Gravitation Theorem vs. Shell Theorem.
Conic Proof.

W. Westenberger

SUMMARY. The gravitation theorem is based on Newton's Gravitation Law and takes into account that each gravitational object does exert gravitation starting from its very position.
The Shell Theorem, on the other hand, sums up all positions of gravitational objects, thus calculations of gravitation are made easier.
A spherically symmetric body of equal density can be modelled as an infinite number of cones within a globe, starting from a point A upon the radius of the globe. If one of these cones can be treated as a point mass (i.e. as a single gravitation point), then a system of cones in the shape of a sphere can also be treated as a point mass.

INTRODUCTION.

Gravitation theorem: The total gravitational force of spherically symmetric distributed masses within a global space affects an object outside of the globe as though all masses were concentrated in a point outside of the very centre of the globe. [1]
That means: The distance between a point A upon the radius and the **single gravitation point** of a globe, representing the sum of the gravitational effects of all masses within the globe,
is **smaller than the distance R** between A and the centre.

Shell theorem: A spherically symmetric body affects external objects gravitationally as though all of its mass were concentrated at a point at its centre. [2]
Therefore: The distance between a point A upon the radius and the **single gravitation point** of a globe, representing the sum of the gravitational effects of all masses within the globe,
is **exactly equal to the distance R**.

Consider the special case of spherically symmetric distribution of masses within a globe of equal density. Gravitational cones, completely filling the globe, are presented as a line of argument. Thus the crucial difference between these two theorems is shown. This way it is possible to get a conclusive judgement. This may be called the conic proof.

GEOMETRIC FACTS.

The following considerations are based on the inverse square relation of Newton's gravitation law: Doubling the distance between two masses you will get a quarter of the gravitational force.

There is another well-known square relation:
Doubling the radius of a circle you will get four times the area.

Presuming equal distribution of objects within the area of a circle, you will get four times the gravitation as well as four times the area by doubling the radius.
Imagine a conical space starting from a point A.
At an arbitrary height above A upon the symmetrical axis of the cone, a right-angled plane will cut the surface of the cone at a certain radius, forming the boundary of the circle area.
Doubling this height you will get a circle of double radius and of four times the circle area. Therefore also four times the mass of the circle area.

Four times the mass of an area at double distance will exert the same gravitation as one time the mass at the distance one.

As the area of each circle within a cone is dependent on the height H above A, and as on the other hand gravitation is inversely dependent on H, both effects are balancing out. Therefore **cutting the cone at any height whatever you will get the identic amount of gravitation.** [3]

LINE OF ARGUMENT.

Imagine a constant globe consisting of many objects at equal density.

Consider a cone starting at A. The centre of the globe is located upon the symmetric axis of the cone. The radius R of the globe is passing through A.
The globe is intersecting the cone at a certain height H above A. The area of the intersected circle is determined by H. Therefore the summed up masses of this upper circle area will exert gravitational force on A dependent on the distance, that's the height H.

Below we first regard gravitational effects and the single gravitation point of any cone whatever.
Second we regard any other cones starting at A and intersecting the constant globe. We will get a definite result on behalf of the single gravitation point of several cones.
At last we regard the summed up gravitational effects of all the globe's mass by using the summed up effects of in-globe-cones.
Of course each mass is effecting gravitation exactly once, so superposed parts of cones have to be subtracted.
Each mass within a globe is a part of at least one cone. And each mass may be assigned to one of the circle areas within a cone.

Dividing any intersected cone into halves by halving H we get another circle area. This circle area at half the height H will be a quarter of the area above, therefore at equal mass distribution there will be only a quarter of standard masses within this area. The gravitational force of each mass of this area on behalf of A will be four times stronger at half H than the same mass in the circle area above. So the summed up gravitational effect of the lower area will be exactly the same as the effect of the upper one.

