore$

$$\begin{aligned}
\hat{\mathbf{i}} &= \hat{\mathbf{i}}'\left(\frac{\Delta_{11}}{\Delta}\right) + \hat{\mathbf{j}}'\left(\frac{-\Delta_{12}}{\Delta}\right) + \hat{\mathbf{k}}'\left(\frac{\Delta_{13}}{\Delta}\right) + \hat{\mathbf{t}}'\left(\frac{-\Delta_{14}}{\Delta}\right) \\
\hat{\mathbf{j}} &= \hat{\mathbf{i}}'\left(\frac{-\Delta_{21}}{\Delta}\right) + \hat{\mathbf{j}}'\left(\frac{\Delta_{22}}{\Delta}\right) + \hat{\mathbf{k}}'\left(\frac{-\Delta_{23}}{\Delta}\right) + \hat{\mathbf{t}}'\left(\frac{\Delta_{24}}{\Delta}\right) \\
\hat{\mathbf{k}} &= \hat{\mathbf{i}}'\left(\frac{\Delta_{31}}{\Delta}\right) + \hat{\mathbf{j}}'\left(\frac{-\Delta_{32}}{\Delta}\right) + \hat{\mathbf{k}}'\left(\frac{\Delta_{33}}{\Delta}\right) + \hat{\mathbf{t}}'\left(\frac{-\Delta_{34}}{\Delta}\right) \\
\hat{\mathbf{t}} &= \hat{\mathbf{i}}'\left(\frac{-\Delta_{41}}{\Delta}\right) + \hat{\mathbf{j}}'\left(\frac{\Delta_{42}}{\Delta}\right) + \hat{\mathbf{k}}'\left(\frac{-\Delta_{43}}{\Delta}\right) + \hat{\mathbf{t}}'\left(\frac{\Delta_{44}}{\Delta}\right)
\end{aligned}$$

$$\begin{aligned}
\Delta_{11} =& \bigg(\bigg(\frac{\gamma^2}{\gamma+1}\bigg)\frac{u^{2\prime}}{c}\frac{u^{2\prime}}{c}+1\bigg)\bigg[\bigg(\bigg(\frac{\gamma^2}{\gamma+1}\bigg)\frac{u^{3\prime}}{c}\frac{u^{3\prime}}{c}+1\bigg)(\gamma) - \bigg(-\frac{u^{3\prime}}{c^2}\gamma\bigg)(-u^{3\prime}\gamma)\bigg] + \\
& -\bigg(\bigg(\frac{\gamma^2}{\gamma+1}\bigg)\frac{u^{2\prime}}{c}\frac{u^{3\prime}}{c}\bigg)\bigg[\bigg(\bigg(\frac{\gamma^2}{\gamma+1}\bigg)\frac{u^{3\prime}}{c}\frac{u^{2\prime}}{c}\bigg)(\gamma) - \bigg(-\frac{u^{2\prime}}{c^2}\gamma\bigg)(-u^{3\prime}\gamma)\bigg] + \\
& + (-u^{2\prime}\gamma)\bigg[\bigg(\bigg(\frac{\gamma^2}{\gamma+1}\bigg)\frac{u^{3\prime}}{c}\frac{u^{2\prime}}{c}\bigg)\bigg(-\frac{u^{3\prime}}{c^2}\gamma\bigg) - \bigg(-\frac{u^{2\prime}}{c^2}\gamma\bigg)\bigg(\bigg(\frac{\gamma^2}{\gamma+1}\bigg)\frac{u^{3\prime}}{c}\frac{u^{3\prime}}{c}+1\bigg)\bigg] \\
=& \bigg(\bigg(\frac{\gamma^2}{\gamma+1}\bigg)\frac{u^{2\prime}}{c}\frac{u^{2\prime}}{c}+1\bigg)\bigg[\bigg(\frac{u^{3\prime}}{c}\frac{u^{3\prime}}{c}\bigg)\bigg[\bigg(\frac{\gamma^2}{\gamma+1}\bigg)\gamma - \gamma^2\bigg] + \gamma\bigg] + \\
& -\bigg(\bigg(\frac{\gamma^2}{\gamma+1}\bigg)\frac{u^{2\prime}}{c}\frac{u^{3\prime}}{c}\bigg)\bigg[\bigg(\frac{u^{3\prime}}{c}\frac{u^{2\prime}}{c}\bigg)\bigg[\bigg(\frac{\gamma^2}{\gamma+1}\bigg)\gamma - \gamma^2\bigg]\bigg] + \\
& + (-u^{2\prime}\gamma)\bigg[\bigg(\frac{u^{3\prime}}{c}\frac{u^{2\prime}}{c}\frac{u^{3\prime}}{c^2}\bigg)\bigg[\bigg(\frac{\gamma^2}{\gamma+1}\bigg)(-\gamma) - (-\gamma)\bigg(\frac{\gamma^2}{\gamma+1}\bigg)\bigg] - \bigg(-\frac{u^{2\prime}}{c^2}\gamma\bigg)\bigg] \\
=& \bigg(\bigg(\frac{\gamma^2}{\gamma+1}\bigg)\frac{u^{2\prime}}{c}\frac{u^{2\prime}}{c}+1\bigg)\bigg[\bigg(\frac{u^{3\prime}}{c}\frac{u^{3\prime}}{c}\bigg)\bigg[-\bigg(\frac{\gamma^2}{\gamma+1}\bigg)\bigg] + \gamma\bigg] + \\
& -\bigg(\bigg(\frac{\gamma^2}{\gamma+1}\bigg)\frac{u^{2\prime}}{c}\frac{u^{3\prime}}{c}\bigg)\bigg[\bigg(\frac{u^{3\prime}}{c}\frac{u^{2\prime}}{c}\bigg)\bigg[-\bigg(\frac{\gamma^2}{\gamma+1}\bigg)\bigg]\bigg] + \\
& + (-u^{2\prime}\gamma)\bigg[-\bigg(-\frac{u^{2\prime}}{c^2}\gamma\bigg)\bigg] \\
=& \bigg(\frac{u^{2\prime}}{c}\frac{u^{2\prime}}{c}\bigg)\bigg(\gamma\bigg(\frac{\gamma^2}{\gamma+1}\bigg)\bigg) + \bigg(\frac{u^{3\prime}}{c}\frac{u^{3\prime}}{c}\bigg)\bigg[-\bigg(\frac{\gamma^2}{\gamma+1}\bigg)\bigg] + \\
& + \bigg(\frac{u^{2\prime}}{c}\frac{u^{2\prime}}{c}\bigg)\bigg(\frac{u^{3\prime}}{c}\frac{u^{3\prime}}{c}\bigg)\bigg[-\bigg(\frac{\gamma^2}{\gamma+1}\bigg)^2\bigg] + \gamma + \\
& -\bigg(\frac{u^{2\prime}}{c}\frac{u^{3\prime}}{c}\bigg)\bigg(\frac{u^{3\prime}}{c}\frac{u^{2\prime}}{c}\bigg)\bigg[-\bigg(\frac{\gamma^2}{\gamma+1}\bigg)^2\bigg] + \bigg(-\frac{u^{2\prime}}{c}\bigg)\bigg[-\bigg(-\frac{u^{2\prime}}{c}\bigg)\gamma^2\bigg] \\
=& \bigg(\frac{u^{2\prime}}{c}\frac{u^{2\prime}}{c}\bigg)\bigg[\gamma\bigg(\frac{\gamma^2}{\gamma+1}\bigg) - \gamma^2\bigg] + \bigg(\frac{u^{3\prime}}{c}\frac{u^{3\prime}}{c}\bigg)\bigg[-\bigg(\frac{\gamma^2}{\gamma+1}\bigg)\bigg] + \gamma
\end{aligned}$$

