Conjecture that states that, beside few definable exceptions, Poulet numbers are either c-primes, m-primes, c-composites or m-composites

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Abstract. In one of my previous paper, “Conjecture that states than any Carmichael number is a cm-composite”, I defined the notions of c-prime, m-prime, cm-prime, c-composite, m-composite and cm-composite. I conjecture that all Poulet numbers but a set of few definable exceptions belong to one of these six sets of numbers.

Conjecture:

All Poulet numbers but a set of few definable exceptions belong to one of the following six sets of numbers: c-primes, m-primes, cm-primes, c-composites, m-composites and cm-composites.

Note:

Because the Poulet numbers with three or more prime factors have a nature which is nearer than the nature of Carmichael numbers (which, all of them, have three or more prime factors), we will verify the conjecture only for 2-Poulet numbers. We highlight that only 2-Poulet numbers can be c-primes, m-primes or cm-primes, because, by definition, these numbers can only be primes or semiprimes. That means that the conjecture implies that all Poulet numbers with three or more prime factors (beside the exceptions mentioned) are c-composites, m-composites or cm-composites.

Verifying the conjecture:
(for the first fifteen 2-Poulet numbers)

For 341 = 11*31 we have:
: 31 – 11 + 1 = 21 = 3*7 and 7 – 3 + 1 = 5, a prime;
: 31 + 11 – 1 = 41, a prime.
The number 341 is a cm-prime.

For 1387 = 19*73 we have:
: 73 – 19 + 1 = 55 = 5*11 and 11 – 5 + 1 = 7, a prime;
: 73 + 19 – 1 = 91 = 7*13 and 7 + 13 – 1 = 19, a prime.
The number 1387 is a cm-prime.

For 2701 = 37*73 we have:
: 73 – 37 + 1 = 37, a prime;
: 73 + 37 – 1 = 109, a prime.
The number 2701 is a cm-prime.

For 3277 = 29*113 we have:
: 113 – 29 + 1 = 85 = 5*17 and 17 – 15 + 1 = 3, a prime;
: 29 + 113 – 1 = 141 = 3*47 and 3 + 47 – 1 = 49 = 7^2 and 7 + 7 – 1 = 13, a prime.
The number 3277 is a cm-prime.

For 4033 = 37*109 we have:
: 109 – 37 + 1 = 73, a prime;
: 37 + 109 – 1 = 145 = 5*29 and 5 + 29 – 1 = 33 = 3*11 and 3 + 11 - 1 = 13, a prime.
The number 4033 is a cm-prime.

For 4369 = 17*257 we have:
: 257 – 17 + 1 = 241, a prime;
: 17 + 257 – 1 = 273 = 3*7*13;
The number 4369 is a c-prime.

For 4681 = 31*151 we have:
: 151 – 31 + 1 = 121 = 11^2, square of prime;
: 151 + 31 – 1 = 181, prime;
The number 4681 is a cm-prime.

For 5461 = 43*127 we have:
: 127 – 43 + 1 = 85 = 5*17 and 17 – 5 + 1 = 13, a prime;
: 127 + 43 – 1 = 169 = 13^2 and 13 + 13 – 1 = 25 = 5^2 and 5 + 5 – 1 = 9 – 3^2 and 3 + 3 – 1 = 5, a prime;
The number 5461 is a cm-prime.

For 7957 = 73*109 we have:
: 109 – 73 + 1 = 37, prime;
: 73 + 109 – 1 = 181, prime;
The number 7957 is a cm-prime.

For 8321 = 53*157 we have:
: 157 – 53 + 1 = 105 = 3*5*7;
: 53 + 157 – 1 = 209 = 11*19 and 11 + 19 – 1 = 29, prime;
The number 4681 is a m-prime.
For 10261 = 31*331 we have:
: 331 – 31 + 1 = 301 = 7*43 and 43 – 7 + 1 = 37, prime;
: 31 + 331 – 1 = 361 = 19^2 and 19 + 19 – 1 = 37, prime;

The number 10261 is a cm-prime.

For 13747 = 59*233 we have:
: 233 - 59 + 1 = 175 = 5^2*7;
: 59 + 233 - 1 = 291 = 3*97 and 3 + 97 - 1 = 99 = 3^2*11;
The number 13747 is not a c-number.

For 14491 = 43*337 we have:
: 337 - 43 + 1 = 295 = 5*59 and 59 - 5 + 1 = 55 = 5*11 and 11 - 5 + 1 = 7, prime;
: 43 + 337 - 1 = 379, prime;
The number 14491 is a cm-prime.

For 15709 = 23*683 we have:
: 683 - 23 + 1 = 661, prime;
: 23 + 683 - 1 = 705 = 3*5*47;
The number 15709 is a c-prime.

For 18721 = 97*193 we have:
: 193 - 97 + 1 = 97, prime;
: 97 + 193 - 1 = 289 = 17^2 and 17 + 17 - 1 = 33 = 3*11 and 3 + 11 - 1 = 13, prime;
The number 18721 is a cm-prime.