Modifying distance H generally means modifying the circle area and its mass by square, and inversely modifying gravitational effect by square of distance.
So any intersected circle area within a certain cone will exert gravitation upon a point A at exactly the same amount.
Therefore the gravitational effect of the summed up masses of the upper half of the cone will be exactly at the same amount as the effect of the lower half.
So we recognize the **single gravitation point of any cone will be at H/2.**

Obviously the **height H** of any cone within a globe **will be smaller than 2R**. Therefore half H will be smaller than R. That means that the summed up gravitational force of a cone is located in a **single gravitation point at H/2 < R.**

Let's regard two superposed cones starting from A. The distance of the single gravitation point of each cone will be smaller than R.
The outer of two cones, characterized by a greater angle at A, will be lower, the second H/2 will be smaller than the first H/2.
Of course the mass of the superposed area will exert gravitation only once. Taking away the inner part of the second cone, that's the superposed part with the first cone, will not change the position of the effective point of the outer cone. [3]
The single gravitation point of two in-globe-cones will be located between the first and the second one. We see that **H/2 < R** for the summed up masses of any two or more **superposed in-globe-cones.**

The total mass of a spherically symmetric body may be enclosed by a sufficient number of in-globe-cones. For any cone H/2 will be < R.
One therefore has the **single gravitation point of a total globe at H/2 < R .**

Now let's **summarize the line of argument**:
The distance between the single gravitation point of the innermost cone and A will be near R, but surely at **< R**.
The single gravitation point of any outer cone of a greater angle at A will have a smaller distance from A than the innermost one. The distance will decrease by increasing angle. One therefore has that the single gravitation point of the outermost cone at an angle near 90° has got a distance of nearly zero, but surely > 0 .
Therefore we know for sure that the **single gravitation point** of the total mass of the considered globe will be at a distance from A of > 0 and **< R .**

That's a proof of the gravitation theorem concerning the case of equal density of mass distribution.

DISCUSSION.

The first objection against the line of argument may be that the mass opposite to A at a distance of 2R is not sufficiently respected. For it is a part of the globe, but not a part of any cone.
Well, you may regard all masses upon the axis of the cones starting from A to the opposite point of the globe at 2R. You will find that the single gravitation point of a line will be at <R. [4]
The single gravitation point of all in-globe-cones outside of the axis will also result in H/2 < R. Therefore the total globe is subject to H/2 <R.

Second disproof of the objection: Any real mass object will be a part of any small enough cone, because the object is located within the global radius of R. The innermost cone as well as the remaining in-globe-cones are subject to the rule H/2 < R.

The second objection may be called the paradigm objection: It is not only a general scientific agreement that the single gravitation point of a globe is located in the very centre of the globe, but The Shell Theorem is proven by many sparkling geometric proofs and is a fundamental component of astronomic science since generations upon generations. One therefore has that the single gravitation point of a globe will be at a distance of exactly = R.
Disproving the second objection: Any proof of the shell theorem does sum up all different positions of objects, and of course the result will be the very centre. (No one will doubt that the geometric centre of a spherically symmetrical mass distribution is indeed the very centre.) But you have to follow the instructions of bracket calculation. The crucial difference is, you first have to consider the gravitational force of any mass at its position, and second you have to sum up the local gravitation of the total mass. Since generations upon generations the instructions of bracket calculation are violated by summing up the geometric positions of all objects instead of summing up the local gravitation. [5]
Sometimes it is crucial to observe the correct sequence.

The conic proof of the gravitation theorem is taking account of the local gravitational force of each object at its very position on behalf of distance and angle, in contrary to the shell theorem. (Calculation by the shell theorem is somewhat easier, but at the end of the day you will not get the correct result.)

In addition to the conic proof there is some further bearing out of the gravitation theorem:

-by calculation of gravitational circles, filling a globe [1, 4], disproving the shell theorem for all cases of spherically symmetric distribution, not only on behalf of equal density,

-as well as by the results of satellite experiments like GRAIL 2012 [6, 7, 8], disproving by observations of utmost precision the centre-referred theories of spherical mass distribution,

-and by calculation of the galactic rotation curve by the so-called method of gravity areas [5, 9, 10], disproving the missing-mass assumption.

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