$\therefore$

$$\Delta_{11} = \left(\frac{u^{2'}}{c}\frac{u^{2'}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right] + \left(\frac{u^{3'}}{c}\frac{u^{3'}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right] + \gamma$$

$$\begin{aligned}
\Delta_{12} = &\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{1'}}{c}\right)\left[\left(\frac{u^{3'}}{c}\frac{u^{3'}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right]+\gamma\right]+\\
&-\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{3'}}{c}\right)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)(\gamma)-\left(-\frac{u^{1'}}{c^2}\gamma\right)(-u^{3'}\gamma)\right]+\\
&+(-u^{2'}\gamma)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)\left(-\frac{u^{3'}}{c^2}\gamma\right)-\left(-\frac{u^{1'}}{c^2}\gamma\right)\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{3'}}{c}+1\right)\right]\\
= &\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{1'}}{c}\right)\left[\left(\frac{u^{3'}}{c}\frac{u^{3'}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)\gamma-\gamma^2\right]+\gamma\right]+\\
&-\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{3'}}{c}\right)\left[\left(\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)\gamma-\gamma^2\right]\right]+\\
&+(-u^{2'}\gamma)\left[\left(\frac{u^{3'}}{c}\frac{u^{1'}}{c}\frac{u^{3'}}{c^2}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)(-\gamma)-(-\gamma)\left(\frac{\gamma^2}{\gamma+1}\right)\right]-\left(-\frac{u^{1'}}{c^2}\gamma\right)\right]\\
= &\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{1'}}{c}\right)\left[\left(\frac{u^{3'}}{c}\frac{u^{3'}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right]+\gamma\right]+\\
&-\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{3'}}{c}\right)\left[\left(\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right]\right]+\\
&+(-u^{2'}\gamma)\left[-\left(-\frac{u^{1'}}{c^2}\gamma\right)\right]\\
= &\left(\frac{u^{2'}}{c}\frac{u^{1'}}{c}\right)\left(\gamma\left(\frac{\gamma^2}{\gamma+1}\right)\right)+\\
&+\left(\frac{u^{2'}}{c}\frac{u^{1'}}{c}\right)\left(\frac{u^{3'}}{c}\frac{u^{3'}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)^2\right]-\left(\frac{u^{2'}}{c}\frac{u^{3'}}{c}\right)\left(\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)^2\right]+\\
&+\left(-\frac{u^{2'}}{c}\right)\left[-\left(-\frac{u^{1'}}{c}\right)\gamma^2\right]\\
= &\left(\frac{u^{2'}}{c}\frac{u^{1'}}{c}\right)\left[\gamma\left(\frac{\gamma^2}{\gamma+1}\right)-\gamma^2\right]
\end{aligned}$$

$\therefore$

$$\Delta_{12} = \left(\frac{u^{2'}}{c}\frac{u^{1'}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right]$$

$$\begin{aligned}
\Delta_{13} = &\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{1'}}{c}\right)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{2'}}{c}\right)(\gamma)-\left(-\frac{u^{2'}}{c^2}\gamma\right)(-u^{3'}\gamma)\right]+\\
&-\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{2'}}{c}+1\right)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)(\gamma)-\left(-\frac{u^{1'}}{c^2}\gamma\right)(-u^{3'}\gamma)\right]+\\
&+(-u^{2'}\gamma)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)\left(-\frac{u^{2'}}{c^2}\gamma\right)-\left(-\frac{u^{1'}}{c^2}\gamma\right)\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{2'}}{c}\right)\right]\\
= &\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{1'}}{c}\right)\left[\left(\frac{u^{3'}}{c}\frac{u^{2'}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)\gamma-\gamma^2\right]\right]+\\
&-\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{2'}}{c}+1\right)\left[\left(\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)\gamma-\gamma^2\right]\right]+\\
&+(-u^{2'}\gamma)\left[\left(\frac{u^{3'}}{c}\frac{u^{1'}}{c}\frac{u^{2'}}{c^2}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)(-\gamma)-(-\gamma)\left(\frac{\gamma^2}{\gamma+1}\right)\right]\right]\\
= &\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{1'}}{c}\right)\left[\left(\frac{u^{3'}}{c}\frac{u^{2'}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right]\right]+\\
&-\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{2'}}{c}+1\right)\left[\left(\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right]\right]\\
= &\left(\frac{u^{2'}}{c}\frac{u^{1'}}{c}\right)\left(\frac{u^{3'}}{c}\frac{u^{2'}}{c}\right)\left(-\left(\frac{\gamma^2}{\gamma+1}\right)^2\right)+\\
&-\left(\frac{u^{2'}}{c}\frac{u^{2'}}{c}\right)\left(\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)\left(-\left(\frac{\gamma^2}{\gamma+1}\right)^2+1\right)-\left(\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right]
\end{aligned}$$

$\therefore$

$$\Delta_{13} = \left(\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)\right]$$

$$\begin{aligned}
\Delta_{14} = &\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{1'}}{c}\right)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{2'}}{c}\right)\left(-\frac{u^{3'}}{c^2}\gamma\right)-\left(-\frac{u^{2'}}{c^2}\gamma\right)\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{3'}}{c}+1\right)\right]+\\
&-\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{2'}}{c}+1\right)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)\left(-\frac{u^{3'}}{c^2}\gamma\right)-\left(-\frac{u^{1'}}{c^2}\gamma\right)\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{3'}}{c}+1\right)\right]+\\
&+\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{3'}}{c}\right)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)\left(-\frac{u^{2'}}{c^2}\gamma\right)-\left(-\frac{u^{1'}}{c^2}\gamma\right)\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{2'}}{c}\right)\right]\\
= &\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{1'}}{c}\right)\left[\left(\frac{u^{3'}}{c}\frac{u^{2'}}{c}\frac{u^{3'}}{c^2}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)(-\gamma)-(-\gamma)\left(\frac{\gamma^2}{\gamma+1}\right)\right]-\left(-\frac{u^{2'}}{c^2}\gamma\right)\right]+\\
&-\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{2'}}{c}+1\right)\left[\left(\frac{u^{3'}}{c}\frac{u^{1'}}{c}\frac{u^{3'}}{c^2}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)(-\gamma)-(-\gamma)\left(\frac{\gamma^2}{\gamma+1}\right)\right]-\left(-\frac{u^{1'}}{c^2}\gamma\right)\right]+\\
&+\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{3'}}{c}\right)\left[\left(\frac{u^{3'}}{c}\frac{u^{1'}}{c}\frac{u^{2'}}{c^2}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)(-\gamma)-(-\gamma)\left(\frac{\gamma^2}{\gamma+1}\right)\right]\right]\\
= &\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{1'}}{c}\right)\left[-\left(-\frac{u^{2'}}{c^2}\gamma\right)\right]-\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{2'}}{c}+1\right)\left[-\left(-\frac{u^{1'}}{c^2}\gamma\right)\right]
\end{aligned}$$

$\therefore$

$$\Delta_{14} = -\frac{u^{1'}}{c^2}\gamma$$

$$\begin{aligned}
\Delta_{21} = {} & \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{2'}}{c}\right)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{3'}}{c}+1\right)(\gamma)-\left(-\frac{u^{3'}}{c^2}\gamma\right)(-u^{3'}\gamma)\right]+ \\
& -\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{3'}}{c}\right)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{2'}}{c}\right)(\gamma)-\left(-\frac{u^{2'}}{c^2}\gamma\right)(-u^{3'}\gamma)\right]+ \\
& +(-u^{1'}\gamma)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{2'}}{c}\right)\left(-\frac{u^{3'}}{c^2}\gamma\right)-\left(-\frac{u^{2'}}{c^2}\gamma\right)\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{3'}}{c}+1\right)\right] \\
= {} & \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{2'}}{c}\right)\left[\left(\frac{u^{3'}}{c}\frac{u^{3'}}{c}\right)\left(\left(\frac{\gamma^2}{\gamma+1}\right)(\gamma)-\gamma^2\right)+\gamma\right]+ \\
& -\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{3'}}{c}\right)\left[\left(\frac{u^{3'}}{c}\frac{u^{2'}}{c}\right)\left(\left(\frac{\gamma^2}{\gamma+1}\right)(\gamma)-\gamma^2\right)\right]+ \\
& +(-u^{1'}\gamma)\left[\left(\frac{u^{3'}}{c}\frac{u^{2'}}{c}\frac{u^{3'}}{c^2}\right)\left(\left(\frac{\gamma^2}{\gamma+1}\right)(-\gamma)-(-\gamma)\left(\frac{\gamma^2}{\gamma+1}\right)\right)-\left(-\frac{u^{2'}}{c^2}\gamma\right)\right] \\
= {} & \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{2'}}{c}\right)\left[\left(\frac{u^{3'}}{c}\frac{u^{3'}}{c}\right)\left(-\left(\frac{\gamma^2}{\gamma+1}\right)\right)+\gamma\right]+ \\
& -\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{3'}}{c}\right)\left[\left(\frac{u^{3'}}{c}\frac{u^{2'}}{c}\right)\left(-\left(\frac{\gamma^2}{\gamma+1}\right)\right)\right]+ \\
& +(-u^{1'}\gamma)\left[-\left(-\frac{u^{2'}}{c^2}\gamma\right)\right] \\
= {} & \left(\frac{\gamma^2}{\gamma+1}\right)\left(\frac{u^{1'}}{c}\frac{u^{2'}}{c}\right)\left(\frac{u^{3'}}{c}\frac{u^{3'}}{c}\right)\left[\left(-\left(\frac{\gamma^2}{\gamma+1}\right)\right)-\left(-\left(\frac{\gamma^2}{\gamma+1}\right)\right)\right]+ \\
& +\left(\frac{\gamma^2}{\gamma+1}\right)\left(\frac{u^{1'}}{c}\frac{u^{2'}}{c}\right)\gamma+\left(\frac{u^{1'}}{c}\frac{u^{2'}}{c}\right)(-\gamma^2)
\end{aligned}$$

$\therefore$

$$\Delta_{21} = \left(\frac{u^{1'}}{c}\frac{u^{2'}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right]$$

$$\begin{aligned}
\Delta_{22} = {} & \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{1'}}{c}+1\right)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{3'}}{c}+1\right)(\gamma)-\left(-\frac{u^{3'}}{c^2}\gamma\right)(-u^{3'}\gamma)\right]+ \\
& -\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{3'}}{c}\right)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)(\gamma)-\left(-\frac{u^{1'}}{c^2}\gamma\right)(-u^{3'}\gamma)\right]+ \\
& +(-u^{1'}\gamma)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)\left(-\frac{u^{3'}}{c^2}\gamma\right)-\left(-\frac{u^{1'}}{c^2}\gamma\right)\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{3'}}{c}+1\right)\right] \\
= {} & \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{1'}}{c}+1\right)\left[\left(\frac{u^{3'}}{c}\frac{u^{3'}}{c}\right)\left(\left(\frac{\gamma^2}{\gamma+1}\right)(\gamma)-\gamma^2\right)+\gamma\right]+ \\
& -\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{3'}}{c}\right)\left[\left(\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)\left(\left(\frac{\gamma^2}{\gamma+1}\right)(\gamma)-\gamma^2\right)\right]+ \\
& +(-u^{1'}\gamma)\left[\left(\frac{u^{3'}}{c}\frac{u^{1'}}{c}\frac{u^{3'}}{c^2}\right)\left(\left(\frac{\gamma^2}{\gamma+1}\right)(-\gamma)-(-\gamma)\left(\frac{\gamma^2}{\gamma+1}\right)\right)-\left(-\frac{u^{1'}}{c^2}\gamma\right)\right] \\
= {} & \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{1'}}{c}+1\right)\left[\left(\frac{u^{3'}}{c}\frac{u^{3'}}{c}\right)\left(-\left(\frac{\gamma^2}{\gamma+1}\right)\right)+\gamma\right]+ \\
& -\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{3'}}{c}\right)\left[\left(\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)\left(-\left(\frac{\gamma^2}{\gamma+1}\right)\right)\right]+ \\
& +(-u^{1'}\gamma)\left[-\left(-\frac{u^{1'}}{c^2}\gamma\right)\right] \\
= {} & \left(\frac{u^{1'}}{c}\frac{u^{1'}}{c}\right)\left(\frac{u^{3'}}{c}\frac{u^{3'}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)^2\right]+ \\
& +\left(\frac{u^{1'}}{c}\frac{u^{1'}}{c}\right)\left(\frac{\gamma^2}{\gamma+1}\right)(\gamma)+\left(\frac{u^{3'}}{c}\frac{u^{3'}}{c}\right)\left(-\left(\frac{\gamma^2}{\gamma+1}\right)\right)+\gamma+ \\
& -\left(\frac{u^{1'}}{c}\frac{u^{3'}}{c}\right)\left(\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)\left(-\left(\frac{\gamma^2}{\gamma+1}\right)^2\right)+\left(\frac{u^{1'}}{c}\frac{u^{1'}}{c}\right)(-\gamma^2)
\end{aligned}$$

$\therefore$

$$\Delta_{22} = \left(\frac{u^{1'}}{c}\frac{u^{1'}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right]+\left(\frac{u^{3'}}{c}\frac{u^{3'}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right]+\gamma$$

$$\begin{aligned}
\Delta_{23} = {} & \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{1'}}{c}+1\right)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{2'}}{c}\right)(\gamma)-\left(-\frac{u^{2'}}{c^2}\gamma\right)(-u^{3'}\gamma)\right]+ \\
& -\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{2'}}{c}\right)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)(\gamma)-\left(-\frac{u^{1'}}{c^2}\gamma\right)(-u^{3'}\gamma)\right]+ \\
& +(-u^{1'}\gamma)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)\left(-\frac{u^{2'}}{c^2}\gamma\right)-\left(-\frac{u^{1'}}{c^2}\gamma\right)\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{2'}}{c}\right)\right] \\
= {} & \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{1'}}{c}+1\right)\left[\left(\frac{u^{3'}}{c}\frac{u^{2'}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)(\gamma)-\gamma^2\right]\right]+ \\
& -\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{2'}}{c}\right)\left[\left(\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)(\gamma)-\gamma^2\right]\right]+ \\
& +(-u^{1'}\gamma)\left[\left(\frac{u^{3'}}{c}\frac{u^{1'}}{c}\frac{u^{2'}}{c^2}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)(-\gamma)-(-\gamma)\left(\frac{\gamma^2}{\gamma+1}\right)\right]\right] \\
= {} & \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{1'}}{c}+1\right)\left[\left(\frac{u^{3'}}{c}\frac{u^{2'}}{c}\right)\left(-\left(\frac{\gamma^2}{\gamma+1}\right)\right)\right]+
\end{aligned}$$

$$-\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{2'}}{c}\right)\left[\left(\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)\left(-\left(\frac{\gamma^2}{\gamma+1}\right)\right)\right]$$
$$=\left(\frac{u^{1'}}{c}\frac{u^{1'}}{c}\right)\left(\frac{u^{3'}}{c}\frac{u^{2'}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right]+$$
$$-\left(\frac{u^{1'}}{c}\frac{u^{2'}}{c}\right)\left(\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right]+\left(\frac{u^{3'}}{c}\frac{u^{2'}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right]$$

$\therefore$

$$\Delta_{23}=\left(\frac{u^{3'}}{c}\frac{u^{2'}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right]$$

$$\Delta_{24}=\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{1'}}{c}+1\right)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{2'}}{c}\right)\left(-\frac{u^{3'}}{c^2}\gamma\right)-\left(-\frac{u^{2'}}{c^2}\gamma\right)\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{3'}}{c}+1\right)\right]+$$
$$-\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{2'}}{c}\right)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)\left(-\frac{u^{3'}}{c^2}\gamma\right)-\left(-\frac{u^{1'}}{c^2}\gamma\right)\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{3'}}{c}+1\right)\right]+$$
$$+\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{3'}}{c}\right)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)\left(-\frac{u^{2'}}{c^2}\gamma\right)-\left(-\frac{u^{1'}}{c^2}\gamma\right)\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{2'}}{c}\right)\right]$$
$$=\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{1'}}{c}+1\right)\left[\left(\frac{u^{3'}}{c}\frac{u^{2'}}{c}\frac{u^{3'}}{c^2}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)(-\gamma)-(-\gamma)\left(\frac{\gamma^2}{\gamma+1}\right)\right]-\left(-\frac{u^{2'}}{c^2}\gamma\right)\right]+$$
$$-\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{2'}}{c}\right)\left[\left(\frac{u^{3'}}{c}\frac{u^{1'}}{c}\frac{u^{3'}}{c^2}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)(-\gamma)-(-\gamma)\left(\frac{\gamma^2}{\gamma+1}\right)\right]-\left(-\frac{u^{1'}}{c^2}\gamma\right)\right]+$$
$$+\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{3'}}{c}\right)\left[\left(\frac{u^{3'}}{c}\frac{u^{1'}}{c}\frac{u^{2'}}{c^2}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)(-\gamma)-(-\gamma)\left(\frac{\gamma^2}{\gamma+1}\right)\right]\right]$$
$$=\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{1'}}{c}+1\right)\left[-\left(-\frac{u^{2'}}{c^2}\gamma\right)\right]-\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{2'}}{c}\right)\left[-\left(-\frac{u^{1'}}{c^2}\gamma\right)\right]$$
$$=\left(\frac{u^{1'}}{c}\frac{u^{1'}}{c}\frac{u^{2'}}{c^2}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)(-(-\gamma))-\left(\frac{\gamma^2}{\gamma+1}\right)-(-\gamma)\right]+\left[-\left(-\frac{u^{2'}}{c^2}\gamma\right)\right]$$

$\therefore$

$$\Delta_{24}=\frac{u^{2'}}{c^2}\gamma$$

$$\Delta_{31}=\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{2'}}{c}\right)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{3'}}{c}\right)(\gamma)-\left(-\frac{u^{3'}}{c^2}\gamma\right)(-u^{2'}\gamma)\right]+$$
$$-\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{3'}}{c}\right)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{2'}}{c}+1\right)(\gamma)-\left(-\frac{u^{2'}}{c^2}\gamma\right)(-u^{2'}\gamma)\right]+$$
$$+(-u^{1'}\gamma)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{2'}}{c}+1\right)\left(-\frac{u^{3'}}{c^2}\gamma\right)-\left(-\frac{u^{2'}}{c^2}\gamma\right)\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{3'}}{c}\right)\right]$$
$$=\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{2'}}{c}\right)\left[\left(\frac{u^{2'}}{c}\frac{u^{3'}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)(\gamma)-\gamma^2\right]\right]+$$
$$-\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{3'}}{c}\right)\left[\left(\frac{u^{2'}}{c}\frac{u^{2'}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)(\gamma)-\gamma^2\right]+\gamma\right]+$$
$$+(-u^{1'}\gamma)\left[\left(\frac{u^{2'}}{c}\frac{u^{2'}}{c}\frac{u^{3'}}{c^2}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)(-\gamma)-(-\gamma)\left(\frac{\gamma^2}{\gamma+1}\right)\right]+\left(-\frac{u^{3'}}{c^2}\gamma\right)\right]$$
$$=\left(\frac{u^{1'}}{c}\frac{u^{2'}}{c}\right)\left(\frac{u^{2'}}{c}\frac{u^{3'}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)^2\right]-\left(\frac{u^{1'}}{c}\frac{u^{3'}}{c}\right)\left(\frac{u^{2'}}{c}\frac{u^{2'}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)^2\right]+$$
$$-\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{3'}}{c}\right)(\gamma)+(-u^{1'}\gamma)\left(-\frac{u^{3'}}{c^2}\gamma\right)$$
$$=\left(\frac{u^{1'}}{c}\frac{u^{3'}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)(\gamma)+\gamma^2\right]$$

$\therefore$

$$\Delta_{31}=\left(\frac{u^{1'}}{c}\frac{u^{3'}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)\right]$$

$$\Delta_{32}=\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{1'}}{c}+1\right)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{3'}}{c}\right)(\gamma)-\left(-\frac{u^{3'}}{c^2}\gamma\right)(-u^{2'}\gamma)\right]+$$
$$-\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{3'}}{c}\right)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{1'}}{c}\right)(\gamma)-\left(-\frac{u^{1'}}{c^2}\gamma\right)(-u^{2'}\gamma)\right]+$$
$$+(-u^{1'}\gamma)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{1'}}{c}\right)\left(-\frac{u^{3'}}{c^2}\gamma\right)-\left(-\frac{u^{1'}}{c^2}\gamma\right)\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{3'}}{c}\right)\right]$$
$$=\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{1'}}{c}+1\right)\left[\left(\frac{u^{2'}}{c}\frac{u^{3'}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)(\gamma)-\gamma^2\right]\right]+$$
$$-\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{3'}}{c}\right)\left[\left(\frac{u^{2'}}{c}\frac{u^{1'}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)(-\gamma)-\gamma^2\right]\right]+$$
$$+(-u^{1'}\gamma)\left[\left(\frac{u^{2'}}{c}\frac{u^{1'}}{c}\frac{u^{3'}}{c^2}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)(-\gamma)-(-\gamma)\left(\frac{\gamma^2}{\gamma+1}\right)\right]\right]$$
$$=\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{1'}}{c}+1\right)\left(\frac{u^{2'}}{c}\frac{u^{3'}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right]+$$
$$-\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{3'}}{c}\right)\left(\frac{u^{2'}}{c}\frac{u^{1'}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right]$$
$$=\left(\frac{u^{1'}}{c}\frac{u^{1'}}{c}\right)\left(\frac{u^{2'}}{c}\frac{u^{3'}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)^2\right]+\left(\frac{u^{1'}}{c}\frac{u^{3'}}{c}\right)\left(\frac{u^{2'}}{c}\frac{u^{1'}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)^2\right]+$$
$$+\left(\frac{u^{2'}}{c}\frac{u^{3'}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right]$$

∴

$$\Delta_{32} = \left(\frac{u^{2\prime}}{c}\frac{u^{3\prime}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right]$$

$$\begin{aligned}
\Delta_{33} = {} & \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1\prime}}{c}\frac{u^{1\prime}}{c}+1\right)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2\prime}}{c}\frac{u^{2\prime}}{c}+1\right)(\gamma)-\left(-\frac{u^{2\prime}}{c^2}\gamma\right)(-u^{2\prime}\gamma)\right]+\\
& -\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1\prime}}{c}\frac{u^{2\prime}}{c}\right)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2\prime}}{c}\frac{u^{1\prime}}{c}\right)(\gamma)-\left(-\frac{u^{1\prime}}{c^2}\gamma\right)(-u^{2\prime}\gamma)\right]+\\
& +(-u^{1\prime}\gamma)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2\prime}}{c}\frac{u^{1\prime}}{c}\right)\left(-\frac{u^{2\prime}}{c^2}\gamma\right)-\left(-\frac{u^{1\prime}}{c^2}\gamma\right)\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2\prime}}{c}\frac{u^{2\prime}}{c}+1\right)\right]\\
= {} & \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1\prime}}{c}\frac{u^{1\prime}}{c}+1\right)\left[\left(\frac{u^{2\prime}}{c}\frac{u^{2\prime}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)\gamma-\gamma^2\right]+\gamma\right]+\\
& -\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1\prime}}{c}\frac{u^{2\prime}}{c}\right)\left[\left(\frac{u^{2\prime}}{c}\frac{u^{1\prime}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)\gamma-\gamma^2\right]\right]+\\
& +(-u^{1\prime}\gamma)\left[\left(\frac{u^{2\prime}}{c}\frac{u^{1\prime}}{c}\frac{u^{2\prime}}{c^2}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)(-\gamma)-(-\gamma)\left(\frac{\gamma^2}{\gamma+1}\right)\right]-\left(-\frac{u^{1\prime}}{c^2}\gamma\right)\right]\\
= {} & \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1\prime}}{c}\frac{u^{1\prime}}{c}+1\right)\left[\left(\frac{u^{2\prime}}{c}\frac{u^{2\prime}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right]+\gamma\right]+\\
& -\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1\prime}}{c}\frac{u^{2\prime}}{c}\right)\left[\left(\frac{u^{2\prime}}{c}\frac{u^{1\prime}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right]\right]+\\
& +(-u^{1\prime}\gamma)\left[-\left(-\frac{u^{1\prime}}{c^2}\gamma\right)\right]\\
= {} & \left(\frac{u^{1\prime}}{c}\frac{u^{1\prime}}{c}\right)\left(\frac{u^{2\prime}}{c}\frac{u^{2\prime}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)^2\right]+\left(\frac{u^{1\prime}}{c}\frac{u^{1\prime}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)\gamma\right]+\\
& +\left(\frac{u^{2\prime}}{c}\frac{u^{2\prime}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right]+\gamma+\\
& -\left(\frac{u^{1\prime}}{c}\frac{u^{2\prime}}{c}\right)\left(\frac{u^{2\prime}}{c}\frac{u^{1\prime}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)^2\right]+\left(\frac{u^{1\prime}}{c}\frac{u^{1\prime}}{c}\right)[-\gamma^2]
\end{aligned}$$

∴

$$\Delta_{33} = \left(\frac{u^{1\prime}}{c}\frac{u^{1\prime}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right]+\left(\frac{u^{2\prime}}{c}\frac{u^{2\prime}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right]+\gamma$$

$$\begin{aligned}
\Delta_{34} = {} & \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1\prime}}{c}\frac{u^{1\prime}}{c}+1\right)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2\prime}}{c}\frac{u^{2\prime}}{c}+1\right)\left(-\frac{u^{3\prime}}{c^2}\gamma\right)-\left(-\frac{u^{2\prime}}{c^2}\gamma\right)\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2\prime}}{c}\frac{u^{3\prime}}{c}\right)\right]+\\
& -\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1\prime}}{c}\frac{u^{2\prime}}{c}\right)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2\prime}}{c}\frac{u^{1\prime}}{c}\right)\left(-\frac{u^{3\prime}}{c^2}\gamma\right)-\left(-\frac{u^{1\prime}}{c^2}\gamma\right)\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2\prime}}{c}\frac{u^{3\prime}}{c}\right)\right]+\\
& +\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1\prime}}{c}\frac{u^{3\prime}}{c}\right)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2\prime}}{c}\frac{u^{1\prime}}{c}\right)\left(-\frac{u^{2\prime}}{c^2}\gamma\right)-\left(-\frac{u^{1\prime}}{c^2}\gamma\right)\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2\prime}}{c}\frac{u^{2\prime}}{c}+1\right)\right]\\
= {} & \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1\prime}}{c}\frac{u^{1\prime}}{c}+1\right)\left[\left(\frac{u^{2\prime}}{c}\frac{u^{2\prime}}{c}\frac{u^{3\prime}}{c^2}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)(-\gamma)-(-\gamma)\left(\frac{\gamma^2}{\gamma+1}\right)\right]+\left(-\frac{u^{3\prime}}{c^2}\gamma\right)\right]+\\
& -\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1\prime}}{c}\frac{u^{2\prime}}{c}\right)\left[\left(\frac{u^{2\prime}}{c}\frac{u^{1\prime}}{c}\frac{u^{3\prime}}{c^2}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)(-\gamma)-(-\gamma)\left(\frac{\gamma^2}{\gamma+1}\right)\right]\right]+\\
& +\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1\prime}}{c}\frac{u^{3\prime}}{c}\right)\left[\left(\frac{u^{2\prime}}{c}\frac{u^{1\prime}}{c}\frac{u^{2\prime}}{c^2}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)(-\gamma)-(-\gamma)\left(\frac{\gamma^2}{\gamma+1}\right)\right]-\left(-\frac{u^{1\prime}}{c^2}\gamma\right)\right]\\
= {} & \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1\prime}}{c}\frac{u^{1\prime}}{c}+1\right)\left[\left(-\frac{u^{3\prime}}{c^2}\gamma\right)\right]+\\
& +\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1\prime}}{c}\frac{u^{3\prime}}{c}\right)\left[-\left(-\frac{u^{1\prime}}{c^2}\gamma\right)\right]
\end{aligned}$$

∴

$$\Delta_{34} = -\frac{u^{3\prime}}{c^2}\gamma$$

$$\begin{aligned}
\Delta_{41} = {} & \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1\prime}}{c}\frac{u^{2\prime}}{c}\right)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2\prime}}{c}\frac{u^{3\prime}}{c}\right)(-u^{3\prime}\gamma)-\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3\prime}}{c}\frac{u^{3\prime}}{c}+1\right)(-u^{2\prime}\gamma)\right]+\\
& -\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1\prime}}{c}\frac{u^{3\prime}}{c}\right)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2\prime}}{c}\frac{u^{2\prime}}{c}+1\right)(-u^{3\prime}\gamma)-\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3\prime}}{c}\frac{u^{2\prime}}{c}\right)(-u^{2\prime}\gamma)\right]+\\
& +(-u^{1\prime}\gamma)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2\prime}}{c}\frac{u^{2\prime}}{c}+1\right)\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3\prime}}{c}\frac{u^{3\prime}}{c}+1\right)-\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3\prime}}{c}\frac{u^{2\prime}}{c}\right)\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2\prime}}{c}\frac{u^{3\prime}}{c}\right)\right]\\
= {} & \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1\prime}}{c}\frac{u^{2\prime}}{c}\right)\left[\left(\frac{u^{2\prime}}{c}\frac{u^{3\prime}}{c}u^{3\prime}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)(-\gamma)-\left(\frac{\gamma^2}{\gamma+1}\right)(-\gamma)\right]-(-u^{2\prime}\gamma)\right]+\\
& -\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1\prime}}{c}\frac{u^{3\prime}}{c}\right)\left[\left(\frac{u^{2\prime}}{c}\frac{u^{2\prime}}{c}u^{3\prime}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)(-\gamma)-\left(\frac{\gamma^2}{\gamma+1}\right)(-\gamma)\right]+(-u^{3\prime}\gamma)\right]+\\
& +(-u^{1\prime}\gamma)\Big[\left(\frac{u^{2\prime}}{c}\frac{u^{2\prime}}{c}\right)\left(\frac{u^{3\prime}}{c}\frac{u^{3\prime}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)^2\right]+\left(\frac{u^{2\prime}}{c}\frac{u^{2\prime}}{c}\right)\left(\frac{\gamma^2}{\gamma+1}\right)+\\
& +\left(\frac{u^{3\prime}}{c}\frac{u^{3\prime}}{c}\right)\left(\frac{\gamma^2}{\gamma+1}\right)+1-\left(\frac{u^{3\prime}}{c}\frac{u^{2\prime}}{c}\right)\left(\frac{u^{2\prime}}{c}\frac{u^{3\prime}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)^2\right]\Big]\\
= {} & \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1\prime}}{c}\frac{u^{2\prime}}{c}\right)[-(-u^{2\prime}\gamma)]-\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1\prime}}{c}\frac{u^{3\prime}}{c}\right)[(-u^{3\prime}\gamma)]+\\
& +(-u^{1\prime}\gamma)\left[\left(\frac{u^{2\prime}}{c}\frac{u^{2\prime}}{c}\right)\left(\frac{\gamma^2}{\gamma+1}\right)+\left(\frac{u^{3\prime}}{c}\frac{u^{3\prime}}{c}\right)\left(\frac{\gamma^2}{\gamma+1}\right)+1\right]
\end{aligned}$$

∴

$$\Delta_{41} = -u^{1'}\gamma$$

$$\begin{aligned}
\Delta_{42} = {} & \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{1'}}{c}+1\right)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{3'}}{c}\right)(-u^{3'}\gamma) - \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{3'}}{c}+1\right)(-u^{2'}\gamma)\right] + \\
& -\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{3'}}{c}\right)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{1'}}{c}\right)(-u^{3'}\gamma) - \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)(-u^{2'}\gamma)\right] + \\
& +(-u^{1'}\gamma)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{1'}}{c}\right)\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{3'}}{c}+1\right) - \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{3'}}{c}\right)\right] \\
= {} & \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{1'}}{c}+1\right)\left[\left(\frac{u^{2'}}{c}\frac{u^{3'}}{c}u^{3'}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)(-\gamma) - \left(\frac{\gamma^2}{\gamma+1}\right)(-\gamma)\right] - (-u^{2'}\gamma)\right] + \\
& -\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{3'}}{c}\right)\left[\left(\frac{u^{2'}}{c}\frac{u^{1'}}{c}u^{3'}\right)(-\gamma) - \left(\frac{\gamma^2}{\gamma+1}\right)(-\gamma)\right] + \\
& +(-u^{1'}\gamma)\left[\left(\frac{u^{2'}}{c}\frac{u^{1'}}{c}\right)\left(\frac{u^{3'}}{c}\frac{u^{3'}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)^2\right] + \left(\frac{u^{2'}}{c}\frac{u^{1'}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)\right] + \right. \\
& \left. -\left(\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)\left(\frac{u^{2'}}{c}\frac{u^{3'}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)^2\right]\right] \\
= {} & \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{1'}}{c}+1\right)[-(-u^{2'}\gamma)] + (-u^{1'}\gamma)\left[\left(\frac{u^{2'}}{c}\frac{u^{1'}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)\right]\right] \\
= {} & \left(\frac{u^{1'}}{c}\frac{u^{1'}}{c}u^{2'}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)(\gamma) + (-\gamma)\left(\frac{\gamma^2}{\gamma+1}\right)\right] + [-(-u^{2'}\gamma)]
\end{aligned}$$

$\therefore$

$$\Delta_{42} = u^{2'}\gamma$$

$$\begin{aligned}
\Delta_{43} = {} & \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{1'}}{c}+1\right)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{2'}}{c}+1\right)(-u^{3'}\gamma) - \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{2'}}{c}\right)(-u^{2'}\gamma)\right] + \\
& -\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{2'}}{c}\right)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{1'}}{c}\right)(-u^{3'}\gamma) - \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)(-u^{2'}\gamma)\right] + \\
& +(-u^{1'}\gamma)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{1'}}{c}\right)\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{2'}}{c}\right) - \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{2'}}{c}+1\right)\right] \\
= {} & \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{1'}}{c}+1\right)\left[\left(\frac{u^{2'}}{c}\frac{u^{2'}}{c}u^{3'}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)(-\gamma) - \left(\frac{\gamma^2}{\gamma+1}\right)(-\gamma)\right] + (-u^{3'}\gamma)\right] + \\
& -\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{2'}}{c}\right)\left[\left(\frac{u^{2'}}{c}\frac{u^{1'}}{c}u^{3'}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)(-\gamma) - \left(\frac{\gamma^2}{\gamma+1}\right)(-\gamma)\right]\right] + \\
& +(-u^{1'}\gamma)\left[\left(\frac{u^{2'}}{c}\frac{u^{1'}}{c}\right)\left(\frac{u^{3'}}{c}\frac{u^{2'}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)^2\right] - \left(\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)\left(\frac{u^{2'}}{c}\frac{u^{2'}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)^2\right] + \right. \\
& \qquad \left. -\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)\right] \\
= {} & \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{1'}}{c}+1\right)[(-u^{3'}\gamma)] + (-u^{1'}\gamma)\left[-\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)\right] \\
= {} & \left(\frac{u^{1'}}{c}\frac{u^{1'}}{c}u^{3'}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)(-\gamma) + (-\gamma) - \left(\left(\frac{\gamma^2}{\gamma+1}\right)\right)\right] + (-u^{3'}\gamma)
\end{aligned}$$

$\therefore$

$$\Delta_{43} = -u^{3'}\gamma$$

$$\begin{aligned}
\Delta_{44} = {} & \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{1'}}{c}+1\right)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{2'}}{c}+1\right)\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{3'}}{c}+1\right) - \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{2'}}{c}\right)\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{3'}}{c}\right)\right] + \\
& -\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{2'}}{c}\right)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{1'}}{c}\right)\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{3'}}{c}+1\right) - \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{3'}}{c}\right)\right] + \\
& +\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{3'}}{c}\right)\left[\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{1'}}{c}\right)\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{2'}}{c}\right) - \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{2'}}{c}+1\right)\right] \\
= {} & \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{1'}}{c}+1\right)\left[\left(\frac{u^{2'}}{c}\frac{u^{2'}}{c}\right)\left(\frac{u^{3'}}{c}\frac{u^{3'}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)^2\right] + \left(\frac{u^{2'}}{c}\frac{u^{2'}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)\right] + \right. \\
& \qquad \left. + \left(\frac{u^{3'}}{c}\frac{u^{3'}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)\right] + 1 - \left(\frac{u^{3'}}{c}\frac{u^{2'}}{c}\right)\left(\frac{u^{2'}}{c}\frac{u^{3'}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)^2\right]\right] + \\
& -\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{2'}}{c}\right)\left[\left(\frac{u^{2'}}{c}\frac{u^{1'}}{c}\right)\left(\frac{u^{3'}}{c}\frac{u^{3'}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)^2\right] + \left(\frac{u^{2'}}{c}\frac{u^{1'}}{c}\right)\left(\frac{\gamma^2}{\gamma+1}\right) + \right. \\
& \qquad \left. -\left(\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)\left(\frac{u^{2'}}{c}\frac{u^{3'}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)^2\right]\right] + \\
& +\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{3'}}{c}\right)\left[\left(\frac{u^{2'}}{c}\frac{u^{1'}}{c}\right)\left(\frac{u^{3'}}{c}\frac{u^{2'}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)^2\right] - \left(\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)\left(\frac{u^{2'}}{c}\frac{u^{2'}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)^2\right] + \right. \\
& \qquad \left. -\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)\right] \\
= {} & \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{1'}}{c}+1\right)\left[\left(\frac{u^{2'}}{c}\frac{u^{2'}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)\right] + \left(\frac{u^{3'}}{c}\frac{u^{3'}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)\right] + 1\right] + \\
& -\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{2'}}{c}\right)\left[\left(\frac{u^{2'}}{c}\frac{u^{1'}}{c}\right)\left(\frac{\gamma^2}{\gamma+1}\right)\right] \\
& +\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{3'}}{c}\right)\left[-\left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{1'}}{c}\right)\right]
\end{aligned}$$

$$
\begin{aligned}
&= \left(\frac{u^{1\prime}}{c}\frac{u^{1\prime}}{c}\right)\left(\frac{u^{2\prime}}{c}\frac{u^{2\prime}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)^2\right] + \left(\frac{u^{2\prime}}{c}\frac{u^{2\prime}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)\right] + \\
&+ \left(\frac{u^{1\prime}}{c}\frac{u^{1\prime}}{c}\right)\left(\frac{u^{3\prime}}{c}\frac{u^{3\prime}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)^2\right] + \left(\frac{u^{3\prime}}{c}\frac{u^{3\prime}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)\right] + \\
&- \left(\frac{u^{1\prime}}{c}\frac{u^{2\prime}}{c}\right)\left(\frac{u^{2\prime}}{c}\frac{u^{1\prime}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)^2\right] + \\
&+ \left(\frac{u^{1\prime}}{c}\frac{u^{1\prime}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)\right] + 1 + \left(\frac{u^{1\prime}}{c}\frac{u^{3\prime}}{c}\right)\left(\frac{u^{3\prime}}{c}\frac{u^{1\prime}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)^2\right] \\
&= 1 + \left(\frac{\gamma^2}{\gamma+1}\right)\frac{|\vec{\mathbf{u}}|^2}{c^2}
\end{aligned}
$$

$\therefore$

$$\Delta_{44} = \gamma$$

$$
\begin{aligned}
\Delta &= \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1\prime}}{c}\frac{u^{1\prime}}{c} + 1\right)(\Delta_{11}) + \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1\prime}}{c}\frac{u^{2\prime}}{c}\right)(-\Delta_{12}) + \\
&\qquad + \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1\prime}}{c}\frac{u^{3\prime}}{c}\right)(\Delta_{13}) + (-u^{1\prime}\gamma)(-\Delta_{14}) \\
&= \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1\prime}}{c}\frac{u^{1\prime}}{c} + 1\right)\left[\left(\frac{u^{2\prime}}{c}\frac{u^{2\prime}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right] + \left(\frac{u^{3\prime}}{c}\frac{u^{3\prime}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right] + \gamma\right] + \\
&\qquad + \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1\prime}}{c}\frac{u^{2\prime}}{c}\right)\left[-\left(\frac{u^{2\prime}}{c}\frac{u^{1\prime}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right]\right] + \\
&\qquad + \left(\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1\prime}}{c}\frac{u^{3\prime}}{c}\right)\left[\left(\frac{u^{3\prime}}{c}\frac{u^{1\prime}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)\right]\right] + \\
&\qquad + (-u^{1\prime}\gamma)\left[-\left(-\frac{u^{1\prime}}{c^2}\gamma\right)\right] \\
&= \left(\frac{u^{1\prime}}{c}\frac{u^{1\prime}}{c}\right)\left(\frac{u^{2\prime}}{c}\frac{u^{2\prime}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)^2\right] + \\
&\qquad + \left(\frac{u^{1\prime}}{c}\frac{u^{1\prime}}{c}\right)\left(\frac{u^{3\prime}}{c}\frac{u^{3\prime}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)^2\right] + \left(\frac{u^{1\prime}}{c}\frac{u^{1\prime}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)\gamma\right] + \\
&\qquad + \left(\frac{u^{2\prime}}{c}\frac{u^{2\prime}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right] + \left(\frac{u^{3\prime}}{c}\frac{u^{3\prime}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right] + \gamma + \\
&\qquad - \left(\frac{u^{1\prime}}{c}\frac{u^{2\prime}}{c}\right)\left(\frac{u^{2\prime}}{c}\frac{u^{1\prime}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)^2\right] + \\
&\qquad + \left(\frac{u^{1\prime}}{c}\frac{u^{3\prime}}{c}\right)\left(\frac{u^{3\prime}}{c}\frac{u^{1\prime}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)^2\right] + \left(\frac{u^{1\prime}}{c}\frac{u^{1\prime}}{c}\right)[-\gamma^2] \\
&= \left[\left(\frac{u^{1\prime}}{c}\frac{u^{1\prime}}{c}\right) + \left(\frac{u^{2\prime}}{c}\frac{u^{2\prime}}{c}\right) + \left(\frac{u^{3\prime}}{c}\frac{u^{3\prime}}{c}\right)\right]\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right] \\
&= -\left(\frac{\gamma^2}{\gamma+1}\right)\frac{|\vec{\mathbf{u}}|^2}{c^2} + \gamma = 1
\end{aligned}
$$

$\therefore$

$$
\begin{aligned}
\hat{\mathbf{i}} &= \hat{\mathbf{i}}'\left(\frac{\Delta_{11}}{\Delta}\right) + \hat{\mathbf{j}}'\left(\frac{-\Delta_{12}}{\Delta}\right) + \hat{\mathbf{k}}'\left(\frac{\Delta_{13}}{\Delta}\right) + \hat{\mathbf{t}}'\left(\frac{-\Delta_{14}}{\Delta}\right) \\
\hat{\mathbf{j}} &= \hat{\mathbf{i}}'\left(\frac{-\Delta_{21}}{\Delta}\right) + \hat{\mathbf{j}}'\left(\frac{\Delta_{22}}{\Delta}\right) + \hat{\mathbf{k}}'\left(\frac{-\Delta_{23}}{\Delta}\right) + \hat{\mathbf{t}}'\left(\frac{\Delta_{24}}{\Delta}\right) \\
\hat{\mathbf{k}} &= \hat{\mathbf{i}}'\left(\frac{\Delta_{31}}{\Delta}\right) + \hat{\mathbf{j}}'\left(\frac{-\Delta_{32}}{\Delta}\right) + \hat{\mathbf{k}}'\left(\frac{\Delta_{33}}{\Delta}\right) + \hat{\mathbf{t}}'\left(\frac{-\Delta_{34}}{\Delta}\right) \\
\hat{\mathbf{t}} &= \hat{\mathbf{i}}'\left(\frac{-\Delta_{41}}{\Delta}\right) + \hat{\mathbf{j}}'\left(\frac{\Delta_{42}}{\Delta}\right) + \hat{\mathbf{k}}'\left(\frac{-\Delta_{43}}{\Delta}\right) + \hat{\mathbf{t}}'\left(\frac{\Delta_{44}}{\Delta}\right)
\end{aligned}
$$

so:

$$
\begin{aligned}
\hat{\mathbf{i}} &= \hat{\mathbf{i}}'\left(\frac{\Delta_{11}}{\Delta}\right) + \hat{\mathbf{j}}'\left(\frac{-\Delta_{12}}{\Delta}\right) + \hat{\mathbf{k}}'\left(\frac{\Delta_{13}}{\Delta}\right) + \hat{\mathbf{t}}'\left(\frac{-\Delta_{14}}{\Delta}\right) \\
&= \hat{\mathbf{i}}'\left(\left(\frac{u^{2\prime}}{c}\frac{u^{2\prime}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right] + \left(\frac{u^{3\prime}}{c}\frac{u^{3\prime}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right] + \gamma\right) + \\
&+ \hat{\mathbf{j}}'\left(-\left(\frac{u^{2\prime}}{c}\frac{u^{1\prime}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right]\right) + \\
&+ \hat{\mathbf{k}}'\left(\left(\frac{u^{3\prime}}{c}\frac{u^{1\prime}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)\right]\right) + \hat{\mathbf{t}}'\left(-\left(-\frac{u^{1\prime}}{c^2}\gamma\right)\right) \\
&= \hat{\mathbf{i}}'\left[\left(\frac{\gamma^2}{\gamma+1}\right)\left(\frac{u^{1\prime}}{c}\frac{u^{1\prime}}{c}\right) + 1\right] + \hat{\mathbf{j}}'\left[\left(\frac{\gamma^2}{\gamma+1}\right)\left(\frac{u^{2\prime}}{c}\frac{u^{1\prime}}{c}\right)\right] + \\
&+ \hat{\mathbf{k}}'\left[\left(\frac{\gamma^2}{\gamma+1}\right)\left(\frac{u^{3\prime}}{c}\frac{u^{1\prime}}{c}\right)\right] + \hat{\mathbf{t}}'\left[\frac{u^{1\prime}}{c^2}\gamma\right]
\end{aligned}
$$

$$
\begin{aligned}
\hat{\mathbf{j}} &= \hat{\mathbf{i}}'\left(\frac{-\Delta_{21}}{\Delta}\right) + \hat{\mathbf{j}}'\left(\frac{\Delta_{22}}{\Delta}\right) + \hat{\mathbf{k}}'\left(\frac{-\Delta_{23}}{\Delta}\right) + \hat{\mathbf{t}}'\left(\frac{\Delta_{24}}{\Delta}\right) \\
&= \hat{\mathbf{i}}'\left(-\left(\frac{u^{1\prime}}{c}\frac{u^{2\prime}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right]\right) +
\end{aligned}
$$

$$
\begin{aligned}
&+ \widehat{\mathbf{j}}'\left(\left(\frac{u^{1'}}{c}\frac{u^{1'}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right] + \left(\frac{u^{3'}}{c}\frac{u^{3'}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right] + \gamma\right) + \\
&+ \widehat{\mathbf{k}}'\left(-\left(\frac{u^{3'}}{c}\frac{u^{2'}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right]\right) + \widehat{\mathbf{t}}'\left(\frac{u^{2'}}{c^2}\gamma\right) \\
&= \widehat{\mathbf{i}}'\left[\left(\frac{\gamma^2}{\gamma+1}\right)\left(\frac{u^{1'}}{c}\frac{u^{2'}}{c}\right)\right] + \widehat{\mathbf{j}}'\left[\left(\frac{\gamma^2}{\gamma+1}\right)\left(\frac{u^{2'}}{c}\frac{u^{2'}}{c}\right) + 1\right] + \\
&+ \widehat{\mathbf{k}}'\left[\left(\frac{\gamma^2}{\gamma+1}\right)\left(\frac{u^{3'}}{c}\frac{u^{2'}}{c}\right)\right] + \widehat{\mathbf{t}}'\left[\frac{u^{2'}}{c^2}\gamma\right]
\end{aligned}
$$

$$
\begin{aligned}
\widehat{\mathbf{k}} &= \widehat{\mathbf{i}}'\left(\frac{\Delta_{31}}{\Delta}\right) + \widehat{\mathbf{j}}'\left(\frac{-\Delta_{32}}{\Delta}\right) + \widehat{\mathbf{k}}'\left(\frac{\Delta_{33}}{\Delta}\right) + \widehat{\mathbf{t}}'\left(\frac{-\Delta_{34}}{\Delta}\right) \\
&= \widehat{\mathbf{i}}'\left(\left(\frac{u^{1'}}{c}\frac{u^{3'}}{c}\right)\left[\left(\frac{\gamma^2}{\gamma+1}\right)\right]\right) + \widehat{\mathbf{j}}'\left(-\left(\frac{u^{2'}}{c}\frac{u^{3'}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right]\right) + \\
&+ \widehat{\mathbf{k}}'\left(\left(\frac{u^{1'}}{c}\frac{u^{1'}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right] + \left(\frac{u^{2'}}{c}\frac{u^{2'}}{c}\right)\left[-\left(\frac{\gamma^2}{\gamma+1}\right)\right] + \gamma\right) + \widehat{\mathbf{t}}'\left(-\left(-\frac{u^{3'}}{c^2}\gamma\right)\right) \\
&= \widehat{\mathbf{i}}'\left[\left(\frac{\gamma^2}{\gamma+1}\right)\left(\frac{u^{1'}}{c}\frac{u^{3'}}{c}\right)\right] + \widehat{\mathbf{j}}'\left[\left(\frac{\gamma^2}{\gamma+1}\right)\left(\frac{u^{2'}}{c}\frac{u^{3'}}{c}\right)\right] + \\
&+ \widehat{\mathbf{k}}'\left[\left(\frac{\gamma^2}{\gamma+1}\right)\left(\frac{u^{3'}}{c}\frac{u^{3'}}{c}\right) + 1\right] + \widehat{\mathbf{t}}'\left[\frac{u^{3'}}{c^2}\gamma\right]
\end{aligned}
$$

$$
\begin{aligned}
\widehat{\mathbf{t}} &= \widehat{\mathbf{i}}'\left(\frac{-\Delta_{41}}{\Delta}\right) + \widehat{\mathbf{j}}'\left(\frac{\Delta_{42}}{\Delta}\right) + \widehat{\mathbf{k}}'\left(\frac{-\Delta_{43}}{\Delta}\right) + \widehat{\mathbf{t}}'\left(\frac{\Delta_{44}}{\Delta}\right) \\
&= \widehat{\mathbf{i}}'(-(-u^{1'}\gamma)) + \widehat{\mathbf{j}}'(u^{2'}\gamma) + \widehat{\mathbf{k}}'(-(-u^{3'}\gamma)) + \widehat{\mathbf{t}}'(\gamma) \\
&= \widehat{\mathbf{i}}'(u^{1'}\gamma) + \widehat{\mathbf{j}}'(u^{2'}\gamma) + \widehat{\mathbf{k}}'(u^{3'}\gamma) + \widehat{\mathbf{t}}'(\gamma)
\end{aligned}
$$

$\therefore$

$\frac{\partial x'}{\partial x} = \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{1'}}{c} + 1$	$\frac{\partial x'}{\partial y} = \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{2'}}{c}$	$\frac{\partial x'}{\partial z} = \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{1'}}{c}\frac{u^{3'}}{c}$	$\frac{\partial x'}{\partial t} = u^{1'}\gamma$
$\frac{\partial y'}{\partial x} = \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{1'}}{c}$	$\frac{\partial y'}{\partial y} = \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{2'}}{c} + 1$	$\frac{\partial y'}{\partial z} = \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{2'}}{c}\frac{u^{3'}}{c}$	$\frac{\partial y'}{\partial t} = u^{2'}\gamma$
$\frac{\partial z'}{\partial x} = \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{1'}}{c}$	$\frac{\partial z'}{\partial y} = \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{2'}}{c}$	$\frac{\partial z'}{\partial z} = \left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{3'}}{c}\frac{u^{3'}}{c} + 1$	$\frac{\partial z'}{\partial t} = u^{3'}\gamma$
$\frac{\partial t'}{\partial x} = \frac{u^{1'}}{c^2}\gamma$	$\frac{\partial t'}{\partial y} = \frac{u^{2'}}{c^2}\gamma$	$\frac{\partial t'}{\partial z} = \frac{u^{3'}}{c^2}\gamma$	$\frac{\partial t'}{\partial t} = \gamma$

Thus, the complete Lorentz Transformations between reference frames using Rectangular Cartesian Coordinate Systems, is given by:

$x^{k'} = x^j\left\{\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^{k'}}{c}\frac{u^{j'}}{c} + \delta^k_j\right\} + \gamma u^{k'} t$
$t' = \gamma\left(t + \frac{u^{j'}}{c^2}x^j\right)$
$x^{k'} = x^j\left\{\left(\frac{\gamma^2}{\gamma+1}\right)\frac{u^k}{c}\frac{u^j}{c} + \delta^k_j\right\} - \gamma u^k t$
$t' = \gamma\left(t - \frac{u^j}{c^2}x^j\right)$